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Nonlinear hydrodynamic response confronts LHC data

Li Yan[a], Subrata Pal[b], and Jean-Yves Ollitrault[a]

[a] *Institut de Physique Théorique, Université Paris Saclay, CEA, CNRS, F-91191 Gif-sur-Yvette, France*
[b] *Department of Nuclear and Atomic Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai, 400005, India*

Abstract

Higher order harmonic flow v_n (with $n \geq 4$) in heavy-ion collisions can be measured either with respect to their own plane, or with respect to a plane constructed using lower-order harmonics. By assuming that higher flow harmonics are the superposition of medium nonlinear and linear responses to initial anisotropies, we propose a set of nonlinear response coefficients χ_n's, which are independent of initial state by construction. In experiments, χ_n's can be extracted as the ratio between higher order harmonic flow measured in the plane constructed by v_2 and v_3, and moments of lower order harmonic flow. Simulations with single-shot hydrodynamics and AMPT model lead to results of these nonlinear response coefficients in good agreement with the experimental data at the LHC energy. Predictions for v_7 and v_8 measured with respect to plane of lower order harmonics are given accordingly.

Keywords: Harmonic flow, nonlinear hydrodynamics response, heavy-ion collisions

1. Introduction

The observed flow phenomena in high energy heavy-ion collisions carried out at RHIC and the LHC provides great opportunities in analyzing the collective dynamics of the strongly-coupled Quark-Gluon Plasma (QGP) (for a recent review, cf. [1]). To a quantitative level, the analysis of harmonic flow V_n, which is defined through a Fourier decomposition of the observed particle spectrum,

$$V_n = v_n e^{in\Psi_n} = \{e^{in\phi_p}\}, \tag{1}$$

has led to strong constraints on the dissipative properties of the QGP medium in various aspects. For instance, correlations among flow harmonics have been studied in terms of the correlations between event-plane Ψ_n [2], which present non-trivial patterns depending on the shear viscosity over entropy ratio η/s of the medium. Recent measurements of harmonic flow V_n have achieved results with high precisions, which extends the studies of harmonic flow to flow fluctuations [3] and higher order harmonic flow ($n \geq 4$) [4]. In particular, higher order flow harmonics have been measured with respect to their own event-plane, and plane constructed by lower order flow harmonics, from which nonlinear medium response to initial eccentricities can be studied. In this work, we focus on the nonlinear generation of higher order flow harmonics in heavy-ion collisions. By assuming that higher harmonics are the superposition of medium nonlinear and linear responses, a new set of nonlinear response coefficients are formulated.

2. Nonlinear hydrodynamic response and χ_n

We expand harmonic flow V_n in a series of initial eccentricities $\mathcal{E}_n$, accounting for the fact that magnitudes of initial eccentricities are small. Note that in this work, $\mathcal{E}_n$, as well as V_n defined in Eq. (1) are taken as complex quantities. For higher order flow harmonics, it has been shown that nonlinear hydro response to initial eccentricities result in significant contributions [5, 6]. Taking the fourth order harmonic flow V_4 as an example, in addition to the component V_4^L which is linearly proportional to $\mathcal{E}_4$, there exists a large fraction induced by hydro response to $\mathcal{E}_2^2$. Therefore, one can write V_4 as,

$$V_4 = V_4^L + \chi_4 V_2^2 \,. \tag{2}$$

$\mathcal{E}_2^2$ has been absorbed into V_2^2 in the second term on the right hand side of Eq. (2), accordingly the coefficient χ_4 is found independent of initial eccentricities by construction. Similar strategy can be applied to other higher order harmonic flow as well. For V_5, V_6 and V_7, the corresponding expansion leads to

$$V_5 = V_5^L + \chi_5 V_2 V_3 \,, \tag{3a}$$

$$V_6 = V_6^L + \chi_{63} V_3^2 + \chi_{62} V_2^3 \,, \tag{3b}$$

$$V_7 = V_7^L + \chi_7 V_2^2 V_3 \,. \tag{3c}$$

The nonlinear terms in the right-hand side are the lowest-order terms involving V_2 and V_3 which are compatible with rotational symmetry. For V_6, there exists a non-negligible component from cubic order hydro response to $\mathcal{E}_2^3$, which has already been noticed through the observed event-plane correlation between V_6 and V_2 [2], thus one must expand V_6 to cubic order, with an extra cubic order coefficient χ_{62}. For V_7, there is no contributions of quadratic order, thus the coefficient χ_7 is defined regarding the cubic order hydro response to $\mathcal{E}_2^2\mathcal{E}_3$.

