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Quasi-satellite dynamics in formation flight

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ABSTRACT

The quasi-satellite (QS) phenomenon makes two celestial bodies to fly near each other (Mikkola et al. 2006) and that effect can be used also to make artificial satellites move in tandem. We consider formation flight of two or three satellites in low eccentricity near Earth orbits. With the help of weak ion thrusters it is possible to accomplish tandem flight. With ion thrusters it is also possible to mimic many kinds of mutual force laws between the satellites. We found that both a constant repulsive force or an attractive force that decreases with the distance are able to preserve the formation in which the eccentricities cause the actual relative motion and the weak thrusters keep the mean longitude difference small. Initial values are important for the formation flight but very exact adjustment of orbital elements is not important. Simplicity is one of our goals in this study and this result is achieved at least in the way that, when constant force thrusters are used, the satellites only need to detect the directions of the other ones to fly in tandem. A repulsive acceleration of the order of 10^{-6} times the Earth attraction, is enough to effectively eliminate the disruptive effects of all the perturbations at least for a timescale of years.

Key words: celestial mechanics – planets and satellites

1 INTRODUCTION

In an attempt to understand better the quasi-satellite phenomenon (Mikkola et al. 2006) we tested what would happen if the force between two co-orbital bodies were different from the normal Newtonian gravity. We found that many forces can produce similar relative motion of the bodies. Even repulsive forces result in tandem motion of the bodies. At that point it became clear that constant weak repulsive force could be used to keep artificial satellites flying near each other. This paper studies that phenomenon mainly numerically but we also present some simple analytical considerations.

In quasi satellite motion an asteroid moves around the Sun co-orbitally with a planet and remains near the planet such that in the rotating coordinate system it looks like moving around the planet in a retrograde orbit. In that system the mutual force of the bodies is attractive. On the other hand it is well known that a spacecraft chasing a satellite in the same orbit must brake to catch it up. Thus, somewhat counter intuitively, tandem flight of two satellites moving around a central body in its gravitational field seems to be possible both with attractive or repulsive mutual acceleration. In the case of two satellites the force between the satellites can been mimicked using ion thrusters. Both an attractive force $\propto 1/\Delta^2$ and repulsive constant force produce similar effects, which may look somewhat unexpected. Above Δ means the distance of the satellites which we assume to be quite small compared with the size of the orbit. We restrict our consideration to low eccentricity orbits. High eccentricity orbits are generally more complicated for formation flying (Roscoe et al. 2013).

The simplest of all methods to make satellites to fly in tandem is to put them into precisely same orbit to some distance from each other. An example of such a real system is NASA's Grail mission in which two spacecrafts are flying in tandem orbits around the Moon to measure its gravity field in detail.

This works perfectly as long as one can consider the gravitational field to be rotationally symmetrical (like the Earth's field modeled without tesseral harmonics) in which case the distance between the satellites varies only due to eccentricity caused speed differences. The simplest way to achieve tandem flight in that situation is possibly making ion thrusters to point towards, or away from, the other satellite(s) and this seems to work according to our simulations. One can keep the satellites to move in the same fashion as the natural quasi-satellites. Even near triangle configuration for three satellites is possible without any complicated

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control, just constant acceleration or an acceleration that depends on the distance of the satellites. No other information is necessary but the relative directions (and distances if the acceleration needs to depend on the distance). Long lasting thrusters are necessary, but such exist as shown e.g. by ESA's SMART-1 mission from a near Earth orbit to the Moon.

The very basic method to consider satellite relative motions near each other and/or rendezvous is naturally the Clohessy-Wiltshire equations (Clohessy and Wiltshire 1960). To get satellites flying in tandem has been considered in numerous publications e.g. precise orbital elements adjustments have been discussed in e.g. (Alfriend et al. 2001; Beichman et al. 2004; Schaub and Alfriend 2001; Gim and Alfried 2005).

Kristiansen et al. (2010) considered the formation flight using differential approximations of the pure Kepler motion, also Prioroc and Mikkola (2015) discuss the computation of relative motion using simple methods. Roscoe et al. (2013) discuss formation flight in highly eccentric orbits using differential mean orbital elements as design variables and the Gim and Alfried (2005) state transition matrix for relative motion propagation. Use of a coordinate transformation was consider by Vallado and Alfano (2014).

We first consider the problem in terms of a simple approximate analytical theory. The assumptions in that theory are based on simulation results since a complete analytical solution seems laborious. After the analytical considerations we present results from numerical simulation. The gravity field that we used in the tests was the spherical function expansion, following Mikkola, Palmer and Hashida (2002), plus Luni-Solar terms, but we also studied the simple J_2 field model. The ion thruster effects we model using different model potentials could be engineered with adjustable thrusts. However, the constant force model is the simplest one and it seems to work.

Finally we remark that this is not an engineering paper, we just study the dynamics.

