Inference and classifications of the Lagrangian dark matter sheet in the SDSS

Florent Leclercq,[1,2,3,4,a)] Jens Jasche,[5] Guilhem Lavaux,[2,3] and Benjamin Wandelt[2,3,6,7]
[1)] *Institute of Cosmology and Gravitation (ICG), University of Portsmouth, Dennis Sciama Building, Burnaby Road, Portsmouth PO1 3FX, United Kingdom*
[2)] *Institut d'Astrophysique de Paris (IAP), UMR 7095, CNRS – UPMC Université Paris 6, Sorbonne Universités, 98bis boulevard Arago, F-75014 Paris, France*
[3)] *Institut Lagrange de Paris (ILP), Sorbonne Universités, 98bis boulevard Arago, F-75014 Paris, France*
[4)] *École polytechnique ParisTech, Route de Saclay, F-91128 Palaiseau, France*
[5)] *Excellence Cluster Universe, Technische Universität München, Boltzmannstrasse 2, D-85748 Garching, Germany*
[6)] *Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, IL 61801, USA*
[7)] *Department of Astronomy, University of Illinois at Urbana-Champaign, 1002 West Green Street, Urbana, IL 61801, USA*

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Whereas previous studies have demonstrated that the shape of the cosmic web can be described by studying the Lagrangian displacement field, state-of-the-art analyses have been limited to cosmological simulations. This letter reports on the possibility to perform a Lagrangian description of cosmic web environments in real data from large-scale structure surveys. Building upon recent Bayesian large-scale inference of initial conditions, we present an analysis of the Lagrangian dark matter sheet in the nearby universe as probed by the Sloan Digital Sky Survey. In particular, we consider its stretchings and foldings and we dissect cosmic structures into four distinct components (voids, sheets, filaments, and clusters), using the Lagrangian classifiers DIVA and ORIGAMI. As a result, identified structures explicitly carry physical information about their formation history. The present study carries potential for profound implications in the analysis of the cosmological large-scale structure, as it opens the way for new confrontations of observational data and theoretical models.

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I. INTRODUCTION

The accurate description of large-scale structure formation, from which the cosmic web originates (Bond, Kofman & Pogosyan, 1996), is one of the major goals of modern cosmology. Traditionally, the Lagrangian and Eulerian descriptions of a discretized fluid of collisionless dark matter are comparatively discussed. In the Eulerian picture, quantities are tracked at fixed positions as particles move. In the Lagrangian framework, the motion and evolution of individual fluid elements is followed. Therefore, particles can be thought of not only as mass tracers, but also as elements of a three-dimensional dark matter sheet, which stretches, folds and distorts in a six-dimensional velocity-position phase space.

Using cosmological simulations, several recent papers have demonstrated the power of working within the dark matter sheet. Tessellations of the initial Lagrangian space allow the identification of stream crossings (Shandarin, Habib & Heitmann, 2012), improve density measurements (Abel, Hahn & Kaehler, 2012), and yield a new approach for simulating the dark matter fluid (Hahn, Abel & Kaehler, 2013). In addition, the study of caustics, the two-dimensional edges along which the dark matter sheet folds, brings in valuable insights into structure formation (Neyrinck, 2012). Strong observational motivations exist for a Lagrangian understanding of the cosmic web. These include studying the dependence of galaxy properties on their evolving environment (e.g. Blanton *et al.*, 2005), probing the effect of the dynamic large-scale structure on the cosmic microwave background (e.g. Planck Collaboration, 2014), testing the standard general-relativistic picture of gravitational instability (Falck, Koyama & Zhao, 2015), and identifying the most promising regions to observe a possible dark matter annihilation signal (e.g. Gao *et al.*, 2012).

Unfortunately, the Lagrangian picture cannot be readily applied to observational data. Difficulties are of two sorts: (i) the need for reconstructions of the dark matter density from galaxies, accounting for typical observational effects such as biases, survey mask and selection functions; and (ii) the need for corresponding initial conditions, which requires the reconstructions to be based on a physical picture of structure formation. For these reasons, most cosmic web analyses have been limited so far to simulations (see e.g. Cautun *et al.*, 2014, for a recent study). Jasche *et al.* (2010) and Leclercq, Jasche & Wandelt (2015) addressed the first issue. Using real data, they presented Eulerian classifications of cosmic environments in constrained realizations of the large-scale structure. Importantly, they demonstrated capability of propagating observational uncertainties to cosmic web classification. Taking advantage of the inclusion of a physical model within the inference process, Leclercq,

[a)] Electronic mail: florent.leclercq@polytechnique.org

Jasche & Wandelt (2015) also presented the first probabilistic analysis of proto-structures present in the initial density field. However, a complete investigation of the problem, including a probabilistic description of the Lagrangian dark matter sheet has not yet been performed in the literature.

