Tracking Dark Energy from Axion-Gauge Field Couplings

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We propose a model of Dark Energy in which the field currently dominating the energy density of the universe is an "axion field" linearly coupled to the Pontryagin density, $\mathrm{tr}(F \wedge F)$, (i.e., the exterior derivative of the Chern-Simons form) of a massive gauge field. We assume that the axion has self-interactions corresponding to a non-trivial (exponential) potential. We argue that a non-vanishing magnetic helicity of the gauge field triggers slow-rolling of the axion at field values far below the Planck scale. Our proposal leads to a "Tracking Dark Energy Scenario" in which the contribution of the axion energy density to the total energy density is constant (and small) during the early radiation phase, until a secular growth term proportional to the Pontryagin density of the gauge field becomes dominant. The initially small contribution of the axion field to the total energy density is related to the observed small baryon-to-entropy ratio. Some speculations concerning the nature of the gauge field are offered.

I. INTRODUCTION

As is well known, the *Strong CP Problem* associated with the vacuum structure of QCD, as described by the θ-angle, can be solved by promoting the vacuum angle to a pseudo-scalar field, the *axion* [1], which gives rise to a new light particle. This field can also be viewed as the *phase* of a complex scalar field related to a $U(1)$-symmetry [2]. A non-trivial vacuum expectation value of the scalar field then leads to the spontaneous breaking of this symmetry. The particles associated with the axion acquire a mass through instanton effects and can be made "invisible" by choosing the symmetry breaking scale to be sufficiently high [3]. The axion can then be a candidate for *dark matter*; (see e.g. [4]). Some time ago, it has been suggested [5] that, besides the QCD axion, there could exist an effective axion field conjugate to the anomalous axial vector current in QED that would give rise to an instability triggering the growth of low-frequency magnetic fields. The time derivative of this axion field would then play the role of a space-time dependent chemical potential for the axial charge density in QED and, through the chiral anomaly, would give rise to a magnetic instability; see also [6]. Possible applications of this suggestion to early universe cosmology, in particular to the issue of the generation of primordial magnetic fields, have been discussed in [7]; (see also [8], [6]).

In this paper, we explore the possibility that an axion field, ϕ, linearly coupled to the Pontryagin density, $\mathrm{tr}(F \wedge F)$, of a massive gauge field (possibly identified with the weak $SU(2)$-gauge field, see [9]) could contribute to the *dark energy* of the universe. We assume that this new axion field acquires a non-trivial potential term $V(\phi)$ describing self-interactions. As was realized in the context of inflationary models in [10, 11] (see also [12]), this coupling can lead to *slow-rolling* of ϕ even for field values much smaller than the Planck mass. We show that this slow-rolling leads to a *tracking* solution in which the energy density of ϕ tracks that of the radiation-dominated background of the early universe until a time t_c when the secular growth term in the magnetic helicity of the gauge field starts to dominate. From that time onwards the contribution of ϕ to the energy density of the universe starts to grow and can begin to dominate it at some late time. Given parameter values motivated by the observed small baryon-to-entropy ratio, we arrive at a scenario in which ϕ explains the currently observed dark energy. Thus, our mechanism could lead to an implementation of the *tracking dark energy* scenario previously discussed in [13]; (see also [14]).

In the following section we introduce key features of our scenario. One of them is related to a secular growth of the electric component of the gauge field tensor, as time increases. This feature is discussed in more detail in Section III, where we derive the gauge field equations of motion in the presence of a term coupling the Pontryagin density, i.e., the derivative of the Chern-Simons 3-form, to the axion field. We then attempt to find homogeneous and isotropic solutions of these equations. In Section IV we consider an exponential potential for ϕ and try to find out under what conditions it is possible to obtain tracking dark energy. In Section V we discuss particle physics connections of our scenario. Some conclusions are discussed in Section VI. An interesting variant of our scenario involving a complex scalar field whose phase plays the role the new axion field introduced in the present paper will be discussed in forthcoming work.

A word on our notation: Our space-time metric has signature $(-, +, +, +)$. We work in units in which the speed of light, Planck's constant and Boltzmann's con-

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stant are all set to 1. The cosmological scale factor is denoted by $a(t)$, where t is physical time. The Hubble expansion rate is $H(t) = \frac{\dot{a}}{a}(t)$.

II. KEY FEATURES OF OUR SCENARIO

In this section we introduce our dark energy model, postponing a discussion of its origins in particle physics to Section V.