2 ANALYTICAL CONSIDERATIONS

2.1 Units and orbital elements

We choose the units such that

$$G = 1,\ R_\oplus = 1,\ m_\oplus = 1$$

where G=the gravitational constant, $R_\oplus$=radius of the Earth (actually 6378.137 km) and $m_\oplus$= the mass of the Earth. The unit of time becomes such that the formal period of a satellite with semi-major-axis $a = 1$ would be $= 2\pi$. In this system one time unit in seconds is $=$ 806.811064922699 sec and the length of a day is $= 107.088$. Despite these choices we write in some formulae the radius of the Earth and its mass explicitly visible. In illustration, to make them easier to understand, we use also other units, e.g. kilometers. For orbital elements we use the standard notations

$$a,\ e,\ i,\ \omega\ ,\Omega,\ M,$$

which are the semi-major axis, eccentricity, inclination, argument of perihelion, longitude of the ascending node and the mean anomaly, respectively. For the position vector we use $\mathbf{r}$ and for velocity $\mathbf{v}$. In the used units the velocity and momentum are the same i.e. $\mathbf{v} = \mathbf{p}$. The semi-major-axes of the satellites are only very slightly different (and equal on the average) and therefore we can in most consideration take them to be practically same although the difference δa has an important role in some analytical considerations.

We need also the difference of the mean longitudes $\lambda = M + \omega + \Omega$. We consider (almost) coplanar orbits which means that $\Omega_1 = \Omega_2$ to high precision and therefore the difference of λ's is essentially the same as the difference of the angles $M+\omega$. This quantity is one of the most important ones in our analytical consideration of the stability of the tandem flight. We use for it the notation

$$\theta = \lambda_2 - \lambda_1 = M_2 - M_1 + \omega_2 - \omega_1, \tag{1}$$

where $\dot{M}_k = n_k = 1/\sqrt{(a_k^3/m_\oplus)}$ are the mean motions. An other important quantity is the eccentricity vector

$$\mathbf{e} = \mathbf{v}\times(\mathbf{r}\times\mathbf{v})/m_\oplus - \mathbf{r}/r, \tag{2}$$

and especially their difference

$$\widetilde{\mathbf{e}} = \mathbf{e}_2 - \mathbf{e}_1, \tag{3}$$

which is a kind of relative eccentricity vector.

2.2 C-W/Hill theory

Clohessy and Wiltshire (1960) presented a theory of satellite rendezvous (C-W hereafter). This theory used the variational equations of the two-body problem

$$\dot{\mathbf{x}} = \mathbf{w}, \quad \dot{\mathbf{w}} = -m_\oplus(\mathbf{x}/r^3 - 3\mathbf{r}\cdot\mathbf{x}\,\mathbf{r}/r^5), \tag{4}$$

where $\mathbf{x}$ is the variation of the position and $\mathbf{w} = \dot{\mathbf{x}}$ is the variation of velocity. For a circular orbit in the rotating coordinate system, where $\mathbf{r}$ is constant, the equations become linear and easily solvable. Much earlier Hill had studied the Lunar orbit in a somewhat similar way (Hill 1878). The difference in these treatments is that in the latter one there are additional nonlinear terms in the equations (4) of motion. As shown e.g. by Prioroc and Mikkola (2015) the C-W-solution and in fact a somewhat more general one can be obtained by differentiating the two-body equations with respect to the orbital elements. Let $q_k \quad k =, 1, 2.., 6$ be a set of elements e.g. $\mathbf{q} = (M_0, i, \Omega, \omega, a, e)$, which are the mean anomaly M at epoch, inclination, ascending node, argument of pericentre, semi-major-axis and eccentricity, respectively. The solution for the variational equation takes the form

$$\mathbf{x} = \sum \frac{\partial \mathbf{r}}{\partial q_k}\delta q_k \ \text{ and } \mathbf{w} = \sum \frac{\partial \mathbf{v}}{\partial q_k}\delta q_k. \tag{5}$$

The constants δq_k represent the element variations. A particular solution of equation (4) is

$$\mathbf{x}_a = 2\mathbf{r} - 3\,t\,\mathbf{v}, \tag{6}$$

which is related to the variation of the semi-major-axis. Because the time t appears as a factor of $\mathbf{v}$ it is clear that without some regulating additional force the distance between the bodies would increase continuously unless the semi-major-axis difference is precisely zero.

Additional information about the stability of the Hill-type and distant retrograde orbits can also be found in Jackson (1913), Scheeres, (1998) and Mikkola et al. (2006).

2.3 Secular dynamics

2.3.1 Assumptions based on numerical results

Here we present some numerically obtained facts that can be used as assumptions in our analytical considerations. The force model we used consisted Earth potential model with the J_2 terms and a weak ion thruster acceleration that was either a repulsive constant acceleration or, for comparison, an attraction that had the $1/\Delta^2$ distance dependence. Also other models were briefly consider (see below).

The following was found to be true in our simulations:

-The orbital elements vary periodically but their mean values remain the same and the J_2 (and higher order) effects have minor influence in the relative motion of the satellites (see also Prioroc and Mikkola (2015)).

-Inclinations i and longitudes of ascending node Ω remain very close to each other if they are so initially. This is due to similar precession rates. Thus the satellites remain almost co-planar. Also the ω's precess at the same rate.

-The difference of the eccentricity vectors $\widetilde{\mathbf{e}}$ remains almost constant in length. This is important in the theory and in accordance with the results obtained in Mikkola et al. (2006). See Fig.6 in this paper.