In this letter, we present a full-scale Bayesian description of the Lagrangian volume covered by galaxies of the Sloan Digital Sky Survey (SDSS) main sample. This chrono-cosmography project builds upon the inference of the past and present cosmic structure in the SDSS (Jasche, Leclercq & Wandelt, 2015), performed using the BORG algorithm (Jasche & Wandelt, 2013). We start by describing our tools for analyzing the dark matter sheet, including the Lagrangian classifiers DIVA and ORIGAMI (section II). Building upon the BORG SDSS analysis, we then infer and classify the Lagrangian dark matter sheet (section III). Finally, we show how to translate these results to Eulerian coordinates, in order to describe the redshift-zero cosmic web with Lagrangian classifiers (section IV).

II. DESCRIPTION OF THE LAGRANGIAN SHEET

In our earlier analysis of the volume covered by the SDSS main sample galaxies (Leclercq, Jasche & Wandelt, 2015), we adopted the Eulerian cosmic web classifier known as the T-WEB (Hahn *et al.*, 2007). In this framework, structures are classified according to the sign of the eigenvalues $\mu_1(\vec{x}) \leq \mu_2(\vec{x}) \leq \mu_3(\vec{x})$ of the tidal field tensor $\mathcal{T}$, defined as the Hessian of the rescaled gravitational potential Φ:

$$\mathcal{T}_{ij} \equiv \mathrm{H}(\Phi)_{ij} = \frac{\partial^2\Phi}{\partial\vec{x}_i\partial\vec{x}_j}, \tag{1}$$

where Φ obeys the reduced Poisson equation

$$\Delta\Phi(\vec{x}) = \delta(\vec{x}), \tag{2}$$

δ being the local density contrast. A void point corresponds to no positive eigenvalue, a sheet to one, a filament to two, and a cluster to three positive eigenvalues.

In this section, we discuss two alternative methods that can be used for producing Lagrangian classifications of the cosmic web: DIVA and ORIGAMI. They both rely on the displacement field $\vec{\Psi}(\vec{q})$, which maps the initial (Lagrangian) position of particles $\vec{q}$ to their final (Eulerian) position $\vec{x}(\vec{q})$ (see e.g. Bernardeau *et al.*, 2002):

$$\vec{x}(\vec{q}) \equiv \vec{q} + \vec{\Psi}(\vec{q}). \tag{3}$$

We denote by $J(\vec{q})$ be the Jacobian of the transformation between Lagrangian and Eulerian coordinates,

$$J(\vec{q}) \equiv \det\left[\frac{\partial\vec{x}}{\partial\vec{q}}\right] = \det\mathcal{D} = \det(\mathcal{I}+\mathcal{R}), \tag{4}$$

where the deformation tensor $\mathcal{D}_{\ell m}$ can be written as the identity tensor $\mathcal{I}_{\ell m} \equiv \delta_{\ell,m}$ plus the shear of the displacement, $\mathcal{R}_{\ell m} \equiv \partial\vec{\Psi}_\ell/\partial\vec{q}_m$.

In analogy with the Zel'dovich (1970) "pancake" theory, Lavaux & Wandelt (2010) propose to define cosmic structures using $\lambda_1(\vec{q}) \leq \lambda_2(\vec{q}) \leq \lambda_3(\vec{q})$, the eigenvalues of the shear of the displacement field, $\mathcal{R}$. In the DIVA (DynamIcal Void Analysis) scheme, a particle is defined as belonging to a Lagrangian void, sheet, filament or cluster, if, respectively, zero, one, two or three of the λs are positive.

The DIVA and T-WEB methods share strong similarity in their formalism. However, a major difference is that the T-WEB operates on *voxels* of the discretized domain, whereas DIVA operates on *Lagrangian patches* (numerically approximated by particles). As shown in Leclercq, Jasche & Wandelt (2015, sections III and IV), the T-WEB can be used to classify both early-time and late-time structures; however the T-WEB classification of early-time structures is performed at the level of voxels of the discretized initial density field, whereas the DIVA classification of Lagrangian structures is performed at the level of particles of the dark matter sheet.

