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Rapid rotational crust-core relaxation in magnetars

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ABSTRACT

If a magnetar interior B-field exceeds 10^{15} G it will unpair the proton superconductor in the star's core by inducing diamagnetic currents which destroy the Cooper pair coherence. Then, the P-wave neutron superfluid in these non-superconducting regions will couple to the stellar plasma by scattering of protons off the quasiparticles confined in the cores of neutron vortices via the strong (nuclear) force. The dynamical time-scales associated with this interaction span from several minutes at the crust-core interface to a few seconds in the deep core. We show that (a) the rapid crust-core coupling is incompatible with oscillation models of magnetars which decouple completely the core superfluid from the crust and (b) magnetar precession is damped by the coupling of normal fluids to the superfluid core and, if observed, needs to be forced or continuously excited by seismic activity.

Key words. magnetars, rotational dynamics, superconductivity, vorticity

1. Introduction

Magnetars are a class of compact stars which exhibit powerful X-ray and soft γ-ray outburst activity which is attributed to the energy release stored in their internal magnetic fields (Thompson & Duncan 1995). Their surface B-fields are measured to be a factor of thousand larger than the fields inferred for rotationally powered pulsars. The interior fields of magnetars are unknown, but could be several orders of magnitude larger than the surface field [for recent reviews see Turolla et al. (2015); Mereghetti et al. (2015)]. For interior fields $B_{16} \sim 1$, where B_{16} is the field value in units of 10^{16} G, the electromagnetic interactions become of the order of characteristic nuclear scales ($\sim$ MeV). As a consequence, the S-wave condensate of protons in the star's core, which contains charged Cooper pairs with opposite spins, becomes affected by the B-field. It is eventually destroyed at the second critical field H_{c2} known from the theory of ordinary superconductivity (Tinkham 1996). This *unpairing effect* arises because of coupling of the charge of Cooper pairs to the electromagnetic field which winds-up the trajectories of protons in strong magnetic field over distances smaller than the coherence length of a Cooper pair.

In the context of magnetars it was shown previously that because of strong density dependence of the proton pairing gaps the quenching of superconductivity is non-uniform: intermediate field magnetars with $0.1 \leq B_{16} \leq 5$ are partially superconducting, whereas high-field magnetars are $B_{16} \geq 5$ are fully non-superconducting (Sinha & Sedrakian 2015a,b). If the fields are by an order of magnitude larger, $B_{16} \geq 10$, the neutron S-wave condensate is unpaired by the magnetic field because of the paramagnetic interaction of spins of neutrons with the B-field (Stein et al. 2015). On the other hand the neutron P-wave superfluid, which features spin-1 Cooper pairs, is unaffected by the magnetic fields at the fundamental level. Being uncharged it cannot show Landau diamagnetism; its paramagnetic response to a B-field is non-destructive because the pairing involves neutrons with parallel spins.

The purpose of this work is to discuss the rotational coupling of neutron superfluid in magnetar cores in the case where the fields are large enough to unpair proton condensate. We show that unpairing opens a new channel of coupling of electron-proton plasma to the neutron vorticity in the core. This new channel (which is suppressed if protons are superconducting) is the scattering of protons off neutron quasiparticles confined in the vortex cores by nuclear force. This process should be contrasted with the scattering of electrons off magnetized neutron vortices by purely electromagnetic forces which provide upper bound on the electron mean-free-path in a type-I superconducting case, *i.e.*, in the absence of proton vorticity (Alpar & Sauls 1988).