-When the initial semi-major-axes are nearly the same they remain so as long as the satellites fly in tandem. Due to the thruster effect the values fluctuate such that the mean angular speeds remain the same. Thus in an approximate theory we can take the (mean) semi-major-axes to be the same, only their difference varies periodically.

Those observations can be used as assumptions in the next section where the analytical treatment is discussed.

2.3.2 Analytical approximations

Consider first a simple case in which we assume the motions to be pure two-body motions and of low eccentricity. Further we assume the motions to be co-planar.

As a first simple example we consider the case in which one of the orbits is circular and the other one has a small eccentricity. In the rotating coordinate system in which the circular orbit satellites coordinates are simply $= a(0, 1)$ when the distance is consider to be the y-coordinate, the eccentric orbit has approximately the coordinates [1]

$$a(2e\sin(M) + \theta, 1 - e\cos(M)),$$

when eccentricity and θ are consider to be quantities of the same order of magnitude (we call this order $O(\varepsilon)$) and so small that the linearized approximation is valid.

The differences of the coordinates of the satellites are thus

$$(\Delta x, \Delta y) = a(2e\sin(M) + \theta, -e\cos(M)) + \vec{O}(\varepsilon^2), \tag{7}$$

from which we see that the relative motion is mainly caused by the eccentricity as long as the mean longitude difference θ remains small. The effect of θ is illustrated in Fig.1. From the above coordinate differences we have for the distance between the satellites the form

[1] The corresponding formulae in Mikkola et al. (2006) are in error but not used.

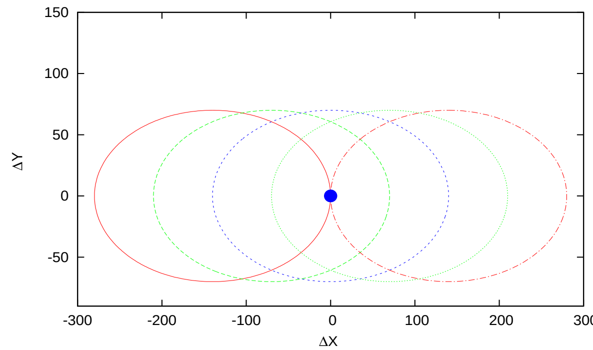

Figure 1. The relative motion of the satellites for the cases when $\theta = \pm 2e$, $\theta = \pm e$ and $\theta = 0$ in the coordinate system that rotates with one of the satellites(=the central dot=satellite 1). Here the eccentricity of satellite 2 is $e = 0.01$, while the satellite 1 has eccentricity 0. The distances were converted to kilometers assuming that $a = 7000$ km.

$$\Delta^2 = a^2((\theta + 2e\sin(M))^2 + e^2\cos^2(M)) + O(\varepsilon^2). \tag{8}$$

The generalization of this to the case of two eccentric orbits is quite simple (see below).

For stable tandem flights it is clearly necessary that the mean longitude difference θ indeed remains small. This is what we study now.

If both satellites are supposed to use the ion thrusters with equal accelerations, it is possible to model the system mathematically in terms of a Hamiltonian

$$H = K_1 + K_2 - R_1 - R_2 - R_{12}, \tag{9}$$

where

$$K_\nu = \mathbf{p}_\nu^2/2 - \frac{m_\oplus}{r_\nu}, \tag{10}$$

and R_k's are the perturbing force functions with high order spherical harmonics, including tesseral ones and the rotation of the Earth, were included. The force function for both satellites was

$$R = \sum_{n=1}^{N_x}\sum_{m=0}^{n}\frac{P_n^m(\cos(\theta))}{r^{n+1}}(C_n^m\cos(m\psi) + S_n^m\sin(m\psi)) + R_{ls}. \tag{11}$$

Here $\psi = \phi - n_E t - \eta_0$, ϕ is the longitude, read eastwards, in the Earth fixed coordinates, t is time, n_E is Earths rotational frequency, η_0 initial phase, while N_x defines the order up to which terms are taken into account (we used $N_x = 36$, like Mikkola, Palmer and Hashida (2002)). The numerical coefficients C_n and S_n define the gravitational potential. P_n^m's are the associated Legendre polynomials. The term R_{ls} naturally signifies the effect due to the Sun and Moon (ls = Luni-Solar).

If we include only the leading term, the J_2 term, then

$$R_\nu = \frac{m_\oplus}{r_\nu}\left(\frac{J_2 R_\oplus^2}{2r_\nu^2}\left(1 - 3\left(\frac{z_\nu}{r_\nu}\right)^2\right)\right), \tag{12}$$

are the J_2 terms ($J_2 = 0.0010826299890519$ in this notation). The 'perturbing' thruster acceleration is in the term $R_{12} = R_{12}(\Delta)$ (which thus depends only on Δ), where

$$\Delta = |\mathbf{r}_2 - \mathbf{r}_1|. \tag{13}$$

The secular (averaged over one period) form of the K_ν Hamiltonians in the J_2 approximation are

$$\langle K_\nu - R_\nu \rangle = -\frac{m_\oplus}{2a_\nu} - \frac{J_2 R_\oplus^2 m_\oplus \left(2 - 3\sin^2(i_\nu)\right)}{4a_\nu^3 \left(1-e_\nu^2\right)^{3/2}}. \tag{14}$$

The perturbing function R_{12} tells how the thrusters are used. For example if constant acceleration is used, then

$$R_{12}(\Delta) = \epsilon\Delta, \tag{15}$$

where the constant ϵ is the satellites acceleration due to the ion thruster. We assume this to be constant (unless otherwise indicated).