Another way of classifying these particles is to consider them as vertices of an initially regular grid, distorted by gravity as the large-scale structure forms. Shell-crossing happens when Lagrangian cells collapse or invert. A particle's ORIGAMI (Order-ReversIng Gravity, Apprehended Mangling Indices) morphology is defined by the number of orthogonal axes along which shell-crossing has occurred (Falck, Neyrinck & Szalay, 2012). More precisely, void, sheet, filament, and cluster particles are defined as particles that have been crossed along zero, one, two, and three orthogonal axes, respectively.

In practice, the computer implementation of ORIGAMI works as follows: a particle ℓ has been crossed along an axis $\hat{n}$ if there exists a particle m in the same row of the Lagrangian grid, such that $(\vec{q}_\ell - \vec{q}_m)\cdot\hat{n}$ and $(\vec{x}_\ell - \vec{x}_m)\cdot\hat{n}$ have opposite sign. Such particle-crossing are checked along the three orthogonal axes of the original Cartesian grid, as well as three triplets of orthogonal axes, obtained by rotating the original grid by 45° along one of its axes. For more details, the reader is referred to section 2.1 in Falck, Neyrinck & Szalay (2012).

III. LAGRANGIAN ANALYSIS OF THE SDSS VOLUME

As mentioned in the introduction, this paper is part of a series following the recent application of the Bayesian inference framework BORG to the SDSS main sample galaxies (Jasche, Leclercq & Wandelt, 2015). We further rely on the set of constrained simulations presented in Leclercq, Jasche & Wandelt (2015). These realizations are obtained, starting from BORG initial conditions, by non-linear filtering with COLA (Tassev, Zaldarriaga & Eisenstein, 2013). The initial density field, defined on a 256^3-voxel grid with side length of 750 Mpc/h, is popu-

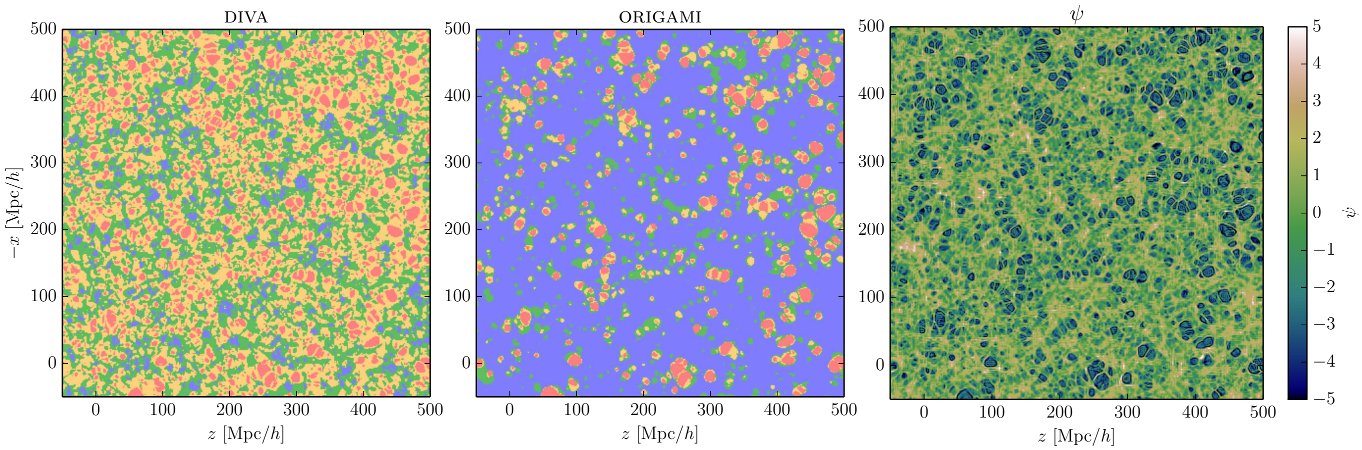

FIG. 1. One sample of the observed Lagrangian dark matter sheet, constrained by the SDSS main sample: DIVA (left panel) and ORIGAMI (middle panel) classifications of particles, and the divergence of the displacement field ψ (right panel). In the first two panels, blue, green, yellow and red correspond to void, sheet filament and cluster particles, respectively.

lated by 512^3 dark matter particles placed on a regular Lagrangian lattice. Particles are evolved with 2LPT to the redshift of $z = 69$ then with 30 COLA timesteps from $z = 69$ to $z = 0$.

