arXiv:1601.00076v1 [hep-th] 1 Jan 2016

Bounds on topological Abelian string-vortex and string-cigar from information-entropic measure

R. A. C. Correa*
CCNH, Universidade Federal do ABC (UFABC), 09210-580, Santo André, SP, Brazil

D. M. Dantas†
Universidade Federal do Ceará (UFC), 60455-760, Fortaleza, CE, Brazil

C. A. S. Almeida‡
Universidade Federal do Ceará (UFC), Departamento de Física, 60455-760, Fortaleza, CE, Brazil

Roldão da Rocha§
Centro de Matemática, Computação e Cognição,
Universidade Federal do ABC (UFABC), 09210-580, Santo André, SP, Brazil

In this work we obtain bounds on topological Abelian string-vortex in six dimensions using a new measure of configurational complexity known as configurational entropy. In this way, the information-theoretical measure of six dimensional braneworlds scenarios are capable to probe situations where the parameters responsible for the thickness are arbitrary. The so-called Configurational Entropy (CE) selects the best value of the parameter in the model. This is accomplished by minimizing the CE, namely, by selecting the most appropriate parameters in the model that correspond to the most organized system, based upon Shannon information theory. This information-theoretical measure of complexity provides a complementary perspective to situations where strictly energy-based arguments are inconclusive. We show that the higher the energy the higher the configurational entropy, what shows an important correlation between the energy of the a localized field configuration and its associated entropic measure.

Keywords: topological Abelian string-vortex, six-dimensional braneworld models, configurational entropy

I. INTRODUCTION

In 1948, in a beautiful and seminal work, Shannon [1] described what it was called "A mathematical theory of communication", which is currently known as "information theory". In that work, Shannon introduced a framework capable of solving the most fundamental problem of communication: the information transmission. The main goal of the information theory in Ref. [1] was to introduce the concepts of entropy and mutual information using the communication theory. In this context, the entropy was defined as a measure of "randomness" of a random phenomenon. Thus, if a little deal of information concerning a random variable is received, the uncertainty decreases. Hence we can measure the decrement in the uncertainty, that can be related to the quantity of transmitted information. This quantity is the so-called mutual information. After that work, a large number of communication systems have been comprehensively analyzed from the information theory, where the various types of information transmission can be studied in the same framework.

Inspired by Shannon, Gleiser and Stamatopoulos (GS) latterly introduced a measure of complexity of a localized mathematical function [2]. GS proposed that the Fourier modes of square-integrable, bounded mathematical functions can be used to construct a measure of what they called configurational entropy (CE): a configuration consisting of a single mode has zero CE (a single wave in space), whereas that one where all modes contribute with equal weight has maximal CE. In order to apply such ideas to physical models, GS used the energy density of a given spatially-localized field configuration, found from the solution of the related partial differential equation (PDE). GS pointed out that the CE can be used to choose the best-fitting trial function in situations where their energies are degenerate.

Although it was recently proposed, the CE has been already employed to acquire the stability bound for compact objects [4], to investigate the non-equilibrium dynamics of spontaneous symmetry breaking [3], to study the emergence of localized objects during inflationary preheating [5] and to discern configurations with energy-degenerate spatial profiles [6]. Moreover, solitons were studied in a Lorentz symmetry violating (LV) framework with the aid of CE [7–10]. The CE for travelling solitons evinces that the most appropriate value of the parameter responsible for breaking the Lorentz symmetry is that one corresponding to the energy density distributed symmetrically with respect to the origin. In this context, the CE associated to travelling solitons in LV frameworks plays a prominent role in probing systems wherein the

* rafael.couceiro@ufabc.edu.br
† davi@fisica.ufc.br
‡ carlos@fisica.ufc.br
§ roldao.rocha@ufabc.edu.br

parameters are somehow arbitrary. Hence the CE determines the value of the parameter in models that present the best physical consistency, providing an additional physical constraint in a physical system. Furthermore, an additional interesting work [11] showed that the CE identified the critical point in the context of continuous phase transitions. Moreover, the CE can be used to measure the informational organization in the structure of the system configuration for some five-dimensional (5D) thick scenarios. In particular, the CE played an important role to decide the most appropriate intrinsic parameters of sine-Gordon braneworld models [12]. Quite interestingly, CE has been studied both in $f(R)$ [13] and $f(R,T)$ [14] theories of gravity. In next, we present a brief discussion of 5D braneworld models to treat the CE in six-dimensional (6D) scenarios.