The strength of the coupling of the superfluid to the unpaired plasma have important implications for the macroscopic observable manifestations of magnetars. We give two specific examples below. Gabler et al. (2013) conducted numerical simulations of axisymmetric, torsional, magneto-elastic oscillations of magnetars with a superfluid core to explain the observed quasiperiodic oscillations of these objects. In doing so, it was assumed that the superfluid is decoupled in the core of the star completely (*i.e.*, the neutron and proton fluids are coupled only by gravity). The assumptions above requires a computation of the coupling time between the neutron superfluid and rest of the plasma in magnetars, in particular, in a situation where protons form a normal fluids, as assumed by Gabler et al. (2013). A second example is the precessional motions of magnetars, more specifically the influence of interior fluid on such motions. Link (2007) discussed the implications of the observation of precession in ordinary neutron stars on the state of proton superconductivity in their cores focusing on the incompatibility of the type-II superconductivity with free precession. An alternative is the type-I superconductivity in the cores of low-magnetic-field neutron stars (Link 2003; Sedrakian 2005; Charbonneau & Zhitnitsky 2007). A natural extension of this discussion to magnetars requires the knowledge of dynamical coupling of the neutron P-wave superfluid, when the proton superfluidity is quenched by magnetic fields and protons form a normal fluid.

This paper is structured as follows. In Sec. 2 we review the unpairing effect in the cores of magnetars and the implied structure of superfluid and superconducting shells. The relaxation times-scales for the coupling of the magnetar core to the crust are computed in Sec. 3. We discuss the implications of our findings in Sec. 4 and provide a summary in Sec. 5.

2. Unpairing effect

As well known [see, for example, Tinkham (1996)] in type-II superconductors the Ginzburg-Landau (GL) parameter, defined as $\kappa = \lambda/\xi_p$, where λ is the London penetration depth of B-field in a superconductor, ξ_p is the coherence length, is in the range $1/\sqrt{2} < \kappa < \infty$. The magnetic field is carried by electromagnetic vortices with quantum flux $\Phi_0 = \pi/e$ (here and below $\hbar = c = 1$) if the B-field is in the range between the lower and upper critical fields, *i.e.*, $H_{c1} \le B \le H_{c2}$. If the B-field is larger than H_{c2} it unpairs the Cooper pairs and destroys the superconductivity.

The unpairing effect in a superfluid neutron and superconducting proton *mixture* was explored within the GL theory on the basis of the following functional (Sinha & Sedrakian 2015a)

$$\mathscr{F}[\phi,\psi] = \mathscr{F}_n[\phi,\psi] + \mathscr{F}_p[\phi,\psi] + \frac{1}{4m_p}|\boldsymbol{D}\psi|^2 + \frac{B^2}{8\pi}, \quad (1)$$

where ψ and ϕ are the proton and neutron condensate wave-functions, m_p is the proton mass, $\boldsymbol{D} = -i\boldsymbol{\nabla} - 2e\boldsymbol{A}$ is the gauge invariant derivative, $\mathscr{F}_n[\phi,\psi]$ and $\mathscr{F}_p[\phi,\psi]$ are the energy-density functionals of neutron and proton condensates. The proximity to H_{c2} guarantees that the proton condensate wave-function is small and its functional can be written as power series

$$\mathscr{F}_p[\phi,\psi] = \alpha\tau|\psi|^2 + \frac{b}{2}|\psi|^4 + b'|\psi|^2|\phi|^2, \quad (2)$$

where $\tau = (T - T_{cp})/T_{cp}$ with T being the temperature and T_{cp} the critical temperature of superconducting phase transition of protons, α and b are the familiar coefficients of GL expansion, whereas b' describes the coupling between the neutron and proton condensates. The equations of motions of the proton condensate associated with the GL functional (1) are given by the variations $\delta\mathscr{F}[\phi,\psi]/\delta\psi = 0$ and $\delta\mathscr{F}[\phi,\psi]/\delta\boldsymbol{A} = 0$. Close to the critical field the GL equations can be linearized assuming further that $\boldsymbol{A}$ is locally linear in coordinates, so that B-field is locally constant. The solution of the pair of linearized GL equations that correspond to non-vanishing ψ provide the maximal value of the field compatible with superconductivity, which is then identified with the upper critical field (Sinha & Sedrakian 2015a)