Lavaux & Wandelt (2010) and Falck, Neyrinck & Szalay (2012) discuss the possibility of smoothing the displacement field at different Lagrangian scales, in order to probe the hierarchical cosmic web. While this would be perfectly feasible in our framework, we chose to focus our analysis on the scale naturally defined by our setup, without an additional smoothing step. This choice corresponds to a resolution of ~ 1.5 Mpc/h for the Lagrangian grid (512^3 particles in a 750 Mpc/h cubic box) and ~ 3 Mpc/h for discretized maps (on a 256^3-voxel grid).

The displacement field is obtained on the Lagrangian lattice by subtracting the final position of particles from their initial position. We then assign a Lagrangian structure type to each of them, according to the DIVA and ORIGAMI procedures. For DIVA, the components of the shear of the displacement field are evaluated numerically by means of fast Fourier transforms on the Lagrangian grid. In figure 1, we represent the Lagrangian grid of particles in one of our constrained samples.[1] The first two panels show the DIVA and ORIGAMI morphologies of the dark matter sheet. For comparison, the last panel shows the divergence of the displacement field $\psi \equiv \vec{\nabla} \cdot \vec{\Psi}$ for the same particles, computed using a fast Fourier transform on the Lagrangian grid.

Our constrained samples exhibit the same behavior as previously observed in simulations. DIVA and ORIGAMI structures correlate well with ψ, which quantifies the angle-average stretching and distortion of the dark matter sheet (Neyrinck, 2013). In particular, DIVA and ORIGAMI clusters nicely correspond to the Lagrangian "lakes" where $\psi \approx -3$. Note that in this analysis, we are taking advantage of the absence of substantial vorticity on large scales (see e.g. Bernardeau *et al.*, 2002). Indeed, since the displacement field in Lagrangian perturbation theory is curl-free up to second order (2LPT), and since BORG relies on 2LPT as a proxy for gravitational dynamics (Jasche & Wandelt, 2013), any vorticity present in our large-scale structure realizations is not constrained by the data. However, as shown by Chan (2014), at $z = 0$ and up to $k \approx 1$ Mpc/h, the contribution to the power spectrum of $\vec{\Psi}$ coming from its curl component remains negligible compared to the non-linear evolution of its scalar part. For the sake of our analysis, it is therefore safe to assume that all the information on $\vec{\Psi}$ is contained in its divergence ψ.

As in our previous work (Jasche, Leclercq & Wandelt, 2015; Leclercq, Jasche & Wandelt, 2015; Lavaux & Jasche, 2016), the variation between constrained samples constitutes a Bayesian quantification of uncertainties (coming in particular from the survey mask and selection effects). This is illustrated in figure 2, where we show the particle-wise posterior mean of different quantities on the Lagrangian grid. The three leftmost panels are the eigenvalues of the displacement shear tensor, $\lambda_1(\vec{q}) \leq \lambda_2(\vec{q}) \leq \lambda_3(\vec{q})$. The right panel shows the Jacobian of the transformation from Lagrangian to Eulerian coordinates, $J(\vec{q}) = (1 + \lambda_1(\vec{q}))(1 + \lambda_2(\vec{q}))(1 + \lambda_3(\vec{q}))$. For each particle, it is the volume of the associated fluid element times its parity (White & Vogelsberger, 2009; Neyrinck, 2012). White/blue particles have the same parity as in the initial conditions, while black/orange particles have swapped parity. Caustics are clearly visible wherever blue and orange regions are juxtaposed, for example in the patches that collapse to form halos and in

[1] In the plots of this paper, we kept the coordinate system of Jasche, Leclercq & Wandelt (2015) and limited the slice to a square of 500 Mpc/h side for improved clarity.