Following Mikkola et al. (2006) for co-planar orbits one can derive the approximation

$$\Delta^2 \approx a^2(\widetilde{e}^2\cos(w)^2 + (\theta + 2\widetilde{e}\sin(w))^2) \tag{16}$$

where

$$\widetilde{e} = |\mathbf{e}_2 - \mathbf{e}_1|, \quad w = M + \text{constant}. \tag{17}$$

Thus this more general approximation differs from the simple equation (8) only in the way that the eccentricity is replaced by the 'relative eccentricity' $\widetilde{e}$ and the angle w differs from M by a constant (of two-body motion).

2.4 Equations of motion

The secular perturbing function for the two-satellite motion is

$$R = \sum_{\nu=1}^{2} \frac{J_2 m_\oplus R_\oplus^2 \left(2 - 3\sin^2(i_\nu)\right)}{4a_\nu^3 \left(1-e_\nu^2\right)^{3/2}} + \epsilon < \Delta > . \tag{18}$$

The usual Lagrange equations for the orbital elements a, e, i, ϖ, Ω and mean anomaly M can be written in the form

$$\begin{aligned}
\dot{a} &= \frac{2}{na}\frac{\partial R}{\partial M}, \\
\dot{e} &= \frac{-\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega} + \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial M} \\
\dot{i} &= \frac{-1}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial \Omega} + \frac{\cos(i)}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial \omega} \\
\dot{\Omega} &= \frac{1}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial i} \\
\dot{\omega} &= \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} - \frac{\cos(i)}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial i} \\
\dot{M} &= n - \frac{2}{na}\frac{\partial R}{\partial a} - \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial e} \qquad (19) \\
& \qquad (20)
\end{aligned}$$

Since the semi-major-axes a_1 and a_2 are equal on the average, we write a for this average, and $n = 1/(a\sqrt{a/m_\oplus})$ for the corresponding mean motion.

The derivative of θ reads

$$\dot{\theta} = \dot{\lambda}_2 - \dot{\lambda}_1 = \dot{M}_2 + \dot{\omega}_2 - \dot{M}_1 - \dot{\omega}_1. \tag{21}$$

Due to the smallness ($O(J2)$) and similarity ($\dot{\omega}_2 \approx \dot{\omega}_1$) of the perturbations the main term in the equation for $\dot{\theta}$ can be taken as

$$\dot{\theta} \approx n_2 - n_1 \approx -\frac{3}{2}n\alpha \tag{22}$$

where $\alpha = (a_2 - a_1)/a$ and because of the expression (above) for $\dot{a}$ and the fact that M is present only in $\theta = M_2 - M_1 + \omega_2 - \omega_1$, we have

$$\ddot{\theta} \approx -\frac{3}{a^2}(\frac{\partial R}{\partial M_2} - \frac{\partial R}{\partial M_1}) = -\frac{6}{a^2}\frac{\partial R}{\partial \theta}, \tag{23}$$

in which the last equality follows from the fact that $\frac{\partial R}{\partial M_1} = -\frac{\partial R}{\partial M_2} = \frac{\partial R}{\partial \theta}$. Note that the inclusion of the $\frac{\partial R}{\partial M_1}$-term is due to our assumption that both satellites have a thruster. At this point one sees that the J_2 term has no effect in the motion of θ. However it has a small indirect effect because the orbits are not Keplerian and the secular theory is not accurate enough. These facts are illustrated in Fig. 2 in which one can see the effect of J_2 as compared with the pure two-body motion. In these experiments the thrusters perturbed the satellite motions in the same way.

In addition we really have the term $\ddot{\omega}_2 - \ddot{\omega}_1$, but here the terms nearly cancel each other and they are very small $=O(J_2^2)$, so that the equation (23) is a good approximation.

The secular perturbation due to the thruster effect can now be obtained by averaging Δ over the angle w as

$$\langle R_{12} \rangle = \frac{1}{2\pi}\int_0^{2\pi} R_{12}(\Delta(w))\, dw \tag{24}$$

As said above, for constant mutual force $R_{12} = \epsilon\Delta$. Due to the structure of the expression for Δ^2 it is clear that $\langle R \rangle$ is an even function of θ, i.e.

$$\langle R_{12} \rangle = R_{12}^{(0)} + R_{12}^{(2)}\theta^2/2 + .., \tag{25}$$

Averaging over the angle w in Eq.(16) and expanding in powers of θ one gets the secular approximation

$$\langle R_{12} \rangle \quad \approx \quad \epsilon a(1.542\widetilde{e} + 0.1426\theta^2/\widetilde{e} + O(\theta^4)), \tag{26}$$

These equations are naturally valid only to second order in θ since that is the case for Eq.(16). For the equation of θ-motion we have

$$\ddot{\theta} = -\frac{6}{a^2}\frac{\partial R_{12}}{\partial \theta} \approx \frac{-1.711\epsilon}{a\widetilde{e}}\theta, \tag{27}$$

which shows that the θ-motion is harmonic oscillation in this approximation. In Fig. 4 the periods are compared for the different models (using Eq.(11), the J_2 only model and pure two-body motion) and the results are quite close to each other, with only a few percent differences. Thus one may conclude that the $R_{12} = \epsilon\Delta$ perturbation mainly determines the motion of the mean longitude difference θ.

