UCRHEP-T562
January 2016

Soft $A_4 \to Z_3$ Symmetry Breaking and Cobimaximal Neutrino Mixing

Ernest Ma

Physics & Astronomy Department and Graduate Division, University of California, Riverside, California 92521, USA

HKUST Jockey Club Institute for Advanced Study, Hong Kong University of Science and Technology, Hong Kong, China

Abstract

I propose a model of radiative charged-lepton and neutrino masses with A_4 symmetry. The soft breaking of A_4 to Z_3 lepton triality is accomplished by dimension-three terms. The breaking of Z_3 by dimension-two terms allows cobimaximal neutrino mixing ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{CP} = \pm\pi/2$) to be realized with only very small finite calculable deviations from the residual Z_3 lepton triality. This construction solves a long-standing technical problem inherent in renormalizable A_4 models since their inception.

For the past several years, some new things have been learned regarding the theory of neutrino flavor mixing. (1) Whereas the choice of symmetry, for example A_4 [1, 2, 3], and its representations are obviously important, the breaking of this symmetry into specific residual symmetries, for example $A_4 \to Z_3$ lepton triality [4, 5], is actually more important. (2) A mixing pattern may be obtained [6] independent of the masses of the charged leptons and neutrinos. (3) The clashing of residual symmetries between the charged-lepton, for example $A_4 \to Z_3$, and neutrino, for example $A_4 \to Z_2$, sectors is technically very difficult to maintain [7]. (4) The essential incorporation of CP transformations [8, 9] may be the new approach [10, 11, 12, 13, 14, 15] which will lead to an improved understanding of neutrino flavor mixing.

In this paper, a model of radiative charged-lepton and neutrino masses is proposed with the following properties. (1) The masses are generated in one loop through dark matter [16], i.e. particles distinguished from ordinary matter by an exactly conserved dark symmetry. This is the so-called scotogenic mechanism. (2) The symmetry $A_4 \times Z_2$ is imposed on all dimension-four terms of the renormalizable Lagrangian with particle content given in Table 1. (3) Dimension-three terms break $A_4 \times Z_2$, but all such terms respect the residual Z_3 lepton triality. (4) Dimension-two terms break Z_3, which is nevertheless retained in dimension-three (and dimension-four) terms with only finite calculable deviations. This solves the problem of clashing residual symmetries. (5) The proposed specific model results in cobimaximal [15] neutrino mixing ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{CP} = \pm\pi/2$), which is consistent with the present data [17, 18]. It is also theoretically sound, because the residual Z_3 is protected, unlike previous proposals. Cobimaximal mixing becomes thus a genuine prediction, robustly supported in the context of a complete renormalizable theory of neutrino mass and mixing.

The dark $U(1)_D$ and Z_2 symmetries are assumed to be unbroken. The other Z_2 symmetry is used to forbid the dimension-four Yukawa couplings $\bar{l}_L l_R \phi^0$ so that charged leptons only

particles	dark $U(1)_D$	dark Z_2	flavor A_4	Z_2
$(\nu, l)_L$	0	+	3	+
l_R	0	+	3	−
(ϕ^+, ϕ^0)	0	+	1	+
$N_{L,R}$	1	+	3	+
(η^+, η^0)	1	+	1	+
χ^+	1	+	1	−
$(E^0, E^-)_{L,R}$	0	−	1	+
F_L^0	0	−	1	+
s	0	−	3	+

Table 1: Particle content under $U(1)_D \times Z_2 \times A_4 \times Z_2$.

acquire masses in one loop as shown in Fig. 1. Whereas this Z_2 is respected by the dimension-

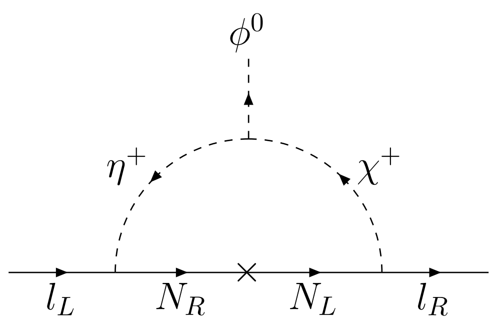

Figure 1: One-loop generation of charged-lepton mass with $U(1)_D$ symmetry.