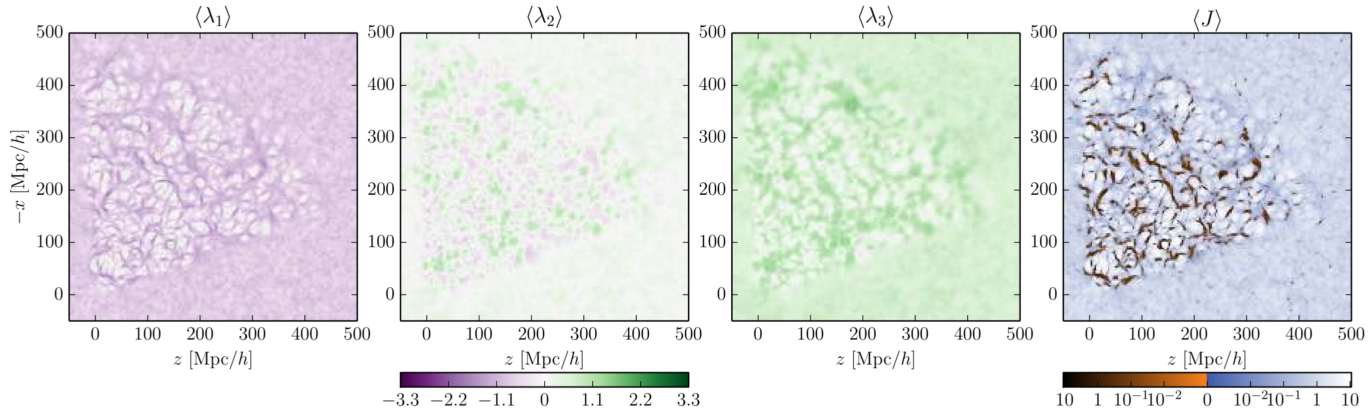

FIG. 2. Uncertainty quantification of the observed Lagrangian dark matter sheet. In the three leftmost panels, the posterior mean of the eigenvalues of the displacement shear tensor, $\lambda_1(\vec{q}) \leq \lambda_2(\vec{q}) \leq \lambda_3(\vec{q})$, are shown. The right panel shows the mean of the Jacobian $J(\vec{q})$. Particles are colored according to their Eulerian volume times their parity. The color scale is streched around zero with the function $x \mapsto \mathrm{argsinh}(10^3\, x)$.

the shell-crossed walls surrounding voids.

Uncertainty quantification can also be self-consistently propagated to Lagrangian structure type classification. Let us denote by ξ one of the Lagrangian classifiers (DIVA or ORIGAMI). In a specific realization, ξ uniquely characterizes the Lagrangian grid, meaning that it provides four particle-wise scalar fields that obey the following conditions for each particle $\vec{q}_\ell$:

$$\mathrm{T}_i(\vec{q}_\ell|\xi) \in \{0,1\} \text{ for } i \in [\![0,3]\!] \quad \text{and} \quad \sum_{i=0}^{3} \mathrm{T}_i(\vec{q}_\ell|\xi) = 1 \tag{5}$$

where T_0 = void, T_1 = sheet, T_2 = filament, T_3 = cluster. By applying ξ to the complete set of BORG-COLA realizations, we can quantify the degree of belief in structure type classification in terms of four particle-wise scalar fields that obey the following conditions for each particle $\vec{q}_\ell$:

$$\mathcal{T}_i(\vec{q}_\ell|\xi) \in [0,1] \text{ for } i \in [\![0,3]\!] \quad \text{and} \quad \sum_{i=0}^{3} \mathcal{T}_i(\vec{q}_\ell|\xi) = 1. \tag{6}$$

Here, $\mathcal{T}_i(\vec{q}_\ell|\xi) \equiv \langle \mathrm{T}_i(\vec{q}_\ell|\xi)\rangle_{\mathcal{P}(\mathrm{T}_i(\vec{q}_\ell)|d,\xi)} = \mathcal{P}(\mathrm{T}_i(\vec{q}_\ell)|d,\xi)$ are the posterior probability distribution functions (pdfs) for particle $\vec{q}_\ell$ to belong to structure T_i, as defined by classifier ξ, conditional on the data d. These are estimated by counting the relatives frequencies at each individual particle position within the set of ξ-realizations (see also Jasche *et al.*, 2010; Leclercq, Jasche & Wandelt, 2015). Therefore, the cosmic web-type pdf mean on the Lagrangian grid is given by

$$\langle \mathcal{P}(\mathrm{T}_i(\vec{q}_\ell)|d,\xi)\rangle = \frac{1}{N}\sum_{n=1}^{N}\sum_{j=0}^{3} \delta^{(\mathrm{K})}_{\mathrm{T}_i(\vec{q}_\ell)\mathrm{T}_j^n(\vec{q}_\ell|\xi)}, \tag{7}$$

where n labels one of the N samples, $\mathrm{T}_j^n(\vec{q}_\ell|\xi)$ is the result of classifier ξ on the n-th sample at particle position $\vec{q}_\ell$ (i.e. a unit four-vector containing zeros except for one component, which indicates the structure type), and $\delta^{(\mathrm{K})}$ is a Kronecker symbol.