In general it is interesting to consider the effect of different kinds of mutual interactions. For a general power n of the distance one may write the average of $R = \epsilon_n\Delta^n$ as

$$\langle R \rangle = \epsilon_n \frac{1}{2\pi}\int_0^{2\pi} \Delta^n(w)\, dw. \tag{28}$$

The sign of the second derivative of $\langle R \rangle$, with respect to θ, tells if the mean longitude difference θ can stay small. In fact is turned out, somewhat unexpectedly, that the second derivative is a positive number for all nonzero powers n, positive or negative. This result is illustrated in Fig.7. Thus the tandem flight seems possible with any mutual acceleration of the satellites if the force is derivable from a potential of the form $\epsilon_n\Delta^n$ for ($n \neq 0$) where $\epsilon_n > 0$. In passing we mention that the perturbing function can in general be anything of the form $R = \sum_{n=-\infty}^{+\infty} c_n\Delta^n$, which may be a finite, or at least convergent, expression still giving positive second derivative [$R_{12}^{(2)}$ in (25)] at $\theta = 0$. This happens at least if the coefficients c_n are non negative.

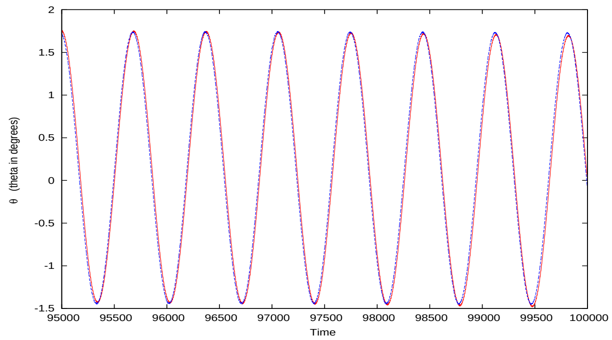

Figure 2. Oscillation of the angle θ in two models: with the entire Earth potential and with J_2-term only. After about 2.5 years (which corresponds near the middle of this figure, in which the total time-span is about 1.5 months) the oscillations are still almost the same within plotting precision. Here $i = 30^o$, $\delta a_0 = 1km$.

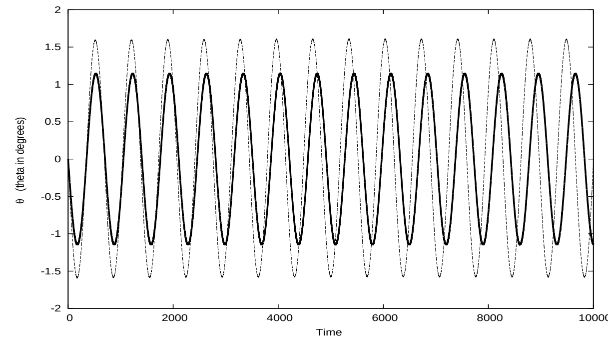

Figure 3. Oscillation of the angle θ in two models: with the entire Earth potential and with pure two-body model. Here the time-span is about three months and the inclination and semi-major-axis difference are correspondingly $i = 30^o$, $\delta a_0 = 1km$.

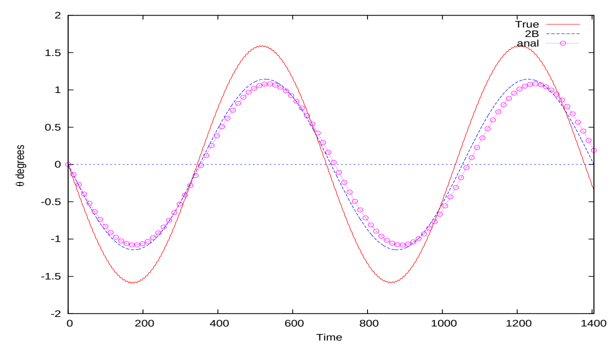

Figure 4. In the different approximations the θ periods are somewhat different, but no more than one could expect. The 'True' curve is from our most accurate modelling (all perturbations included), the '2B' means two-body motion model without any of the Earth harmonics and 'anal' points to the result computed using the above equation (27). The differences in the period length are only a few percent.

3 NUMERICAL EXPERIMENTS

In this section we first discuss the results with two-satellite experiments and later this is extended to three satellites moving essentially in a (flattened) triangle configuration.