$$H_{c2} = \frac{\Phi_0}{2\pi\xi_p^2}\left[1 + \beta(b')\right], \quad (3)$$

where $m_p|\alpha\tau| = (2\xi_p)^{-2}$. The critical value of the field is enhanced by $\beta \simeq 0.2$ due to the density-density coupling between neutron and proton condensates. The dependence of H_{c2} field on density is illustrated in Fig. 1, where we adopted the same input physics as described in Sinha & Sedrakian (2015a). It was found that independent of the details of microphysical input the maximum of H_{c2} is attained close to the crust-core interface (corresponding to $n_b = 0.5n_0$, where n_0 is the nuclear saturation density). As a consequence, magnetars with approximately constant interior fields below H_{c2} will contain two physically distinct regions: (a) the inner core which is void of superconductivity; (b) outer core where protons are superconducting and, consequently, proton flux-tubes (vortices) are present along with the neutron vortex lattice induced by the rotation. The magnetic B-field in type-II superconductor forms quantized electromagnetic vortices with density $N_p = B/\Phi_0$. These phases are enveloped by the crust which is threaded by non-quantized magnetic field.

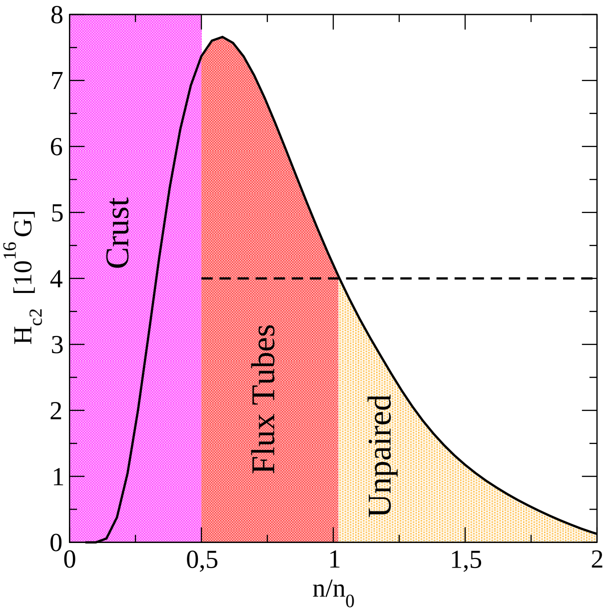

Fig. 1. Dependence of the critical unpairing field H_{c2} on baryonic density (solid line). The density is normalized to the nuclear saturation density $n_0 = 0.16$ fm^{-3} and the equation of state and composition of matter is the same as in Sinha & Sedrakian (2015a). This arrangment of flux-free and flux-featuring phases arises in the case of intermediate field magnetars with average constant field $B < H_{c2}$, which is indicated by the dashed line.

3. Rotational crust-core coupling time-scales

Neutron superfluid rotates by forming an array of quantized vortices. The areal number density of neutron vortices is given by

$$N_n = \frac{2\Omega}{\omega_0}, \qquad \omega_0 = \frac{\pi}{m_n}, \quad (4)$$

where m_n is the bare neutron mass, Ω is the rotation frequency of the star, ω_0 is the quantum of neutron circulation.

Any variation in the angular velocity of the magnetar causes the free vortices to move and leads to their new quasiequilibrium distribution. Hence the vortex distribution depends on their velocity field $\boldsymbol{v}_L$. This velocity field is determined by the equation of motion of a vortex, which because of the negligible vortex mass reduces to the requirement that the sum of forces acting on its unit segment vanishes

$$\rho_n\omega_0[(\boldsymbol{v}_S - \boldsymbol{v}_L)\times\boldsymbol{\nu}] - \eta(\boldsymbol{v}_L - \boldsymbol{v}_N) = 0. \quad (5)$$

Here the first term is the Magnus force and the second term is the friction force between the vortices and the normal liquid, ρ_n is the mass density of the superfluid component, $\boldsymbol{v}_N$ is the velocity of the normal component, and η is the coordinate-dependent longitudinal (with respect to $\boldsymbol{v}_L - \boldsymbol{v}_N$) friction coefficient. We first consider the flux-tube free (unpaired region) and demonstrate that it is coupled to the plasma of the star on short dynamical coupling time-scales. First note that the non-superconducting proton fluid will couple to the electron fluid on plasma timescales, which are much shorter than the hydrodynamical timescales. Therefore, the unpaired core of a magnetar can be considered as a two-fluid system with neutron condensate forming the superfluid component and the proton plus electron fluids forming the normal component. Neutron vortices (and the