Using the above definitions, we obtain probabilistic maps of structures on the Lagrangian grid describing the early Universe in the volume probed by SDSS main sample galaxies. More precisely, for each classifier ξ, we obtain a probability mass function (pmf) at each particle position on the Lagrangian grid, indicating the possibility to encounter a specific structure type (cluster, filament, sheet, void) at that position. These pmfs consist of four numbers in the range $[0,1]$, $\mathcal{P}(\mathrm{T}_i(\vec{q}_\ell)|d,\xi)$, that sum up to one for each particle. They are approximated by the mean of each structure pdf (see equation 7).

In figure 3, we show slices through these web-type posterior probabilities. The rows represent the structures inferred using DIVA and ORIGAMI, respectively. For comparison, the reader is referred to figure 6 in Leclercq, Jasche & Wandelt (2015), which shows structures inferred in the initial density field using the T-WEB. Note that the DIVA and ORIGAMI results are defined on the Lagrangian grid of 512^3 particles, whereas the T-WEB result is defined on the 256^3-voxel grid on which initial density fields are characterized. The plot shows the anticipated behavior, with values close to certainty (i.e. zero or one) in regions covered by data, while the unobserved regions (at high redshift or out of the survey boundary) approach a uniform value corresponding to the prior. It is worth noting that different classifiers can have very different prior probabilities for the same structure types.

IV. TRANSLATION TO EULERIAN COORDINATES

In the previous section, we have considered classifications on the Lagrangian dark matter sheet. In order to classify structures at redshift zero, it is possible to translate the DIVA and ORIGAMI results into Eulerian co-