The first basic feature of our model is the presence of a pseudo-scalar axion field, ϕ, that couples linearly to the Pontryagin density, $\mathrm{tr}(F \wedge F)$, of a massive gauge field. Specifically, we consider the following action functional for the cosmological dynamics of the field ϕ:

$$S \,=\, \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \mathcal{L}_m\right] \,, \tag{1}$$

where the matter Lagrangian is given by

$$\begin{aligned}\mathcal{L}_m \;&=\; \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \\ &\quad -\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} - \frac{\lambda}{f}\phi F_{a\mu\nu}\tilde{F}_a^{\mu\nu} + \text{mass terms}\,,\end{aligned} \tag{2}$$

and the second but last term, henceforth called *"Chern-Simons- (or magnetic helicity) term"*, can sometimes be understood as arising from coupling the gradient of ϕ to an anomalous axial vector current (via the chiral anomaly [15]). We discuss possible particle physics origins of the field ϕ, of an anomalous axial vector current, and of a heavy gauge field (with field strength denoted by F) in Section V. In Eq. (2), the index a is a gauge group index, μ and ν are space-time indices, λ is a dimensionless coupling constant, and f is a reference field value entering the expression for the axion potential $V(\phi)$. In this paper we consider an exponential potential

$$V(\phi) \,=\, \mu^4 e^{\phi/f} \,, \tag{3}$$

where μ sets the energy scale of the potential. This choice of $V(\phi)$ leads to an explicit breaking of parity and time-reversal invariance. To avoid this, one may replace $\exp(\phi/f)$ by $\cosh(\phi/f)^{-1}$ in Eq. (3). A more natural choice of self-interactions not breaking these symmetries explicitly will be considered in forthcoming work.

The second basic feature of our model is related to the assumption that ϕ is very slowly rolling at sub-Planckian field values, due to its coupling to the gauge field. The scalar field equation of motion is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) \,=\, \frac{\lambda}{8f}\vec{E}_a \cdot \vec{B}_a \,, \tag{4}$$

where the prime denotes a derivative of V with respect to ϕ. Following a hypothesis introduced in the context of inflationary models in [10] and in [11], we assume that the term proportional to the Pontryagin density generates slow-rolling of ϕ, in the sense that the terms in (4) proportional to first and second time derivatives of ϕ are negligible as compared to the two remaining terms. If this assumption is justified the equation of motion for ϕ reduces to

$$V'(\phi) \,\simeq\, \frac{\lambda}{8f}\vec{E}_a \cdot \vec{B}_a \,, \tag{5}$$

an equation that determines the time-dependence of ϕ, once we know the time-dependence of $\vec{E}_a \cdot \vec{B}_a$. We note that slow rolling can happen for sub-Planckian field values, [10], in contrast to the usual slow-roll in large-field inflationary scenarios, which requires super-Planckian values. In the context of inflation, a scenario based on the two basic features introduced so far is sometimes called *chromo-natural* inflation [11].

The third key feature of our scenario is related to secular growth of the electric field E_a, in excess of its usual dynamics. This growth is induced by the coupling of the gauge field to the axion field ϕ, as in (2). As we will see, the secular growth of E_a, when combined with Eq. (5), is responsible for the axion field ϕ to give rise to *tracking dark energy.*

The main point is that a non-vanishing magnetic helicity, which originates from the coupling of the gauge field to the axion as expressed by the "Chern-Simons term," acts as an extra friction term that ensures that ϕ will slowly roll down its potential – even for sub-Planckian field values. Thus, the resulting equation of state for the field energy of the axion is dominated by the potential energy term, which can thus act as a contribution to dark energy. Equations (3) and (5) then tell us that if the Pontryagin density $\vec{E}_a \cdot \vec{B}_a$ exhibits secular growth, the contribution of ϕ to the total energy density of the universe can become important at late times.

The last key feature of our scenario is related to the circumstance that the *initial value* of the energy density of ϕ is proportional to a small number in cosmology, such as the baryon to entropy ratio n_b/s, (n_b and s being the baryon and photon number densities, respectively). As far as relating a late-time cosmological observable to the small baryon to entropy ratio (via a term in the Lagrangian coupling the axion to an anomalous current) is concerned there are similarities of our work to the one in [16], where the tensor-to-scalar ratio, (i.e., the ratio of the strength of gravitational waves to that of scalar cosmological fluctuations), is related to n_b/s.

III. GAUGE FIELD DYNAMICS IN THE PRESENCE OF THE "ANOMALY TERM"

The equation of motion for the field strength tensor of the gauge field in the presence of a Chern-Simons term (but neglecting mass terms, which will turn out to be unimportant) is given by

$$D_\alpha^{ab}F^{b\beta\alpha} - \frac{4\lambda}{f}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha^{ab}(\phi F_{\mu\nu}^b) = 0\,, \tag{6}$$

where the operator D is defined by

$$D^{ab}_\alpha \equiv \delta^{ab}\nabla_\alpha + g f^{acb} A^c_\alpha \tag{7}$$

with ∇_α the space-time covariant derivative.