Our numerical experiments confirm clearly the main point of the theory part: the mean longitude difference θ fluctuates around zero and remains small. The amplitude depends on the difference in the initial values of the semi-major-axis, but the effect of the thruster makes this difference fluctuate around zero according to the equation (27) and so the mean values are same. Due to the fact that the two orbits are very similar, all the perturbations in them are similar and do not affect noticeably the relative motion of the satellites.

The initial values in the numerical experiments were produced using two-body formulae. We set initially the inclinations $i = 10°, 30°, 50°, 70°, 90°$ and longitudes of ascending nodes Ω to same value for the satellites. For the semi-major-axes we used typically one kilometer difference in the initial axis value. The initial eccentricities were set to the values of $e = 0.01$. The differences in the initial positions of the satellites were obtained by adding $180°$ to the value of ω and subtracting that amount from the mean anomaly M. For the experiments with three satellites we used $(k-1)*120°$ $(k = 1,\ 2,\ 3)$ ω differences and removed an equal amount from M. These operations made the mean longitudes equal in the beginning.

3.1 Two satellites

The approximate analytical theory suggest that to study the relative motion of the satellites and the correctness of the theory, it is enough to select some small eccentricity and a small thruster power. Scaling for other values is easy using the theory. In this section we present some results of numerical experiments and display them graphically.

We used for eccentricity the value $e = .01$ and initial semi-major-axis difference of 1km. The size of the radial oscillation is about $2ea$ as one derives from the two-body motion and the along-orbit motion is twice this in the coordinate system in which one of the satellites is in the origin and the coordinate system rotates with that satellite.

In Fig.(2) we compare the oscillations of the θ angle in a system with all the Earth potential term [Eq.(11)] with the model that includes only the J_2-term. The time span shown in the figure corresponds approximately to 1.5 months in the end of a total interval of nearly 2.6 years. We see that the θ motion is still very similar in both models after this long

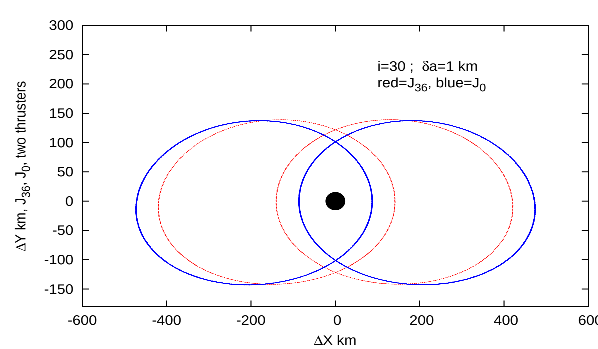

Figure 5. Comparison of relative orbits at extreme θ-values for the case of full perturbations (red) and in the two-body model. Here $i = 30^o$, $\delta a_0 = 1km$.

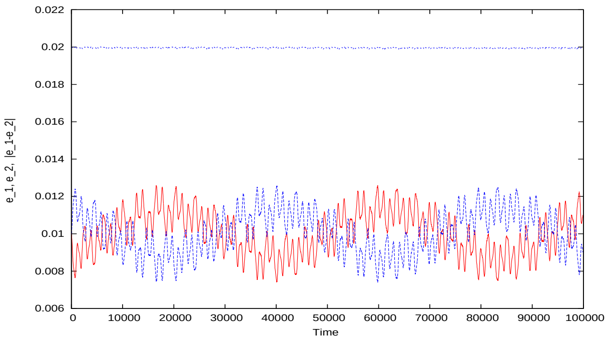

Figure 6. Evolution of individual eccentricities and the relative eccentricity. While the individual eccentricities e_k fluctuate considerably, the relative one $\widetilde{e} = |\mathbf{e}_1 - \mathbf{e}_2|$ remains almost constant. In this example the initial semi-major axis difference was one kilometer, $e_k = .01$ and ω-difference was set to be $= 180^o$.

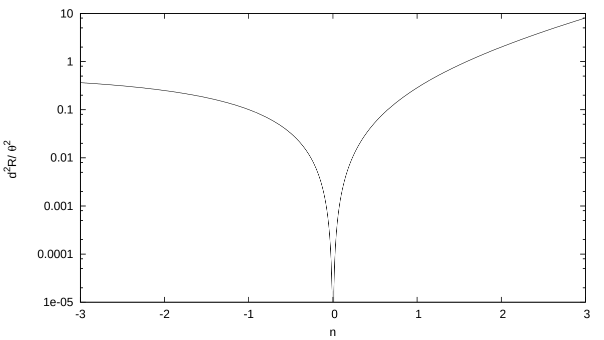

Figure 7. Second derivative of $e^{2-n}\langle\Delta^n\rangle$ with respect to θ (at $\theta = 0$) as function of the power n. One sees that it is positive for all non-zero powers n, thus predicting possibility for stable motion near each other.

integration. Thus the effect of the potential terms higher than J_2 seems minor at least in the θ angle.

Figure (3) we compare the θ motion in the full Earth potential and pure two-body motion for about three moths. There is clear difference in amplitude and also a phase difference. Due to the similarity of the motions one may conclude that these differences are mainly due to the J_2 term that makes the energy values of the satellites different and the initial derivative values, as computed using the two-body formulae, somewhat erroneous. In any case the differences are not very large and thus the two-body motion based estimates remain qualitatively correct.