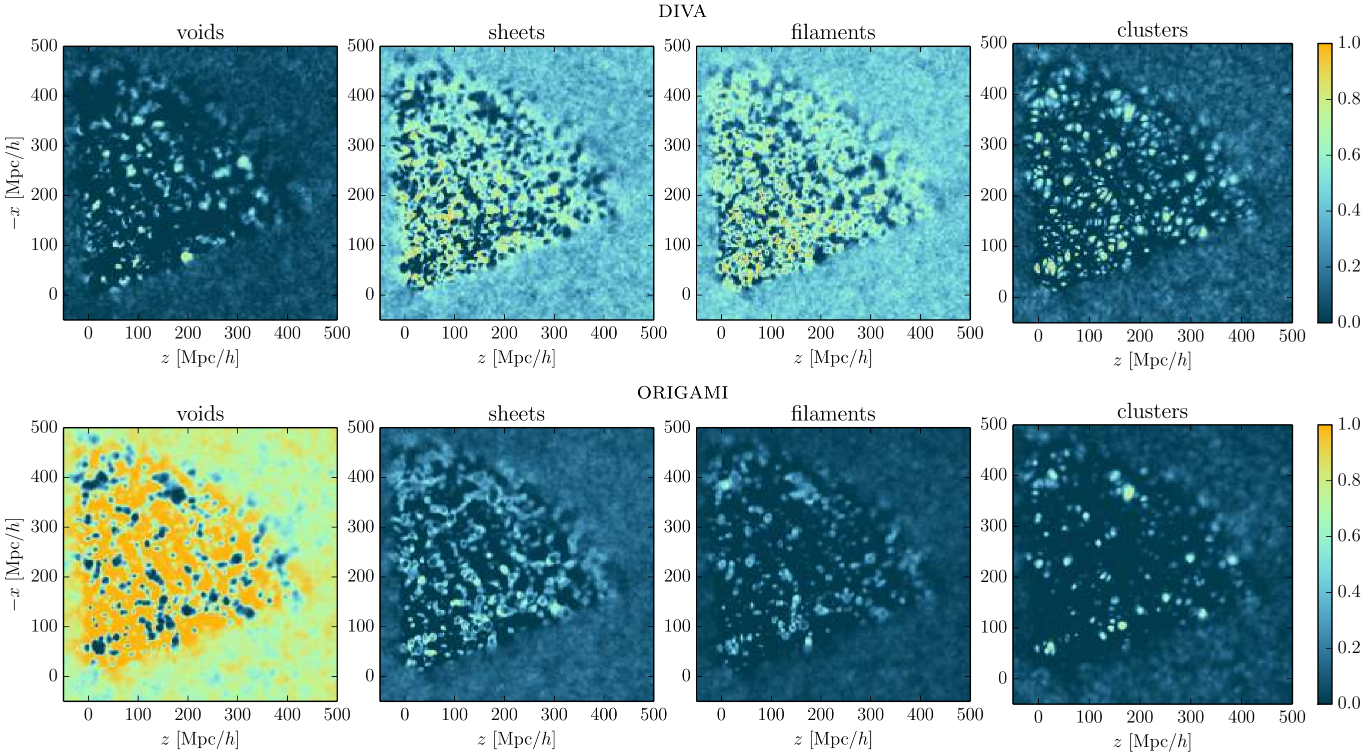

FIG. 3. Slices through the posterior probabilities for different structure types (from left to right: void, sheet, filament, and cluster), in the primordial large-scale structure in the Sloan volume ($a = 10^{-3}$). These four three-dimensional probabilities sum up to one at each location. Structures are defined on the Lagrangian grid of 512^3 particles with DIVA (first row) or ORIGAMI (second row). For comparison, the reader is referred to figure 6 in Leclercq, Jasche & Wandelt (2015), where the primordial density field, defined on a 256^3-voxel grid, is classified according to the T-WEB procedure.

ordinates. To do so, we consider particles at their final positions and assign them to a 256^3-voxel grid using the cloud-in-cell (CiC) scheme. For structure type T_i, each particle gets a weight of 1 if its Lagrangian classification is T_i, and 0 otherwise. In each voxel, we then count the relative frequencies for each mass-weighted structure type. In this fashion, in each realization n, we obtain four probabilities, in the range $[0, 1]$, for a final Eulerian voxel to belong to each of the structure types. We denote them by $\mathcal{P}_n(\mathrm{T}_i(\vec{x}_k)|d, \xi)$. In general, we have $\mathcal{S}_n(\vec{x}_k) \equiv \sum_{i=0}^{3} \mathcal{P}_n(\mathrm{T}_i(\vec{x}_k)|d, \xi) = 1$; this sum is 0 if and only if, in realization n, no particle was assigned to the cell $\vec{x}_k$ by the CiC scheme (i.e. it is empty as well as its eight neighbors). These voxels are flagged as noisy and discarded.

It is then possible to go from these per-sample pdfs to a global pdf for the entire analysis, in a similar way as before: we count the relative frequencies of the probabilities at each Eulerian spatial position within the set of samples. With this definition, the web-type posterior mean is given by

$$\langle \mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d, \xi) \rangle = \frac{1}{N_{\vec{x}_k}} \sum_{n=1}^{N} \mathcal{P}_n(\mathrm{T}_i(\vec{x}_k)|d, \xi), \quad (8)$$

where n labels one of the N samples, and the normalization $N_{\vec{x}_k}$ in voxel $\vec{x}_k$ is

$$N_{\vec{x}_k} = \sum_{n=1}^{N} \delta^{(\mathrm{K})}_{\mathcal{S}_n(\vec{x}_k),1}. \quad (9)$$

In figure 4, we show slices through the redshift-zero web-type posterior mean for the four different structures as defined by DIVA and by ORIGAMI. As before, the plot demonstrates that we are able to propagate typical observational uncertainty to structure type classification. At this point, the reader is referred to figure 3 in Leclercq, Jasche & Wandelt (2015) for our map of the same volume, inferred using the T-WEB definition. The T-WEB and DIVA maps are visually similar, with an overall smoother structure for the voids defined by DIVA, which are sharply separated by walls. DIVA also visually excels in highlighting the filamentary structure of the cosmic web. In contrast, with ORIGAMI, most of the volume is filled by voids and more complex, shell-crossed structures are rarely identified.

V. CONCLUSIONS

In this letter, we presented the first probabilistic maps of the Lagrangian dark matter sheet in a volume covered