We write the equation of motion for the gauge field in terms of the "electric" and "magnetic" fields,

$$\begin{aligned} E^a_\mu &= F^a_{\mu\nu} u^\nu , \\ B^a_\mu &= -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{a\rho\sigma} u^\nu \end{aligned} \tag{8}$$

where $u^\mu = (1,0,0,0)$ is the four-velocity of a comoving observer in an FRW spacetime. In manifestly covariant form, the equations of motion are

$$\begin{aligned} u^\alpha D^{ab}_\alpha E^{b\sigma} &+ 2HE^{a\sigma} - u_\mu \epsilon^{\mu\sigma\alpha\beta} D^{ab}_\alpha B^b_\beta \\ &= -\frac{8\lambda}{f}\left(u_\mu \epsilon^{\mu\sigma\alpha\beta}\partial_\alpha \phi B^a + u^\alpha \partial_\alpha \phi B^{a\sigma}\right), \\ D^{ab}_\alpha E^{b\alpha} &= \frac{8\lambda}{f}\partial_\alpha \phi B^{a\alpha} \end{aligned} \tag{9}$$

In contravariant three-vector form, the equations are

$$\begin{aligned} D^{ab}_0 \mathbf{E}^b &+ 2H\mathbf{E}^a - \frac{1}{a}\mathbf{D}^{ab}\times \mathbf{B}^b \\ &= -\frac{8\lambda}{f}\left(\frac{1}{a}\nabla\phi \times \mathbf{E}^a + \dot{\phi}\mathbf{B}^a\right) \end{aligned} \tag{10}$$

$$\mathbf{D}^{ab}\cdot \mathbf{E}^b = \frac{8\lambda}{f}\nabla\phi\cdot \mathbf{B} \tag{11}$$

where $\mathbf{D}^{ab}$ is the spatial part of D^{ab}_α.

Using the general definition of the electric and magnetic fields,

$$E_{ai} = \partial_0 A^a - D^{ab}_i(A) A^b_0 \tag{12}$$

and

$$B_{ai} = \epsilon_{ijk}(\partial_j A^a_k - \frac{g}{2}\epsilon^{abc} A^b_j A^c_k) \tag{13}$$

we get:

$$\begin{aligned} \frac{\partial}{\partial t}E^i_a &+ \frac{g}{2}\epsilon^{abc} A^c_0 E^i_b + 2HE^i_a - \nabla_j B^i_a \\ &= -\frac{\lambda}{f}[\dot{\phi}B^i_a - \nabla\phi\times E^i_a] \end{aligned} \tag{14}$$

and

$$\nabla_i E^i_a = -\frac{\lambda}{f}\vec{\nabla}\phi \nabla_j B^i_a , \tag{15}$$

with

$$\nabla \times E^i_a = -\frac{\partial}{\partial t}B^i_a \tag{16}$$

We seek homogenous and isotropic solutions of these equations. An ansatz of curl-free electric and magnetic fields and of a background field ϕ only depending on time is then appropriate. We assume that, at an initial time when we start to study the time evolution of our system, the gauge field configuration is described by some spatially constant electric and magnetic fields, E_{0a} and B_{0a}. The above equations of motion then simplify to:

$$\frac{\partial}{\partial t}E^i_a + 2HE^i_a = -\frac{\lambda}{f}[\dot{\phi}B^i_a] \tag{17}$$

and

$$\frac{\partial}{\partial t}B^i_a + 2HB^i_a = 0 \tag{18}$$

In the absence of the Chern-Simons term, these equations lead to the scaling (from now on we will drop the gauge index a which amounts to considering the group to be $U(1)$)

$$E(t) \sim a^{-2}(t) \tag{19}$$
$$B(t) \sim a^{-2}(t) , \tag{20}$$

which implies that the energy density scales as radiation, namely $\propto a(t)^{-4}$.

The equations discussed above would be valid at all times for an unbroken $U(1)$-gauge symmetry. We are interested, however, in a gauge symmetry that is spontaneously broken at a large mass scale m. The gauge field then acquires its mass after the symmetry breaking phase transition, a transition which occurs when the temperature, $T(t)$, of the Universe is of the order of m. The energy density of the gauge field will then scale as matter, i.e., $\rho_{gauge} \sim a(t)^{-3}$, for times greater than t_m, where t_m is determined by

$$H(t_m) \simeq m \tag{21}$$

This corresponds to the scaling

$$E(t), B(t) \sim a(t)^{-3/2} \tag{22}$$

Next, we explain why, due to its coupling to the axion field ϕ, the electric field decays less fast than it would without the presence of ϕ. This effect will lead to an energy density in the E- and B- fields that initially scales as that of radiation, but, at late times, grows relative to the energy density of radiation.