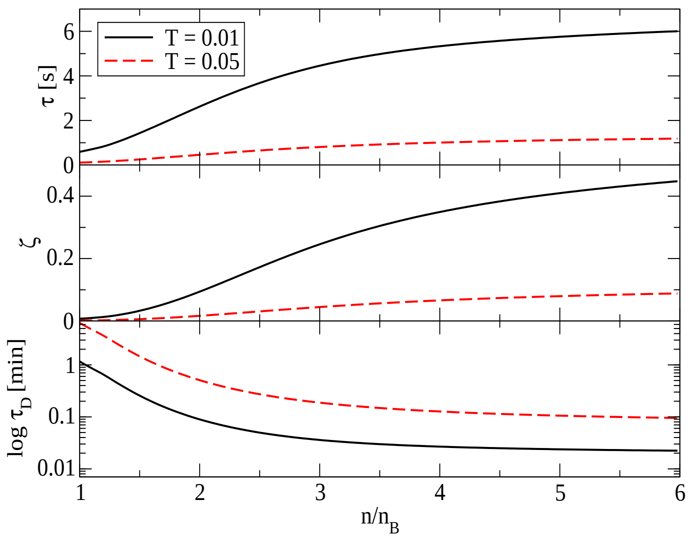

Fig. 2. Relaxation time-scales for protons scattering off the quasiparticles confined in the neutron vortex core (upper panel) the drag-to-lift ratio (middle panel) and dynamical relaxation time (lower panel) for $T = 0.01$ MeV (solid line) and $T = 0.05$ MeV (dashed line) and rotation frequency $\Omega = 1$ Hz. The shown results are valid in the entire density domain if the constant field in the core satisfies $B > \max H_{c2}$; for fields below H_{c2} the results are valid to the right of the superconducting-normal boundary (assuming the arrangment of phases as in Fig 1).

neutron superfluid) couples to this normal component electromagnetically (Sauls et al. 1982). However, because protons are unpaired (*i.e.*, excitations out of Fermi surface can be created without the energy cost of breaking a Cooper pair) they will scatter efficiently off neutron vortex core quasiparticle by the nuclear force. The solution of the Boltzmann equation for protons in the relaxation time approximation leads to the microscopic relaxation timescale (Sedrakian 1998)

$$\tau^{-1} = 13.81 \frac{N_n T}{\varepsilon^0_{1/2} m^*_p \xi^2_n} \left(\frac{\epsilon_{Fn}}{\epsilon_{Fp}}\right)^2 e^{-\frac{\varepsilon^0_{1/2}}{T}} \frac{d\sigma}{d\Omega}, \tag{6}$$

where ϵ_{Fn} and ϵ_{Fp} are the Fermi energies of neutrons and protons, $\varepsilon^0_{1/2} = \pi\Delta_n^2/(4\epsilon_{Fn})$ is the lowest energy state of a neutron quasiparticle confined in the vortex, Δ_n is the pairing gap, $d\sigma/d\Omega$ is the differential neutron-proton scattering cross-section, ξ_n is the neutron condensate coherence length. Here and below, for simplicity, we do not distinguish between the neutron and proton effective masses, *i.e.*, we set $m^*_p = m^*_n$.

The force exerted by proton quasiparticles *per single vortex* is given by (Bildsten & Epstein 1989)

$$\boldsymbol{F} = \frac{2}{\tau N_n} \int f(\boldsymbol{p}, \boldsymbol{v}_L) \boldsymbol{p} \frac{d^3 p}{(2\pi\hbar)^3} = -\eta \boldsymbol{v}_L, \tag{7}$$

where $f(\boldsymbol{p}, \boldsymbol{v}_L)$ is the non-equilibrium distribution function, which we expand assuming small perturbation about the equilibrium distribution function f_0, *i.e.*, $f(\boldsymbol{p}, \boldsymbol{v}_L) = f_0(\boldsymbol{p}) + (\partial f_0/\partial\epsilon)(\boldsymbol{p}\cdot\boldsymbol{v}_L)$. In the low-temperature limit $\partial f_0/\partial\epsilon \simeq -\delta(\epsilon - \epsilon_{Fp})$. The friction coefficient, after phase space integrations in (7), is given by