The Fig.(4) compares θ behavior for a shorter time in the two models as above with the analytical results following from the Eq.(27). One can see that the analytical approximation predicts the period with a few percent accuracy. One concludes that the analytical estimates, although based only on the Keplerian motion formulae, are useful approximations.

The difference in the relative motion curves in the full perturbation and the two-body model are compared in Fig.(5). Again the differences remain acceptably small.

In the theory the relative eccentricity $\widetilde{e} = |\mathbf{e}_1 - \mathbf{e}_2|$ is an important quantity. In Fig.(6) we demonstrate that this quantity is almost constant all the time although the individual eccentricities in the satellite orbits vary considerably.

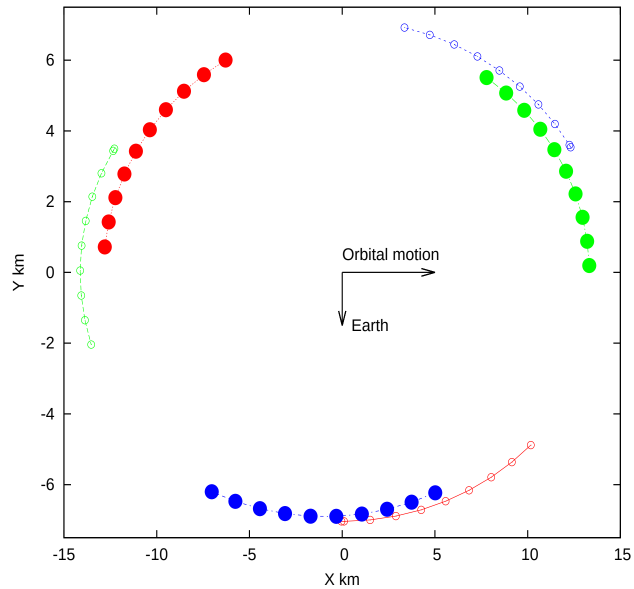

Figure 8. Three satellites keep an almost triangle configuration from the start to the end of this simulation after about 2.6 years. In this system the motion perpendicular to the mean orbital plane is only $\leqslant 1\%$ of the size of the system. The colors show the identity of the satellites and size of dots go from small to bigger according to time.

This result is the same what was observed long ago in quasi-satellite orbits (Mikkola et al. 2006). This numerical result is quite complicated to prove analytically and we ignore such considerations.

We computed the second derivative of $\langle\Delta^n\rangle$ with respect to θ for different powers n. The result is that this quantity is positive for any value of n, positive or negative. Thus one may conclude that any mutual force between the satellites that are derivable from a power-potential could result into stable quasi-satellite like motion. In fact there are many other such forces since at least any sum of power-potentials with positive coefficients would do here. In practice, however, the simple constant force is probably the most useful for real satellites. The only interesting point here is that in the case of real world quasi-satellite motion the force is r^{-2} (plus the indirect term which has only a small effect). Thus the dynamics with mutual forces, derivable from any power potential, has similar features.

In Figures (9) – (12) the relative motions of the satellites are illustrated by plotting it at every maximum of $|\theta|$ for one period around the Earth. The integration lasted over the time interval up to 10^5 time units in our system. This corresponds to about 2.6 years for near Earth satellites. The results look like just one plotted curve, but are actually more that 1400 curves. Thus one sees the stability of the system. The results are very similar in the full potential model (marked J_{36}, which includes also Luni-Solar terms) and the J_2 models with one or two thrusters. (One thruster meaning that only one of the satellites has a thruster). In the Fig.(12). the motion with respect to the osculating orbital plane is illustrated for the full-potential two-thruster computation. Here clearly (like in fact in all the cases) this xz-motion motion is very small compared with the xy-motion.

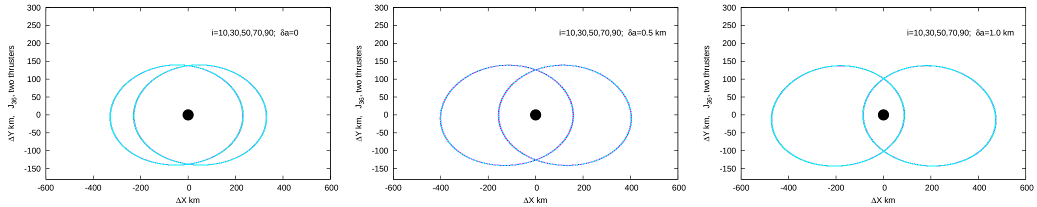

Figure 9. Potential up to J_{36}, xy-motion, two thrusters. The number of curves is larger than 1400.

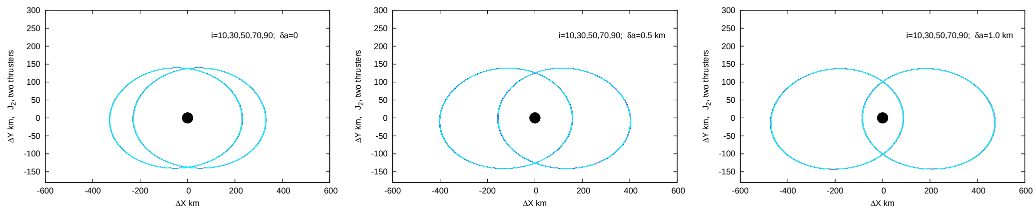

Figure 10. Potential only to J_2, xy-motion, two thrusters. The number of curves is larger than 1400.