On the other hand, Randall-Sundrum (RS) models [15, 16] propose a warped braneworld model, wherein the gauge hierarchy problem is explained and the gravity zero mode is localized as well, reproducing four-dimensional (4D) gravity on the brane. The 5D bulk gravitons provide a small correction in the Newton law [16]. However, this thin model presents singularities and drawbacks concerning the non localization of spin 1 gauge fields and fermions of spin 1/2 and 3/2 as well [17]. To solve these problems some thick models were proposed. For a comprehensive review, see Ref. [18].

Soon after the works of RS and thick 5D models, an axially symmetric warped 6D model was proposed by Gergheta-Shaposhnikov [19], called *string-like defect*. This model represents a local string defect constructed as a warped product between a 3-brane placed at origin and a 2-cycle representing the transverse space. This scenario further provides the resolution of the mass hierarchy. Moreover, it has some advantages, like a smaller correction to the Newtonian potential [19] and the non requirement of fine tuning between the bulk cosmological constant and the brane tension, for the cancellation of the 4D cosmological constant [19]. Besides, the localization of gauge zero modes is spontaneous even in the thin brane case if any scalar field interaction is required [20, 21]. On the other hand, fermions fields are trapped through a minimal coupling with an $U(1)$ gauge background field [22, 23]. Later, other 6D models, spherically symmetric, were employed to explain the generations of fundamental fermions [24, 25] and the resolution of the mass hierarchy of neutrinos as well [26].

Nevertheless, the Gergheta-Shaposhnikov model is a thin model based on a exterior solution of a string. The regularity conditions in the thin core limit leads to a non-dominant energy condition on the stress-energy tensor [27]. Due to it, some 6D thick models were proposed to handle these energy conditions [28–41]. In Ref. [28], a topological abelian Higgs vortex was used to construct a regular scenario in which the dominant energy conditions hold, however solely numerical solutions have been found. Similarly, Ref. [31, 32], looking for an exact vortex solution, shows that the energy density and the angular pressure are similar. The weak energy condition is likewise verified for the Resolved Conifold scenario [37–39], when the resolution parameter is larger. For the String-Cigar [33–36], the transverse space is represented by a cigar soliton, which is a stationary solution for the Ricci flow [43–45]. In this String-Cigar scenario the dominant energy condition holds still and the maxima of energy and pressures are displaced from the origin, as further numerically observed in Ref. [28].

Therefore, in this paper we investigate the entropic measure both in the Torrealba topological Abelian string (TA) [31, 32] and String-Cigar (HC) [33–36] in 6D scenarios, due to analytic properties of these scenarios. In particular, one of the main aims of our work is to find bounds for 6D string defects based upon the CE concept. In this way, we can establish a value for the thickness of the configuration responsible for minimizing the CE. As a consequence, we have a structure with minimal energy, and the field configuration has a more ordered state in an informational sense, in analogy with the Shannon information theory.

This paper is organized as follows: in Section II a briefly review string-like defects is present, whereas in Section III the CE bounds the parameters of TA and HC scenarios. We expose the conclusions and perspectives in Section IV.

II. STRING-LIKE DEFECT IN WARPED SIX DIMENSIONS

In succinct form, the string-like spacetime is composed by a 3-brane ($\mathcal{M}_4$) and a 2-dimensional (2D) transverse cycle ($\mathcal{M}_2$), where $\mathcal{M}_6 = \mathcal{M}_4 \times \mathcal{M}_2$ [35]. We work in this letter with asymptotically anti-de Sitter (AdS_6) spaces.