First, we study the equations of motion for the E- and B- fields in the early (high-temperature) phase, where the gauge field is effectively massless, i.e., for $t < t_m$. For homogeneous and isotropic field configurations, the equations of motion for the electric and magnetic fields are (suppressing the gauge group index a)

$$\begin{aligned} \dot{E}^i + 2HE^i &= -\frac{\lambda}{f}\dot{\phi}B^i , \\ \dot{B}^i + 2HB^i &= 0 \end{aligned} \tag{23}$$

In the absence of the "Chern-Simons term" proportional to $\dot{\phi}$ (or if the ϕ-field is at rest), these equations lead to the behavior $E^i \sim a^{-2}$ and $B^i \sim a^{-2}$ and thus to an energy density in the E- and B- fields that scales as radiation. However, when ϕ is slowly rolling down its potential hill (i.e., if $\dot{\phi} < 0$) then the E- field decays less fast.

Once the mass of the gauge field becomes important, i.e., for $t > t_m$, the energy density in the E- and B- fields decays like that of ordinary matter. This corresponds to a scaling of the fields proportional to $a^{-3/2}$.

We can determine the effects of the "Chern-Simons term" with the help of the Green function method. The magnetic field B^i continues to scale as $a(t)^{-2}$. Hence, the equation for E^i can be written as

$$\dot{E}^i \,+\, 2HE^i \,=\, S(t)^i \,, \tag{24}$$

with a source $S(t)^i$ scaling as $a(t)^{-2}$, for $t < t_m$, and as $a(t)^{-3/2}$, for $t > t_m$, and given by

$$S^i(t) \,=\, \frac{\lambda}{f}|\dot{\phi}|a^2(t_i)a^{-2}(t)B(t_i)^i \,, \tag{25}$$

where t_i is the initial time.

During the radiation period and for $t < t_m$, the fundamental solution of (24) scales as t^{-1}. Hence, the solution of (24) becomes

$$E(t) \,=\, \int_{t_i}^{t} dt' \frac{t'}{t} S(t') + E(t_i)\frac{t_i}{t} \,, \tag{26}$$

where we are suppressing the superscript on the E- field. Inserting the form of the source $S(t')$ and assuming (in the spirit of the slow-roll approximation, which we will justify later) that $\dot{\phi}$ is constant in time we obtain

$$E(t) \,=\, E(t_i)\frac{t_i}{t}\big[1 + \frac{\lambda}{f}|\dot{\phi}|\frac{B(t_i)}{E(t_i}(t - t_i)\big] \tag{27}$$

It can be checked easily that, for $t > t_m$, too, the source term $S(t)$ induces a secular growth linear in t. Thus, for $t > t_m$, we obtain

$$E(t) \,=\, E(t_m)\big(\frac{a(t_m)}{a(t)}\big)^{3/2}\big[1 + \frac{\lambda}{f}|\dot{\phi}|\frac{B(t_m)}{E(t_m)}(t - t_m)\big] \tag{28}$$

For $t > t_{eq}$, where t_{eq} is the time of equal matter and radiation, the fundamental solution of (24) scales as $a(t)^{-3/2} \sim t^{-1}$. The further evolution of $E(t)$ is then given by

$$E(t) \,=\, \int_{t_{eq}}^{t} dt' \big(\frac{a(t')}{a(t)}\big)^{3/2} S(t') + E(t_{eq})\big(\frac{a(t_{eq})}{a(t)}\big)^{3/2} \,, \tag{29}$$

where now

$$S(t) \,=\, \frac{\lambda}{f}|\dot{\phi}|a^{-3/2}(t)B(t_{eq}) \tag{30}$$

Inserting the solution for $E(t_{eq})$ from (27), we find that, for $t \gg t_{eq}$,

$$E(t) \,\simeq\, E(t_i)\big(\frac{a(t_i)}{a(t_m)}\big)^2\big(\frac{a(t_m)}{a(t)}\big)^{3/2}\big[1 + \frac{\lambda}{f}|\dot{\phi}|\frac{B(t_i)}{E(t_i)}t\big] \,. \tag{31}$$