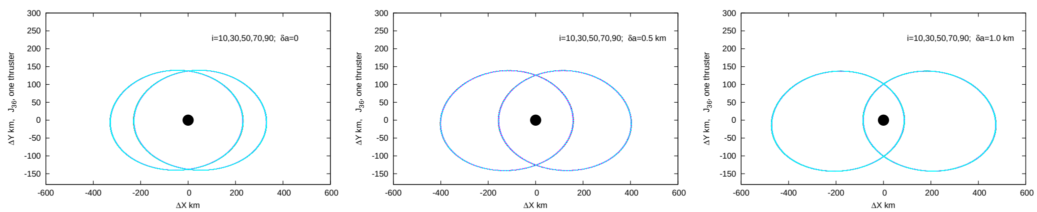

Figure 11. Potential up to J_{36}, xy-motion, one thruster. The number of curves is larger than 1400.

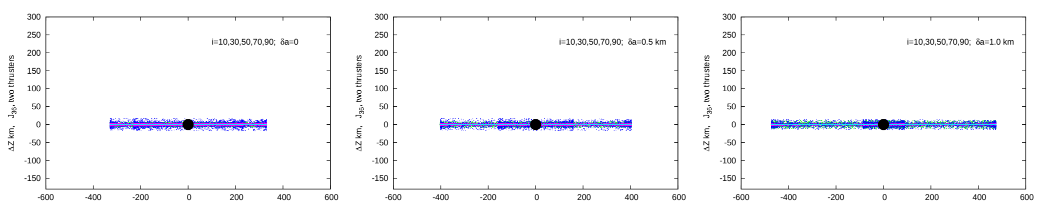

Figure 12. Potential up to J_{36}, xz-motion, two thrusters. The number of curves is larger than 1400.

3.2 More satellites

For three satellite experiments we used the initial values as explained in the beginning of this section. The thruster accelerations for all the satellites were directed according to the mean direction of the two other satellites. In some experiments we randomized these by modifying the direction cosines randomly by 10% (and then we renormalized the direction vector). These operations did not have any noticeable effect.

The result was that the satellites move in a kind of triangle configuration. Actually the orbits with respect to the center of the triple are approximately ellipses, but if the scale of the variation of the distance from the Earth is scaled by a factor of 2 the configuration looks like a circle. This is illustrated in Fig. 8 where the smaller dots represent the situation in the beginning and the larger ones show it after about 2.6 years.

Very similar results were obtained also with more satellites, for example systems of six satellites seems to behave in a similar way. Thus it seems possible to keep a cluster of satellites close to each other just by using constant repulsive forces. This is probably simpler and other thinkable ways for formation flight of satellites. However this method just provides a way to keep satellites in a kind of elliptic like configuration.

4 DISCUSSION AND CONCLUSION

The motions of the satellites in low Earth orbits are essentially just elliptic motions so that the relative motions are due to differences in two-body motion. Our numerical experiments demonstrate that the tandem flight has the effect that the perturbations from the Earth harmonics, which are orders of magnitude larger than the needed thruster force, affect the motions of the satellites in the same way and do not make much difference in their relative motion. This was also confirmed to be the case when high order expansion for the Earth gravitational potential (equation (11)) is used. In other words the relative motion of satellites is mainly due to eccentricities so that in the orbital coordinate system they seem to move approximately along a small ellipse in which the axis ratio is $1:2$. The only thing the thrusters must do is to keep the mean longitudes nearly same. For this purpose very small thrusts are enough when the orbits of the satellites are close to each other. Little differences in the orbits are, however, not important as we found with simulations.

The method woks as well for two or three or even more satellites.

Surprisingly even near triangle configuration (in case of three satellites) is stable in the sense that the relative motion of the satellites keep the same configuration for very long times.

In our experiments, with the constant acceleration model, the value of relative acceleration of order $\epsilon = 10^{-6}g$ [g=the standard gravity value ($\sim$ 10m/sec^2)] is enough. Writing this in physical units we can say that $\epsilon \sim 10^{-5}$m/sec^2 = .01mm/sec^2 is enough, but even smaller values may do. Only if the initial difference in semi-major-axis is more than a kilometer or so, a somewhat stronger force is required, but even in such cases the forces needed are surprisingly small. In any case, the required acceleration is easy to find by numerical simulations. The experiments with the high order expansions, which contains the relevant perturbations for the satellite motion, leads to same results as the ones with only the J_2 term, at least within the precision of our illustrations. This result suggests that the relative motion under the action of the thruster force is not sensitive to perturbations. As mentioned in section 3.2, even somewhat randomized directions of the thruster force does not have an effect in the relative motion of the satellites.

One advantage of this method is that the satellites need only information about the directions of other satellite(s).

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