$$\eta = \frac{m^*_p n_p}{\tau N_n}, \tag{8}$$

where n_p is the proton number density and N_n is defined in Eq. (4). The quantity characterizing the macroscopic relaxation of superfluid is the ratio of the strengths of viscous friction force and the Magnus force or the *drag-to-lift ratio*

$$\zeta = \frac{\eta}{\rho_n \omega_0} = \frac{1}{2\Omega\tau} \frac{n_p}{n_n}. \tag{9}$$

Finally, the macroscopic dynamical coupling time of the superfluid to the plasma is given by

$$\tau_D = \frac{1}{2\Omega} \left(\zeta + \zeta^{-1}\right). \tag{10}$$

We adopt the same equation of state and nucleonic composition as in Sinha & Sedrakian (2015a) to compute the numerical values of the quantities of interest and we do not repeat the details of the input here. We do not consider hyperonic or deconfined quark degrees of freedom. The hyperonic scattering contribution will be subdominant or of the same order of magnitude as the proton scattering because of comparable abundances of these species in hyperon-rich matter. The relevant relaxation time-scales in two-flavor quark matter have been computed for color-magnetic flux tubes interacting via Aharonov-Bohm effect with leptons and unpaired quarks, in which case again the strong force is involved (Alford & Sedrakian 2010).

Figure 2 shows the key results of this study: the relaxation time (6), the drag-to-lift ratio (9), and the dynamical coupling timescale (10) as a function of baryon density in the star's core for $T = 0.01$ and 0.05 MeV, or equivalently for $T = 1.2 \times 10^8$ K and $T = 5.8 \times 10^8$ K. An average energy and angle independent neutron-proton cross-section $\sigma \simeq 60$ fm^2 and rotation frequency $\Omega = 1$ Hz have been assumed. The relaxation time increases with decreasing temperature mainly due to the exponential Boltzmann-factor in (6). The results shown in Fig. 2 are relevant in the entire density range if the field in the star satisfies the condition $B > \max H_{c2}$, *i.e.*, the unpairing effect acts in the entire core. If $B < \max H_{c2}$, then the results are valid above certain density threshold (see Fig. 1) Below this threshold density the dynamics of the core is determined by the vortex-flux interactions and the coupling of the electron liquid to this conglomerate, which is not well understood. For typical magnetar periods of order of 10 sec, *i.e.*, the spin rotations are of order of 1 Hz, Fig. 2 implies that the unpaired core couples to the plasma on short dynamical timescales, which lie in the range from several minutes at the crust-core boundary to a few seconds deep in the magnetar core.

4. Implications for superfluid oscillations and precession

We now briefly comment on some applications of the results above. Superfluid oscillations where studied by Gabler et al. (2013) under assumption that protons form a normal fluid (in line with unpairing effect, which is alluded by these authors), but assuming that the superfluid core is completely decoupled from the crust. This leads to higher Alfven speed in the core because only protons take part in magneto-elastic oscillations and stronger penetration of these modes into the crust. In addition less massive core takes part in the magneto-elastic oscillations which means that the coupling to the crust is stronger. The rapid relaxation times obtained in Sec. 3 imply that the two assumptions above are incompatible, because once protons are normal they will couple the proton-electron normal fluid to the neutron superfluid by scattering off the neutron vortex core quasiparticles; note that for rotation periods characteristic for magnetars