In the hydro response formalism, χ_n's are interpreted as ratios between nonlinear and linear flow response coefficients, which are independent of the initial density profile for a given centrality class. Each χ_n can be readily evaluated in a single-shot hydrodynamic simulation [6] by choosing an initial density profile such that only the term involving χ_n is nonvanishing in the expansion of Eqs. (2) and (3). If one analyses a set of events in a centrality class, where the flow fluctuates event to event, as in actual heavy-ion experiments and AMPT simulations [7], χ_n's can be isolated using Eqs. (2) and (3) under the assumption that the terms in the right-hand side are mutually uncorrelated [8]:

$$\chi_4 = \frac{\langle V_4 (V_2^*)^2 \rangle}{\langle |V_2|^4 \rangle} = \frac{v_4\{\Psi_2\}}{\sqrt{\langle |V_2|^4 \rangle}} \tag{4a}$$

$$\chi_5 = \frac{\langle V_5 V_2^* V_3^* \rangle}{\langle |V_2|^2 |V_3|^2 \rangle} = \frac{v_5\{\Psi_{23}\}}{\sqrt{\langle |V_2|^2 |V_3|^2 \rangle}} \tag{4b}$$

$$\chi_{62} = \frac{\langle V_6 (V_2^*)^3 \rangle}{\langle |V_2|^6 \rangle} = \frac{v_6\{\Psi_2\}}{\sqrt{\langle |V_2|^6 \rangle}} \,, \qquad \chi_{63} = \frac{\langle V_6 (V_3^*)^2 \rangle}{\langle |V_3|^4 \rangle} = \frac{v_6\{\Psi_3\}}{\sqrt{\langle |V_3|^4 \rangle}} \tag{4c}$$

$$\chi_7 = \frac{\langle V_7 (V_2^*)^2 V_3^* \rangle}{\langle |V_2|^4 |V_3|^2 \rangle} = \frac{v_7\{\Psi_{23}\}}{\sqrt{\langle |V_2|^4 |V_3|^2 \rangle}} . \tag{4d}$$

The expressions on the right hand side of Eqs. (4) involve the higher order harmonic flow measured in the plane of lower order harmonics. For example, V_4 can be measured in experiments in its own event-plane Ψ_4 which is defined in Eq. (1), as well as the event-plane Ψ_2 which is determined by V_2. More explicitly, V_4 measured with respect to Ψ_2 is

$$v_4\{\Psi_2\} \equiv \frac{Re\langle V_4 (V_2^*)^2 \rangle}{\sqrt{\langle |V_2|^4 \rangle}} = \langle \cos 4(\Psi_4 - \Psi_2) \rangle_w \times v_4\{\Psi_4\} \,. \tag{5}$$

It is worth mentioning that measuring higher order flow harmonics in the event plane of V_2 and/or V_3 is equivalent to the corresponding measurement of event plane correlations [2], as demonstrated by the second

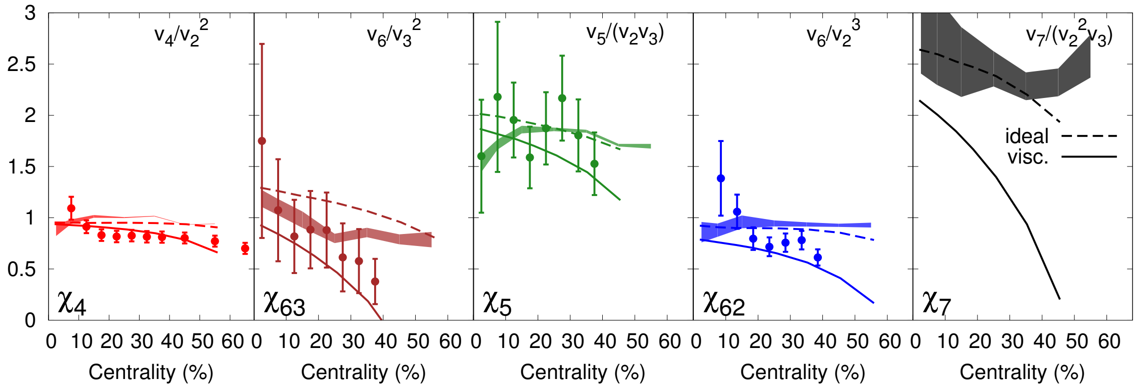

Fig. 1. (Color online) Nonlinear response coefficients χ_n as a function of centrality percentile for $\sqrt{s_{NN}} = 2.76$ TeV PbPb at the LHC. Symbols with errors are extracted results from CMS and ATLAS collaborations. Lines are from single-shot hydro simulations with $\eta/s = 1/4\pi$ (solid line) or $\eta/s = 0$ (dashed line). Shaded bands are results from AMPT simulations with parton elastic cross-section $\sigma = 1.5$ mb.