The 6D Einstein equation is expressed by [19, 20]

$$R_{MN} - \frac{R}{2} g_{MN} = \kappa \left(\Lambda g_{MN} + T_{MN} \right) , \qquad (1)$$

where R_{MN} represents the Ricci tensor, R denotes the scalar curvature, g_{MN} is the metric tensor, Λ denotes the 6D (negative) cosmological constant and T_{MN} stands for the 6D energy-momentum tensor. The 6D gravitational constant $\kappa = \frac{8\pi}{M_6^4}$ is related to the bulk 6D Planck scale.

A metric *ansatz* for 6D string-like models reads [19, 20]

$$ds_6^2 = \sigma(r)\eta_{\mu\nu}dx^\mu dx^\nu - dr^2 - \gamma(r)d\theta^2 \qquad (2)$$

where the signature for the $\mathcal{M}_4$ Minkowski metric $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$ is adopted hereupon. The radial coordinate is limited to $r \in [0, \infty)$, whereas the angular coordinate is restricted to $\theta \in [0, 2\pi)$. The $\sigma(r)$ represents the dimensionless warp factor and $\gamma(r)$ has length squared dimension.

The 4D Planck mass (M_4) and the bulk Planck mass (M_6) are related through the volume of the transverse of space as [19, 33, 35, 36]:

$$M_P^2 = 2\pi M_6^4 \int_0^\infty \sigma(r)\sqrt{\gamma(r)}dr \ . \qquad (3)$$

In addition, the energy-momentum tensor $T_M^N = diag\,(t_0, t_0, t_0, t_0, t_r, t_\theta)$ components are given by [19, 33]

$$t_0(r) = -\frac{1}{\kappa}\left(\frac{3\sigma''}{2\sigma} + \frac{3\sigma'\gamma'}{4\sigma\gamma} + \frac{\gamma''}{2\gamma} - \frac{\gamma'^2}{4\gamma^2}\right) - \Lambda, \quad (4a)$$

$$t_r(r) = -\frac{1}{\kappa}\left(\frac{3\sigma'^2}{2\sigma^2} + \frac{\sigma'\gamma'}{\sigma\gamma}\right) - \Lambda, \quad (4b)$$

$$t_\theta(r) = -\frac{1}{\kappa}\left(\frac{2\sigma''}{\sigma} + \frac{\sigma'^2}{2\sigma^2}\right) - \Lambda, \quad (4c)$$

where the prime denotes the derivative with respect to the radial coordinate r.

To obtain a regular geometry we have the conditions [19, 28, 33, 42]

$$\sigma(r)\Big|_{r=0} = cte, \qquad \sigma'(r)\Big|_{r=0} = 0,$$
$$\gamma(r)\Big|_{r=0} = 0, \qquad \left(\sqrt{\gamma(r)}\right)'\Big|_{r=0} = 1\,. \quad (5)$$

Theses conditions are also correlated to the energy conditions.

For the vacuum solution, the warp factor for the *String Like Defect* (SD) is proposed as [19–23]:

$$\sigma_{SD}(r) = \mathrm{e}^{-cr}, \quad \gamma_{SD}(r) = R_0^2\sigma_{SD}(r) \quad (6)$$

where the parameters $c = \sqrt{\frac{2\kappa}{5}(-\Lambda)} > 0$ connects the Newtonian constants and the cosmological constant in 6D. And R_0 is the radius of compactification of the angular coordinate. Hence in the limit that $r \to 0$, only the first condition of Eq. (5) holds. Besides, the mass hierarchy of Eq. (3) in this model yields

$$M_P^2 = \frac{4\pi R_0}{3c}M_6^4\,. \quad (7)$$

The result of $M_p \gg M_6$ is verified when $c \to 0$.

Following the perspective pointed by Ref. [19], Giovannini in Ref. [28] adopts a 6D action where the matter Lagrangian is an Abelian-Higgs model and the transverse space obeys the Abrikosov-Nielsen-Olesen *ansatz* [28, 31, 32]:

$$\phi(r,\theta) = v f(r)\,\mathrm{e}^{-il\theta} \qquad l \in \mathbb{Z}\,, \quad (8)$$

$$\mathcal{A}_\theta(r) = \frac{1}{q}\left[l - P(r)\right], \quad \mathcal{A}_\mu = \mathcal{A}_r = 0\,, \quad (9)$$

where ϕ and $\mathcal{A}_M$ are scalar and gauge fields, respectively. The condition $v = 1$ is a length dimension L^{-2} constant. The functions $f(r)$ and $P(r)$ are such that $f(r \to 0) = 0$, $f(r \to \infty) = 1$ whereas $P(r \to 0) = l$ and $P(r \to \infty) = 0$.