The second term in the parentheses in (27), (28) and (31) corresponds to a secular growth term caused by the coupling of the gauge field to the axion. There will be a critical time t_c below which the secular growth term is negligible. Its value is

$$t_c \,=\, \frac{f}{\lambda}\frac{E(t_i)}{B(t_i)}\frac{1}{|\dot{\phi}|} \,. \tag{32}$$

Based on the above analysis we are able to determine the scaling of the magnetic helicity term. Taking into account the fact that $B(t)$ scales as $a(t)^{-2}$, for $t < t_m$, and as $a(t)^{-3/2}$, for $t > t_m$, we find that

$$\vec{E} \cdot \vec{B} \,\sim\, a(t)^{-4} \tag{33}$$

for $t < t_m$, as

$$\vec{E} \cdot \vec{B} \,\sim\, a(t)^{-3} \tag{34}$$

for $t_m < t < t_c$, and as

$$\vec{E} \cdot \vec{B} \,\sim\, a(t)^{-3} t \tag{35}$$

for $t > t_c$. Here we should emphasize, once again, that the linear scaling in t is a consequence of our assumption that $\dot{\phi}$ is constant in time. Later on, we will consider the corrections arising from a relaxation of this assumption.

IV. LATE TIME ACCELERATION FOR AN EXPONENTIAL POTENTIAL

Next, we turn to analyzing what a gauge field configuration with non-vanishing magnetic helicity implies for the dynamics of the axion ϕ. We recall that the equation of motion of ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) \,=\, \frac{\lambda}{8f}\vec{E} \cdot \vec{B} \,, \tag{36}$$

where the prime indicates a derivative of V with respect to ϕ. As in the context of inflationary models in [10], we assume that the term on the right side of (36) generates slow-rolling of ϕ, in the sense that the terms $\ddot{\phi}$ and $3H\dot{\phi}$ in (36) are negligible as compared to the two remaining terms. We will check the self-consistency of this assumption below. The evolution of ϕ is then determined by

$$V'(\phi) \,=\, \frac{\lambda}{8f}\vec{E} \cdot \vec{B} \tag{37}$$

For an exponential potential,

$$V(\phi) \,=\mu^4 e^{\phi/f} \,, \tag{38}$$

Eq. (37) yields

$$V(\phi) \,=\, \frac{\lambda}{8}\vec{E} \cdot \vec{B} \tag{39}$$

The proportionality of $V(\phi)$ to $\vec{E}\cdot\vec{B}$ is a special feature of the exponential potential. For power-law and periodic potentials, $V(\phi)$ ends up being proportional to a power of $\vec{E}\cdot\vec{B}$ greater than 1, and hence decays faster in time in an expanding universe. This would appear to make it harder to interpret ϕ as a dark energy candidate. We will study these types of potentials in the context of a slightly different, more general model in a follow-up paper.

Under the assumption that the slow-rolling conditions are indeed satisfied, Eq. (39) immediately leads to an expression for the contribution, Ω_ϕ, of the ϕ- field to the total energy density of the universe. From the above discussion it follows that, for $t < t_m$, the energy density of ϕ scales like that of radiation and hence leads to a constant contribution to Ω_ϕ. For $t_m < t < t_{eq}$, the potential energy of ϕ decreases less fast than the background radiation density, leading to a contribution to Ω_ϕ that grows linearly in $a(t)$. Once $t > t_{eq}$, but before $t = t_c$, both the background density and the energy density of ϕ scale as $a(t)^{-3}$, and hence the contribution of ϕ to Ω is constant. Finally, once $t > t_c$, Ω_ϕ increases linearly in time. Specifically, for late times $t > t_{eq}$, we obtain that

$$\begin{aligned}\Omega_\phi(t) \;&\simeq\; \frac{V(\phi(t))}{\rho_0(t)} \\ &=\; \frac{\lambda}{8}\frac{(\vec{E}\cdot\vec{B})(t_i)}{\rho_r(t_i)}\big(\frac{a(t_{eq})}{a(t_m)}\big)\big[1+\frac{\lambda}{f}|\dot{\phi}|\frac{B(t_i)}{E(t_i)}t\big]\,,\end{aligned} \tag{40}$$

where $\rho_0(t)$ is the background energy density at time t, and $\rho_r(t_i)$ is the energy density of radiation at the initial time t_i, (which is approximately equal to the total energy density at that time, since we have assumed that t_i is in the radiation period). Figure 1 presents a sketch of the time evolution of Ω_ϕ.