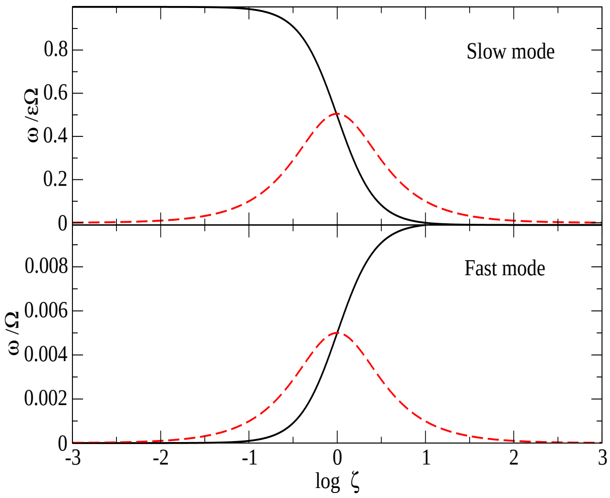

Fig. 3. Eigenfrequencies of precessional modes (solid lines) and their damping (dashed lines) of a compact star with superfluid component (Sedrakian et al. 1999). The slow mode (upper panel) is normalized to classical precession frequency $\epsilon\Omega$, where ϵ is the eccentricity. The fast mode (lower panel) is a fraction of rotational frequency.

the core is threaded by a mesh of neutron vortices with areal density given by $N_n \sim 3.3\times 10^2(\Omega/1\,\mathrm{Hz})\,\mathrm{cm}^{-2}$ according to (4).

In the region where P-wave superfluidity vanishes the coupling between neutron and proton fluids will be faster that our result above by many order of magnitude. The agreement of numerical finds of Gabler et al. (2013) with the data on oscillations of magnetars may indicated a type-II proton superconductivity rather than normal proton fluid, in which case the coupling of neutron and proton fluids depends on the complex ways the vortex lattices pin in the core of the star [see, for example, Link (2014) and references therein]. For models of magnetar oscillations in the case of type-II superconducting matter see, for example, Andersson et al. (2009); van Hoven & Levin (2008).

Free precession of magnetars will be possible in a range of couplings between the superfluid neutron fluid and the normal proton-electron component in the core and the crust material. The eigenmodes of precessional motion for a star with superfluid interior were derived for arbitrary drag-to-lift ratios in Sedrakian et al. (1999) and these are illustrated in Fig. 3. Observationally interesting is the slow precessional modes with eigenfrequency $\sim \epsilon\Omega$, where ϵ is the eccentricity and Ω is the rotation frequency. This mode is precisely the counterpart of ordinary precession in astronomical bodies. The fast mode with eigenfrequency $\sim \Omega$ is observationally irrelevant in electromagnetic spectrum, but could be an important source of gravitational waves (Jones 2010). It is seen that for $\zeta > 1$ the damping of the slow mode exceeds the eigenmode frequency, which implies that the precession is damped within a cycle. For $\zeta \ll 1$ precession is undamped by the superfluid component. A comparison with the values of ζ in Fig. 2 shows that the low-density outer core ($\zeta \simeq 0.2$) does not affect free precession, whereas the high-density inner core ($\zeta \simeq 0.4$) can cause significant damping of precession over a cycle or so. Thus, precession of magnetars is sensitive to the crust-coupling in the case when protons are non-superconducting, because the drag-to-lift ratios we find are within the range of the crossover from undamped to damped precession. Because the inner core unpairs at lower fields (see Fig. 1) we may conclude that unpairing will lead to damping of free precession in magnetars. Note that our arguments apply to free precession; it can still be observed if there is a continuous source of excitation, such as magnetic energy which can induce seismic activity (Lander et al. 2015).

5. Summary

The key result of this work is the demonstration that *if the B-field in the interior of a magnetar is large enough to unpair the proton condensate (unpiaring effect) the magnetar's core will couple to the crust on short dynamical time-scales*. We have also computed the relevant values of the drag-to-lift ratio which measure the influence of superfluid on the dynamics of normal fluid plasma. The obtained range of this parameter lies in the region of the crossover from undamped precession to its complete damping, therefore an observation of precession in magnetars can shed light on the dynamical coupling mechanism of their core to the crust. Our results indicate that long-term precession is unlikely in magnetars and, if observed, needs to be induced by seismic activity. We anticipate that the results above should be useful in other contexts, such as the quasiradial oscillations of magnetars, their glitch and anti-glitch relaxations, vortex shear modes, etc.

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