From constraints by this *ansatz* and the regular conditions in the Eq. (5), the solutions of fields and warp factors are numerically obtained in Ref. [28]. On the other hand, by imposing conditions on the function $P(r) \equiv 0$, Torrealba [31, 32] obtained an analytical solution, named *Topological Abelian Higgs string* (TA):

$$\sigma_{TA}(r) = \cosh^{-2\delta}\left(\frac{\beta r}{\delta}\right), \quad \gamma_{TA}(r) = R_0^2\sigma_{TA}(r)\,, \quad (10)$$

where $\beta = \frac{c}{2} = \sqrt{\frac{(-\Lambda)\kappa}{10}}$, being δ a thickness parameter which, for small values, reproduces the Gergheta-Shaposhnikov model in Eq. (6). Moreover, Ref. [31] concludes that for the localization of gauge fields zero mode the thickness of the model can not exceed the value

$$\delta < \frac{5\beta}{4\pi}q^2v^2\,. \quad (11)$$

Now, in the TA (10) string, two of the conditions (5) are verified.

In another approach, the transverse space can also be built for a cigar soliton solution of Ricci flow [33–36]

$$\frac{\partial}{\partial\lambda}g_{MN}(\lambda) = -2R_{MN}(\lambda)\,, \quad (12)$$

with λ being a metric parameter Ref. [33–36] constructed the geometry named *Hamilton String Cigar* (HC) where the warp factors read

$$\sigma_{HC}(r) = \mathrm{e}^{-cr+\tanh(cr)}, \quad \gamma_{HC}(r) = \frac{\tanh^2 cr}{c^2}\sigma_{HC}(r)\,. \quad (13)$$

In this case, all conditions of Eq. (5) do hold. The relation present in Eq. (3) that $c \to 0$ is also valid in HC model.

To observe the correspondence between the regular condition in Eq. (5) and the energy momentum tensor we plot the $\sigma(r)$ the warp factors (6), (10) and (13) in Fig. 1 and $\gamma(r)$ in Fig. 2, whereas the energy momentum tensor in Fig. II for TA and HC in Fig. 4. Concerning the HC scenario, wherein all metric conditions (5) hold, the dominant energy condition $t_0 \geq |t_i|$, $(i = r, \theta)$ [29, 30, 40] is satisfied.

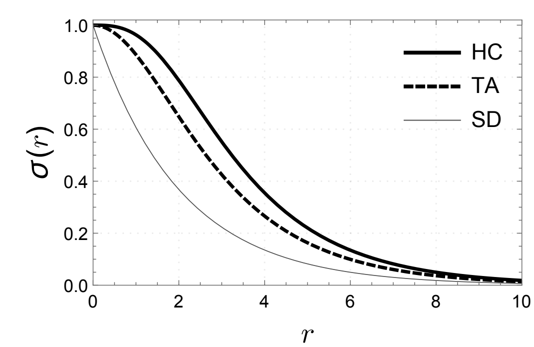

FIG. 1. $\sigma(r)$ warp-factors with $c = 2\beta = 0.5$ and $\delta = 0.5$. In the TA (dashed lines) and HC model (thick lines) the regularity conditions (5) are satisfied for this factor.

In the next section, we shall analyze the vortex-string models from the CE point of view.

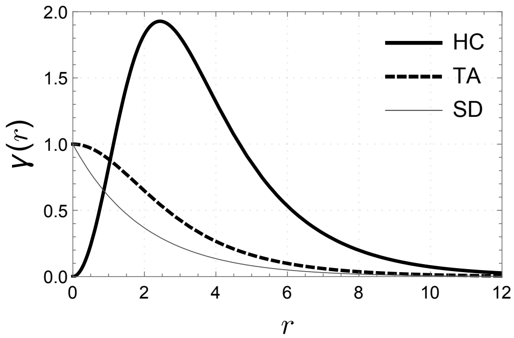

FIG. 2. $\gamma(r)$ angular factors with $c = 2\beta = 0.5$, $R_0 = 1$ and $\delta = 0.5$. Only in the HC model (thick lines) the regularity conditions (5) hold still.