In the above formula for the energy density of ϕ we have assumed that the slow-roll conditions

$$\ddot{\phi} \ll V'(\phi) \quad \text{and} \quad 3H\dot{\phi} \ll V'(\phi) \tag{41}$$

are satisfied, and that the equation of state of ϕ leads to acceleration. To check the self-consistency of these conditions, note that from (38), (39) and (31) the value of ϕ for $t > t_{eq}$ and $t > t_c$ is given by

$$\phi(t) \;\simeq\; f\log(\kappa t^{-1}) \tag{42}$$

with

$$\kappa \;=\; \frac{\lambda}{8}\frac{(\vec{E}\cdot\vec{B})(t_i)}{\mu^4}\big(\frac{a(t_i)}{a(t_m)}\big)^4\frac{\lambda}{f}|\dot{\phi}|\frac{B(t_i)}{E(t_i)}t_0^2\,. \tag{43}$$

Hence

$$\dot{\phi}^2(t) \;\simeq\; \frac{f^2}{t^2} \tag{44}$$

and

$$\ddot{\phi} \;\simeq\; \frac{f}{t^2}\,. \tag{45}$$

It then immediately follows that the slow-roll conditions are satisfied provided that

$$f \;\ll\; m_{pl}\,. \tag{46}$$

It is also easy to check that the equation of state for ϕ is dominated by the potential energy if the condition (46) is satisfied. Thus, the field ϕ is indeed a candidate for tracking dark energy.

Finally, we study the magnitude of the contribution of ϕ to the dark energy budget. Evaluating (40) at the present time t_0 and assuming $t > t_c$ we obtain

$$\Omega_\phi(t_0) \;\simeq\; \frac{\lambda}{8}\frac{(\vec{E}\cdot\vec{B})(t_i)}{\rho_r(t_i)}\big(\frac{a(t_{eq})}{a(t_m)}\big)\frac{\lambda}{f}|\dot{\phi}|\frac{B(t_i)}{E(t_i)}t_0 \tag{47}$$

If the gauge field appears in the anomaly equation of an anomalous matter current then

$$\frac{(\vec{E}\cdot\vec{B})(t_i)}{\rho_r(t_i)} \;\sim\; \frac{n_B}{s}(t_i), \tag{48}$$

where n_B is the baryon number density and s the entropy density. Hence, the smallness of the initial contribution of ϕ to dark energy is guaranteed by the observed small baryon to entropy ratio. This factor is believed to be of the order 10^{-10}.

The third factor on the right hand side of (47) is given by the ratio of the mass of the weak gauge bosons and the current temperature. Assuming, as before, that t_m is the time when $H(t_m) = m$ we have that

$$\big(\frac{a(t_{eq})}{a(t_m)}\big) \;\sim\; \big[\frac{m}{T_{eq}}\frac{m_{pl}}{T_{eq}}\big]^{1/2} \tag{49}$$

(where T_{eq} is the temperature at the time of equal matter and radiation) which is of the order of 10^{18} if the mass m of the gauge field is taken to be of the order of the mass of the weak gauge field.

Inserting these two factors into (47) we see that if we take $|\dot{\phi}|$ to be given by its late time value, then a value

$$\lambda \;\sim\; 10^{-4} \tag{50}$$

is required in order to explain the currently observed value of the dark energy density. This still constitutes a small hierarchy problem, but a much smaller one than the usual hierarchy problem one faces.

As seen from Eq. (44), $\dot{\phi}$ is not constant in time but scales as t^{-1}. This means that it is not self-consistent to neglect the time-dependence of $\dot{\phi}$ in the derivation of the secular growth term in $E(t)$. Inserting the time-dependence of $\dot{\phi}$ we see that the time-dependence of the secular growth term changes from being linear in t to being logarithmic. This does, however, not change the qualitative features of our analysis.

V. PARTICLE PHYSICS CONNECTIONS

A) An axion coupling to an anomalous matter current:

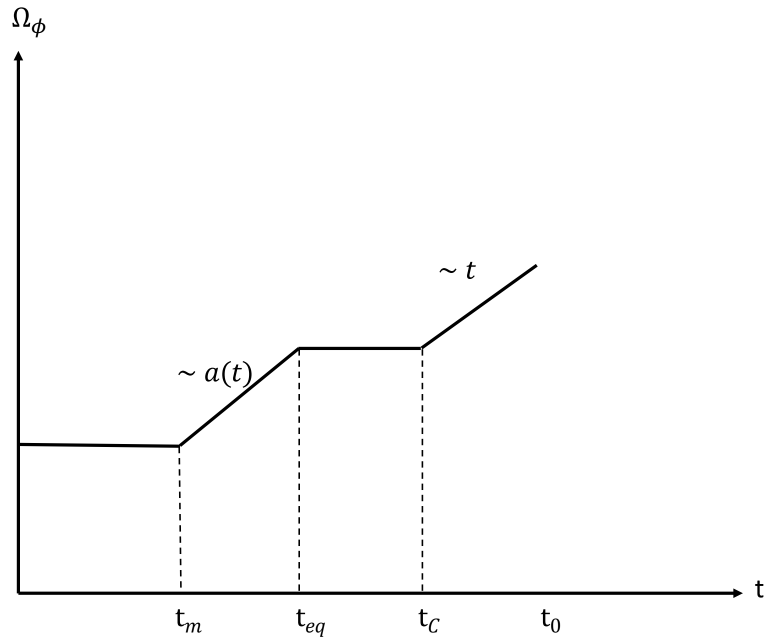

FIG. 1: Sketch of the time evolution of the fractional contribution Ω_ϕ of the ϕ field to energy density of the Universe. The horizontal axis is time, the vertical axis is the value of Ω_ϕ. The contribution is constant until the time t_m when the gauge field mass becomes important. It then rises as the scale factor, to become constant again for $t > t_{eq}$. Once the secular growth of the E field becomes dominant at the time t_c the contribution of ϕ to Ω once again begins to rise, this time linearly in time. Note that t_0 is the present time.

The standard axion field, $a(x,t)$, along with the Peccei-Quinn symmetry has been introduced to solve the strong CP problem of QCD; see [2]. The mechanism leading to the spontaneous breaking of the Peccei-Quinn symmetry involves a complex scalar field with a standard symmetry breaking potential whose angular variable is the axion field a [1]. The coefficient of the Pontryagin density, $\mathrm{tr}(F \wedge F)$, in the QCD Lagrangian then becomes a dynamical variable. At the perturbative level, the axion has a flat potential. Non-perturbative instanton effects create however a non-trivial potential, $V(a)$, for the axion. This potential is periodic in a, which is an "angular variable." The periodicity of the potential is not unproblematic, since it could give rise to an axion domain-wall problem.

The axion of QCD is a candidate for dark matter [4], but cannot be a candidate for dark energy, since it interacts too strongly with electromagnetism. Any viable candidate scalar field for dark energy needs to couple very weakly to standard model matter [17].

The idea underlying our proposal is that the field ϕ responsible for dark energy could be a new axion field conjugate to an anomalous matter current [15]. It is well known, see, e.g., [18], that $B-L$, where B and L stand for baryon- and lepton number, can be coupled to a neutral vector boson, Z', somewhat heavier than the Z^0- boson in an anomaly-free gauge theory extending the standard model. Introducing such a gauge field would again lead to a CP problem that can be solved by introducing a new axion field, which we tentatively identify with the axion field ϕ considered in previous sections. The gradient of ϕ can then be linearly coupled to the anomalous $(B-L)$-axial vector current, introducing a term proportional to

$$\partial_\mu \phi \cdot J^\mu_{5,B-L} \tag{51}$$

in the Lagrangian of the theory. Apparently, the time derivative of ϕ then plays the role of a space-time dependent axial chemical potential for the pseudo-scalar $(B-L)$-density [5]. This may furnish an ingredient in a mechanism leading to matter-antimatter asymmetry. Thanks to the anomaly equation [15], the term (51) is equivalent to a term proportional to

$$\phi(F \wedge F + \frac{1}{f} \sum_j m_j \bar{\psi}_j \gamma_5 \psi_j)\,, \tag{52}$$

where F is the field strength of Z', j labels fermion species, and species j has mass m_j and is described by a spinor field ψ_j.

Instanton effects are usually expected to lead to a potential for ϕ that is periodic in ϕ, and this possibility is studied in forthcoming work. One may imagine, however, that axion shift-symmetry breaking effects might generate an exponential potential. The value of the parameter f is related to the symmetry breaking scale, and the energy-scale parameter μ is set by the strength of the instanton effects.

B) Universal axion of string theory:
Axions arise naturally in superstring theory [19]. Specifically, string compactifications generate Peccei-Quinn type symmetries often broken at the string scale [20]. For example [21], there is an axion field a that is in the same chiral superfield S as the four dimensional dilaton φ

$$S \,=\, e^{-\varphi} \,+\, ia\,. \tag{53}$$

In addition, there is an axion field $\tilde{a}$ in the superfield $\tilde{S}$ of the volume scalar ρ:

$$\tilde{S} \,=\, e^{\rho} \,+\, i\tilde{a}\,. \tag{54}$$

The Peccei-Quinn symmetries of string theory are always broken by stringy instanton effects, leading to a coupling of the axion to some $\mathrm{tr}(F \wedge F)$- term. This can be shown explicitly by reducing the ten-dimensional supergravity action to four space-time dimensions via compactification on some internal Calabi-Yau manifold; (see e.g. [21]). Such a compactification also generates potentials for the superfields to which the axions belong. These potentials are typically exponential in the radial direction, but a remnant of the exponential potential may also affect the potential in the axion direction; especially if stringy effects lead to a breaking of the shift symmetry in the axion direction, as happens in axion-monodromy models [22, 23]. For some explicit constructions of exponential potentials see [24].