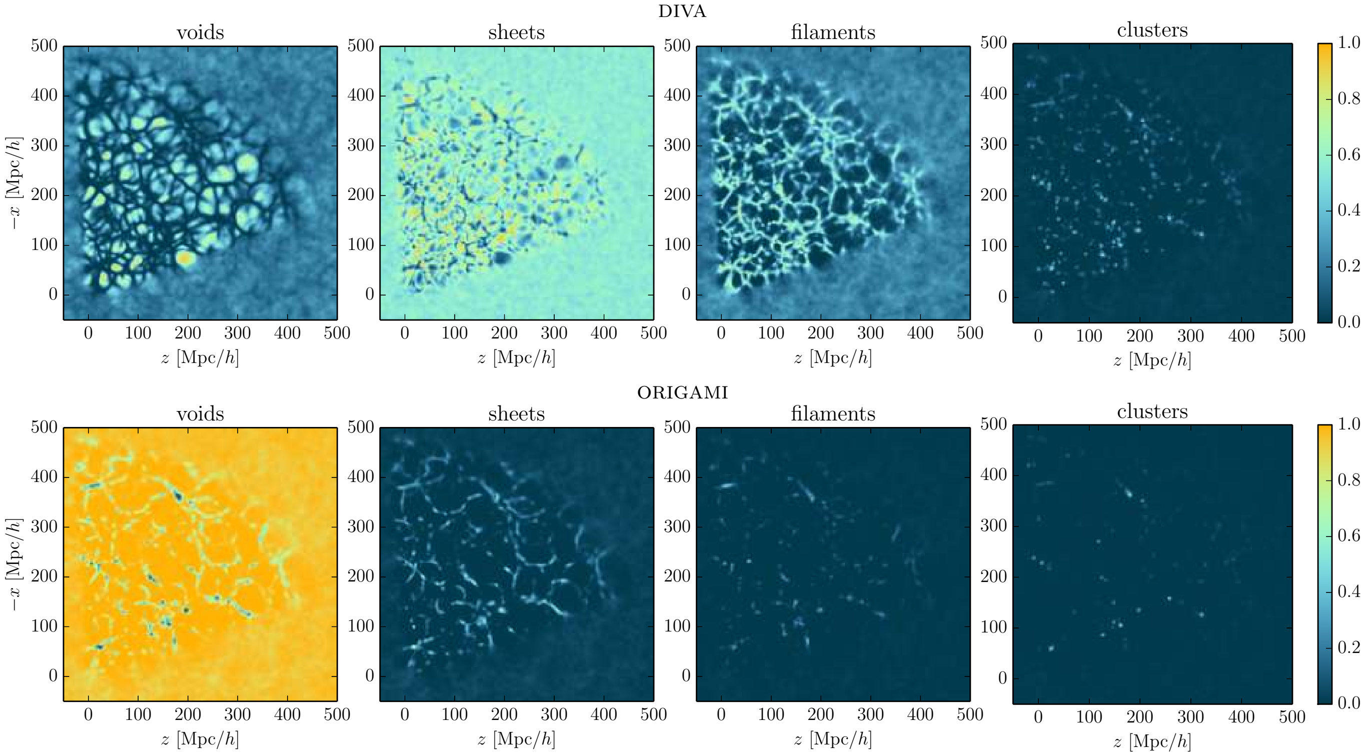

FIG. 4. Slice through the posterior probabilities for different structure types (from left to right: void, sheet, filament, and cluster), in the late-time large-scale structure in the Sloan volume ($a = 1$). These four three-dimensional probabilities, defined on a 256^3-voxel grid, sum up to one on a voxel basis. Structure types are classified using DIVA (first row) or ORIGAMI (second row). For comparison, the reader is referred to figure 3 in Leclercq, Jasche & Wandelt (2015), where structures are classified according to the T-WEB procedure.

by the SDSS main galaxy sample. Beyond this specific region, our study demonstrates the possibility of mapping the dark matter sheet in the real Universe. This possibility, so far limited to simulations, is uniquely offered by Bayesian physical inference of the initial conditions from large-scale structure surveys, which allows a self-consistent treatment of the necessary aspects: quantification of observational uncertainties and description of structure formation. We also showed how to translate the result of Lagrangian classifiers into redshift-zero Eulerian maps. With these definitions, identified structures have a direct physical interpretation in terms of the entire history of matter flows toward and inside them.

Our result may have important implications for large-scale structure data analyses, as they allow new confrontations of observational data and theoretical models, and therefore novel tests of the standard paradigm of cosmic web formation and evolution.

STATEMENT OF CONTRIBUTION

FL implemented COLA and the required cosmic web analysis tools, performed the study, produced the maps and wrote the paper. JJ developed BORG and lead the SDSS analysis; GL developed DIVA; JJ and GL contributed to the development of computational tools. BW was involved in the conception and design of Bayesian large-scale structure inference and contributed to the interpretation of results. All the authors read and approved the final manuscript.

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