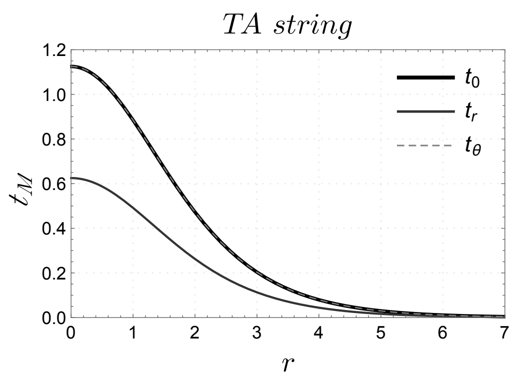

FIG. 3. $t_M(r)$ energy-momentum tensor in TA model with $\beta = 0.25$, $R_0 = 1$ and $\delta = 0.5$. The energy and angular component are identical.

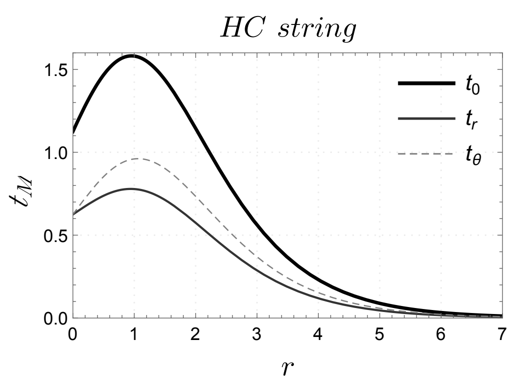

FIG. 4. $t_M(r)$ energy-momentum tensor in HC model with $c = 0.5$. The dominant energy condition is satisfied and the maxima of t_M are displaced from origin as also observed in Ref. [28].

III. CONFIGURATIONAL ENTROPY IN THE VORTEX-STRING SCENARIO

As previously argued in the Introduction, the so-called Configurational Entropy (CE) [2] represents an original concept employed to quantify the existence of non-trivial spatially localized solutions in field configuration space. The CE allows to set up prominent correlations on the parameters of several models. For instance, in a very recent work by Gleiser and Jiang [47], bounds on the stability of various self-gravitating astrophysical objects were obtained. The information-entropic measure moreover states bounds in LV scenarios [7], in compact objects like Q-balls [4], and in modified theories of gravity as well [13].

Here, it is worth to emphasize that the CE corresponds to an extension of Shannon's information entropy [1] to localized energy configurations based upon their respective Fourier transforms. In fact, by considering a set of fields modes, it is possible to acquire information regarding the system configuration. Hence the system informational content can be quantified. As showed in Ref. [2], there is an important correspondence between CE and the energy of a localized field configuration, where low energy systems are correlated with small entropic measures.

Thus, armed with the ideas introduced by GS [2] regarding information-entropic measure, in this section we apply the concept of CE to refine the values of thickness parameters δ in the analytic string vortex model.

The CE can be obtained [2] by the Fourier transform of the energy density $t_0(r)$ [12, 13], yielding

$$\mathcal{F}(\omega) = -\frac{1}{\sqrt{2\pi}} \int_0^\infty t_0(r)\, \mathrm{e}^{i\omega r}\, dr.$$

At this point, it is important to remark that we will consider structures with spatially localized energy densities, which are square-integrable bounded functions $t_0(r) \in L^2(\mathbb{R})$. Hence, using the Plancherel theorem, it follows that

$$\int_0^\infty |\mathcal{F}(\omega)|^2 d\omega = \int_0^\infty |t_0(r)|^2\, dr. \tag{14}$$

Now, the so-called modal fraction [2–4, 6, 7, 12, 13] is given by the following expression

$$f(\omega) = \frac{|\mathcal{F}(\omega)|^2}{\int_0^\infty d\omega |\mathcal{F}(\omega)|^2}\,.$$