identity in Eq. (5). The denominators in Eqs. (4) involve various moments of the distributions of V_2 and V_3. There is no direct measurement of flow moments up to date in experiments, though it can be done in a generalized scalar-product method, with sufficient rapidity gap [9]. In this work, we extract flow moments from flow cumulants [10] which are measured. For instance, the fourth order moment of V_2 is related to $v_2\{2\}$ and $v_2\{4\}$ by

$$\langle |V_2|^4 \rangle = 2v_2\{2\}^4 - v_2\{4\}^4 \tag{6}$$

3. Results and discussions

The CMS collaboration has measured $v_4\{\Psi_2\}$, $v_6\{\Psi_2\}$ and cumulants of V_2 distributions [4][1], therefore we are able to assess χ_4 and χ_{62} according to Eqs. (4). To evaluate χ_5 and χ_{63}, we estimate $v_5\{\Psi_{23}\}$ and $v_6\{\Psi_3\}$ from the event-plane correlations $\langle \cos(5\Psi_5 - 2\Psi_2 - 3\Psi_3)\rangle_w$ and $\langle \cos 6(\Psi_6 - \Psi_3)\rangle_w$ measured by the ATLAS collaboration [2], in addition to the cumulants of V_3 distributions from the CMS collaboration. χ_n's from the $\sqrt{s_{NN}} = 2.76$ TeV PbPb at the LHC are shown as symbols in Fig. 1. To make comparisons, we calculate these nonlinear response coefficients from single-shot hydro simulations [6] as well as AMPT [7]. There is no event-by-event fluctuations implemented in our hydro simulations, where the initial condition is taken by perturbing a smooth and azimuthally symmetric Gaussian density profile with specific initial eccentricities. The normalization of the Gaussian profile is adjusted to fit the values of dN_{ch}/dy of LHC PbPb in a given centrality class. Results from ideal and viscous (with $\eta/s = 1/4\pi$) hydro simulations are depicted as dashed and solid lines respectively in Fig. 1, which present an overall agreement comparing with the experimental data. AMPT simulations contain non-trivial event-by-event fluctuations at the nucleonic and partonic levels, and the parton elastic cross-section is taken to be $\sigma = 1.5$ mb. It is worth mentioning that ideal hydrodynamics predicts quantitative relations among these nonlinear response coefficients due to Cooper-Fyer freeze-out [11]: $\chi_4 \sim \chi_{63} \sim \frac{1}{2}\chi_5$, and $\chi_{62} \sim \frac{1}{3}\chi_7$, which are consistent with the experimental data in Fig. 1 as well as the results obtained from AMPT.

Although there is no experimental data available so far for the extraction of nonlinear response coefficients χ_n for the flow harmonics of order $n > 6$, we make predictions in our model simulations for V_7 and V_8. For V_8, there exist a cubic order term and a quartic order term allowed by rotational symmetry,

$$V_8 = V_8^L + \chi_{8(23)}\mathcal{E}_2\mathcal{E}_3^2 + \chi_{8(2)}\mathcal{E}_2^4\,, \tag{7}$$

[1] Cumulants of V_2 distributions from the CMS collaboration have so far been published up to $v_2\{4\}$. In this work we approximately take $v_2\{8\} \approx v_2\{6\} \approx v_2\{4\}$, as being implied from the measurements by the ATLAS collaboration [3].

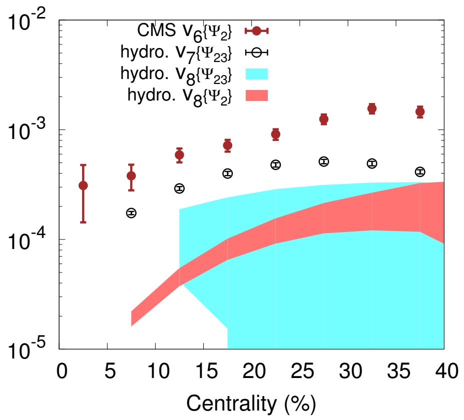

Fig. 2. (Color online) Hydro predictions for v_7 and v_8 measured in the event-plane constructed by v_2 and v_3. Boundaries of the colored bands are determined by ideal and viscous ($\eta/s = 1/4\pi$) hydro calculations respectively.

which correspond in experiments to the measurements of V_8 in the event-plane constructed by V_2 and V_3, and event-plane of V_2 respectively,

$$\chi_{8(23)} = \frac{v_8\{\Psi_{23}\}}{\sqrt{\langle|V_2|^2\rangle\langle|V_3|^4\rangle}}, \qquad \chi_{8(2)} = \frac{v_8\{\Psi_2\}}{\sqrt{\langle|V_2|^8\rangle}} \tag{8}$$

χ_7 from our model simulations are presented in Fig. 1. In Fig. 2, V_7 and V_8 measured in the event-plane consturcted by V_2 and V_3 are predicted with single-shot hydro simulations according to Eqs. (4d) and (8).

4. Conclusions

Under fairly general assumptions, we have proposed a new set of nonlinear response coefficients χ_n based on the measurements of higher order harmonic flow with respect to the event-plane constructed by v_2 and v_3. These coefficients are independent of the detailed information of initial state by construction. Model simulations with single-shot hydrodynamics and AMPT give rise to predictions in good agreement with experimental data. We noticed that the relative ratios among these coefficients are consistent with an ideal hydro expectation based on the analysis of freeze-out. Nonlinear response coefficients associated with v_7 and v_8 are calculated as well in our theoretical models as predictions.

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