C) Axion monodromy:
Indeed, it has recently been realized that stringy effects break the shift symmetry of the axion. The axion ceases to be an angular variable and, instead, has an infinite range of values. Monodromy induces an axion potential rising without bound, as ϕ increases to ∞; see, e.g., [23]. At large field values, the axion potential may be linear. To make contact with our scenario we need to assume that the potential is exponential at small field values.

We are not the first to connect an axion with a potential induced by stringy monodromy effects with dark energy. In [25] it was in fact suggested that a stringy axion may play the role of a quintessence field. The construction in [25] makes use of standard slow-roll inflation and thus requires super-Planckian field values, that is field values that, even in the case of axion monodromy models, may not be consistent from the point of view of string theory [26]. In our construction, the axion field values are sub-Planckian, because slow-rolling is induced by the coupling of the axion to the Chern-Simons term of a gauge field.

VI. CONCLUSIONS AND DISCUSSION

We have studied a model of *tracking dark energy* in which dark energy arises from an axion field ϕ linearly coupled to the Pontryagin density of a gauge field, i.e., to a term $\mathrm{tr}(F \wedge F)$. Thanks to this coupling, the axion is rolling slowly even for sub-Planckian field values, It thus has the right equation of state to account for dark energy. We have considered the example of an exponential potential for the axion. The coupling between the axion and the gauge fields leads to a secular growth term in the electric field. At early times, the energy density in ϕ tracks that of the background; but when the secular growth term becomes important the contribution of ϕ to the density parameter Ω starts to increase. We have studied the evolution of Ω_ϕ (the fraction of the total energy density required for a spatially flat universe due to the axion field ϕ) as a function of time and found that it is constant for early times $t < t_m$, where t_m is the time when the mass of the gauge field becomes important. It grows linearly in the scale factor between time t_m and the time, t_{eq}, of equal matter and radiation. After time t_{eq}, the value of Ω_ϕ ceases to grow until the time when the secular growth term becomes dominant, after which it will start to grow again.

In order for ϕ to be a successful candidate for dark energy, the time when $\Omega(\phi)$ approaches $\Omega = 1$ has to be close to the present time t_0. This is only the case if, at the initial time, the ϕ- field energy is a small contribution to the total energy density. Our proposal is that this small initial value of $\Omega(\phi)$ is linked to the small value of the lepton to entropy ratio. This would imply that the secular growth term becomes important only at rather late times. Thus, our model represents an implementation of the "tracking quintessence" scenario of [13]. We have shown that, in order to obtain the currently observed value of dark energy in our model, it suffices to require a fairly mild tuning of dimensionless coupling constants.

In this paper we have neglected the coupling of the axion field ϕ to pseudo-scalar mass terms, $m_j \bar{\psi}_j \gamma_5 \psi_j$, of fermionic matter fields; see Eq. (52). Taking such couplings into account would lead to extra terms on the right side of the axion equation of motion (36). For $H > \mathrm{max}_j m_j$, these terms will decay as radiation, and, for $H < \mathrm{min}_j m_j$, they decay as matter. If $\bar{m} \equiv \mathrm{max}_j m_j < m$, there is a time interval $t_m < t < t_{\bar{m}}$

when the contribution due to the mass terms on the right side of Eq. (52) decays rapidly, relative to the one of the $F \wedge F$ term. Hence, as long as $m > \bar{m}$, the extra terms in (52) will not change our conclusions.

It has been pointed out that if the field responsible for dark energy is a pseudo-scalar field then it could couple to visible matter, and this leads to rather stringent constraints. The earliest discussion of the coupling of an axion to visible matter has been given in [17], where it has been assumed that the axion couples to the $\vec{E} \cdot \vec{B}$-term of electromagnetism. This would lead to a rotation of the direction of polarization of light emitted by distant radio sources. The constraints resulting from this effect are quite restrictive and would potentially rule out our model if our axion were to couple to the electromagnetic field. However, we have assumed that our axion does *not* interact with the photon and thus evades the bounds presented in [17] and in related work. In a future paper, we will investigate collider signals due to a possible coupling of the axion field ϕ to W- and Z- bosons.

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