Next, the normalized modal fraction is defined as the ratio of the normalized Fourier transformed function and its maximum value f_{max}:

$$\tilde{f}(\omega) = \frac{f(\omega)}{f_{max}}.$$

Now, the CE was motivated by the Shannon's information theory [2]. Indeed, the CE was originally defined by $S_C[f] = -\sum f_n \ln(f_n)$, describing an absolute limit of the best lossless compression of any communication [1]. Hence the CE originally provides the informational content of configurations compatible to bounds on any physical system. A number N of modes having equal weight yields $f_n = 1/N$ and hence the discrete CE has a maximum at $S_C = \ln N$. Instead, if the system is constituted by just one mode, then $S_C = 0$ [2]. Thus, a localized and continuous function $\tilde{f}(\omega)$ yields the following definition for the CE:

$$S(\tilde{f}) = -\int_0^\infty d\omega \tilde{f}(\omega)\ln\left[\tilde{f}(\omega)\right]. \qquad (15)$$

Therefore, we now use this concept to obtain the CE in the Abelian string-vortex and the string-cigar contexts. By substituting the warp factor (10) in the energy density given by Eq. (4a), it yields

$$t_0(r) = \left(\frac{5}{2} + \frac{1}{\beta}\right)\left[2\beta\,\text{sech}\left(\frac{\beta r}{\delta}\right)\right]^2. \qquad (16)$$

It represents a localized density of energy, as can be verified in Fig. II. Now, the Fourier transform of Eq. (16) reads

$$\mathcal{F}(\omega) = \sqrt{2\pi}\delta\omega(5\delta + 2)\text{csch}\left(\frac{\pi\delta\omega}{2\beta}\right), \qquad (17)$$

which is also a localized function and has the normalized modal fraction:

$$\tilde{f}(\omega) = \left[\frac{\pi\delta\omega}{2\beta}\ \text{csch}\left(\frac{\pi\delta\omega}{2\beta}\right)\right]^2. \qquad (18)$$

Hence the profile of CE in Eq. (15) for the function in Eq. (18) is presented in Fig. 5. It is verified that the maximum of CE occurs for $\delta_{crit} \approx 0.09\beta$. The intrinsic braneworld models parameters have been further constrained by analyzing the experimental, phenomenological and observational aspects in, e. g., [12, 46]. In particular, Ref. [12] provides a refined analysis wherein the CE further restricts the range parameters of a 5D sine-Gordon thick braneworld model, namely, the AdS bulk curvature and the braneworld thickness. Here this procedure is applied to the 6D braneworld models and the constraint Eq. (19) below can be obtained for the case here studied:

$$0.09\beta < \delta < 0.40\beta. \qquad (19)$$

Moreover, the maximum CE – corresponding to the best information organization of the analogue system corresponding to this braneworld model – imposes a critical thickness $\delta_{crit} \approx 0.09\beta$. Thus, we have the lower bounds of the thickness parameter provided by δ_{crit}, whereas an upper bound provided Eq. (11) is depicted in Eq. (19).

For the HC model, the density of energy of Eq. (4a) yields

$$t_0(r) = \frac{c^2}{\kappa}\left[7\text{sech}^2(cr) + \frac{13}{2}\tanh(cr)\text{sech}^2(cr)\right.,$$
$$\left.-\frac{5}{2}\text{sech}^4(cr)\right]. \qquad (20)$$

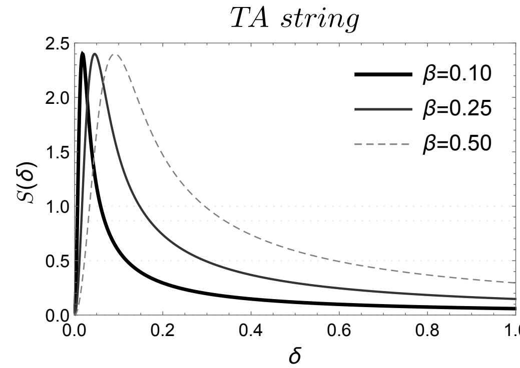

FIG. 5. $S(\delta)$ Configurational entropy for different values of the parameter β, as a function of the thickness parameter δ.

Again, the energy density is localized as can be verified in the Fig. 4. The Fourier transforms of the warp factors in Eq. (13) is moreover localized and its normalized modal fraction reads

$$\tilde{f}(\omega) = \frac{\pi^2\omega^2\left(4096c^4 + 881c^2\omega^2 + 25\omega^4\right)}{16384c^6}\text{csch}^2\left(\frac{\pi\omega}{2c}\right), \qquad (21)$$

The plot of $S(c)$ is present in Figure 6, where the relation of $c \to 0$ that solves the hierarchy problem in Eq. (3) is verified.

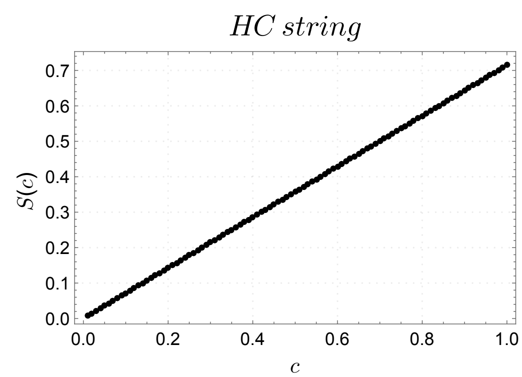

FIG. 6. $S(\delta)$ Configurational entropy of the HC string model, as a function of the parameter c.

IV. DISCUSSION AND CONCLUSIONS

In this work we have investigated the CE in the context of the topological abelian string-vortex and string-cigar scenarios. We have shown that the information-theoretical measure of 6D dimensional braneworld models opens new possibilities to physically constrain, for example, parameters that are related to the brane thickness. The CE provides the most appropriate value of this parameter that is consistent with the best organizational structure, from the Shannon information theory point of view. The information measure regarding the system organization is related to modes regarding the braneworld model. Hence the constraints of the parameters that we obtained for the TA and the HC string models provide the range of the parameters associated to the most organized braneworld models, with respect to the information content of these models, in the context of Shannon information theory. It provides further physical aspects to models where strictly energy-based arguments do not provide further conclusions of the physical parameters.

V. ACKNOWLEDGMENTS

The authors thank the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), and Fundação Cearense de apoio ao Desenvolvimento Científico e Tecnológico (FUNCAP) for financial support. DMD thanks to Projeto CNPq UFC-UFABC 304721/2014-0. RdR is grateful to CNPq grants No. 473326/2013-2 and No. 303027/2012-6, and to Fapesp Grant No. 2015/10270-0. RACC also acknowledges Universidade Federal do Ceará (UFC) for the hospitality.

[1] C. E. Shannon, The Bell System Technical J. **27**, 379 (1948).
[2] M. Gleiser and N. Stamatopoulos, Phys. Lett. B **713**, 304 (2012).
[3] M. Gleiser and N. Stamatopoulos, Phys. Rev. D **86**, 045004 (2012).
[4] M. Gleiser, D. Sowinski, Phys. Lett. B **727**, 272 (2013).
[5] M. Gleiser, N. Graham, Phys. Rev. D **89**, 083502 (2014).
[6] R. A. C. Correa, A. de Souza Dutra, and M. Gleiser, Phys. Lett. B **737**, 388 (2014).
[7] R. A. C. Correa, R. da Rocha, and A. de Souza Dutra, Ann. Phys. **359**, 198 (2015).
[8] A. de Souza Dutra and R. A. C. Correa, Phys. Rev. D **83**, 105007 (2011).
[9] R. A. C. Correa and A. de Souza Dutra, Adv. High Energy Phys. **2015**, 673716 (2015).
[10] R. A. C. Correa, R. da Rocha, and A. de Souza Dutra, Phys. Rev. D **91**, 125021 (2015).
[11] N. Stamatopoulos, M. Gleiser, Phys. Lett. B **747**, 125 (2015).
[12] R. A. C. Correa and R. da Rocha, Eur. Phys. J. C **75**, 522 (2015).
[13] R. A. C. Correa, P. H. R. S. Moraes, A. de Souza Dutra, and R. da Rocha, Phys. Rev. D **92**, 126005 (2015).
[14] R. A. C. Correa and P. H. R. S. Moraes, *Configurational entropy in $f(R,T)$ brane models* [arXiv: arXiv:1509.00732].
[15] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[16] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[17] B. Bajc, G. Gabadadze, Phys. Lett. B **474**, 282 (2000).
[18] V. Dzhunushaliev, V. Folomeev, and M. Minamitsuji, Rept. Prog. Phys. **73**, 066901 (2010).
[19] T. Gherghetta and M. E. Shaposhnikov, Phys. Rev. Lett. **85**, 240 (2000).
[20] I. Oda, Phys. Lett. B **496**, 113 (2000).
[21] I. Oda, Phys. Rev. D **62**, 126009 (2000).
[22] Y. -X. Liu, L. Zhao, and Y. -S. Duan, JHEP **0704**, 097 (2007).
[23] Y. X. Liu, L. Zhao, X. H. Zhang, and Y. S. Duan, Nucl. Phys. B **785**, 234 (2007).
[24] S. Aguilar and D. Singleton, Phys. Rev. D **73**, 085007 (2006).
[25] M. Gogberashvili, P. Midodashvili, and D. Singleton, JHEP **0708**, 033 (2007).
[26] J. M. Frère, M. Libanov, S. Mollet and S. Troitsky, JHEP **1308**, 078 (2013).
[27] P. Tinyakov and K. Zuleta, Phys. Rev. D **64**, 025022 (2001).
[28] M. Giovannini, H. Meyer, and M. E. Shaposhnikov, Nucl. Phys. B **619**, 615 (2001).
[29] R. Koley and S. Kar, Class. Quant. Grav. **24**, 79 (2007).
[30] L. J. S. Sousa, W. T. Cruz and C. A. S. Almeida, Phys. Rev. D **89**, 064006 (2014).
[31] R. S. Torrealba, Phys. Rev. D **82**, 024034 (2010).
[32] R. S. Torrealba, Gen. Rel. Grav. **42**, 1831 (2010).
[33] J. E. G. Silva, V. Santos, and C. A. S. Almeida, Class. Quant. Grav. **30**, 025005 (2013).
[34] F. W. V. Costa, J. E. G. Silva, D. F. S. Veras, and C. A. S. Almeida, Phys. Lett. B **747**, 517 (2015).
[35] D. F. S. Veras, J. E. G. Silva, W. T. Cruz, and C. A. S. Almeida, Phys. Rev. D **91**, 065031 (2015).
[36] D. M. Dantas, D. F. S. Veras, J. E. G. Silva, and C. A. S. Almeida, Phys. Rev. D **92**, 104007 (2015). D. M. Dantas, R. da Rocha and C. A. S. Almeida, arXiv:1512.07888 [hep-th].
[37] J. E. G. Silva and C. A. S. Almeida, Phys. Rev. D **84**, 085027 (2011).
[38] F. W. V. Costa, J. E. G. Silva, and C. A. S. Almeida, Phys. Rev. D **87**, 125010 (2013).
[39] D. M. Dantas, J. E. G. Silva, and C. A. S. Almeida, Phys. Lett. B **725**, 425 (2013).
[40] J. C. B. Araújo, J. E. G. Silva, D. F. S. Veras, and C. A. S. Almeida Eur. Phys. J. C **75**, 127 (2015).
[41] L. J. S. Sousa, C. A. S. Silva, D. M. Dantas, and C. A. S. Almeida, Phys. Lett. B **731**, 64 (2014).
[42] I. Navarro and J. Santiago, JHEP **0502**, 007 (2005).
[43] P. P. Orth, P. Chandra, P. Coleman, and J. Schmalian, Phys. Rev. Lett. **109**, 237205 (2012).
[44] N. Lashkari and A. Maloney, Class. Quant. Grav. **28**, 105007 (2011).
[45] M. Headrick and T. Wiseman, Class. Quant. Grav. **23**, 6683 (2006).
[46] J. M. Hoff da Silva and R. da Rocha, Europhys. Lett., **100**, 11001 (2012).
[47] M. Gleiser and N. Jiang, Phys. Rev. D **92**, 044046 (2015).