four $\bar{l}_R N_L \chi^-$ terms, it is broken softly by the dimension-three trilinear $\eta^+\chi^-\phi^0$ term to complete the loop. This guarantees the one-loop charged-lepton mass to be finite. Note that a dark $U(1)_D$ symmetry [19, 20] is supported here with χ^+, (η^+, η^0), and $N_{L,R}$ all transforming as 1 under $U(1)_D$. The dimension-three soft terms $\bar{N}_L N_R$ are assumed to break A_4 to Z_3 through the well-known unitary matrix [1, 21, 22] U_ω, i.e.

$$\mathcal{M}_N = U_\omega^\dagger \begin{pmatrix} m_{N_1} & 0 & 0 \\ 0 & m_{N_2} & 0 \\ 0 & 0 & m_{N_3} \end{pmatrix} U_\omega, \tag{1}$$

where

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{2}$$

In the A_4 limit, $\mathcal{M}_N$ is proportional to the identity matrix. With three different mass eigenvalues, the residual symmetry is Z_3 lepton triality. Let the (η^+, χ^+) mass eigenvalues be $m_{1,2}$ with mixing angle θ, then each lepton mass is given by [19]

$$m_l = \frac{f_L f_R \sin\theta \cos\theta m_N}{16\pi^2}[F(x_1) - F(x_2)], \tag{3}$$

where $F(x) = x \ln x/(x-1)$, with $x_{1,2} = m_{1,2}^2/m_N^2$.

The dark $U(1)_D$ symmetry forbids the quartic scalar term $(\Phi^\dagger \eta)^2$, so that a neutrino mass is not generated as in Ref. [16]. It comes instead from Fig. 2, where the scalars $s_{1,2,3}$ are assumed real [10, 23, 24] to enable cobimaximal mixing, hence a separate dark Z_2 symmetry is required. Let the $\bar{F}_L E_R$ mass term be m_D and assumed to be much smaller than m_E, m_F,

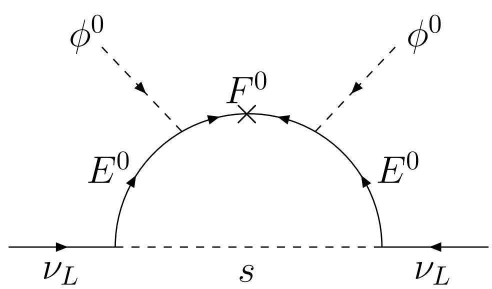

Figure 2: One-loop generation of neutrino mass from s.

then each neutrino mass is given by

$$m_\nu = \frac{h^2 m_D^2 m_F}{16\pi^2(m_F^2 - m_s^2)}[G(x_F) - G(x_s)], \tag{4}$$

where

$$G(x) = \frac{x}{1-x} + \frac{x^2 \ln x}{(1-x)^2}, \tag{5}$$

with $x_F = m_F^2/m_E^2$, $x_s = m_s^2/m_E^2$. The dimension-two $s_i s_j$ terms are allowed to break Z_3 arbitrarily. However, since this mass-squared matrix is real, it is diagonalized by an orthogonal matrix $\mathcal{O}$, hence the neutrino mixing matrix is given by [10, 25, 26]

$$U_{l\nu} = U_\omega \mathcal{O}, \tag{6}$$

resulting in $U_{\mu i} = U_{\tau i}^*$, thus guaranteeing cobimaximal mixing: $\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{CP} = \pm\pi/2$.

In a previous proposal [10], instead of Fig. 1, the radiative charged-lepton masses also come from scalars, i.e. $x_i^+ \sim \underline{3}$, $y_i^+ \sim \underline{1}, \underline{1}', \underline{1}''$ under A_4. The $A_4 \to Z_3$ breaking is accomplished by rotating x_i^+ through U_ω so that $x_{1,2,3}^+$ now correspond to $y_{1,2,3}^+$ under Z_3, and allowing the (x_1, y_1), (x_2, y_2), (x_3, y_3) sectors to have separate arbitrary masses. Now the quartic scalar coupling $(x_1^+ s_1 + x_2^+ s_2 + x_3^+ s_3)(x_1^- s_1 + x_2^- s_2 + x_3^- s_3)$ is allowed under A_4. If the $s_i s_j$ mass-squared terms break Z_3 as in Fig. 2, then the $s_1 s_2 (x_1^+ x_2^- + x_2^+ x_1^-)$ term from the above will induce a quadratic $x_1 x_2$ term as shown in Fig. 3. Whereas this diagram is

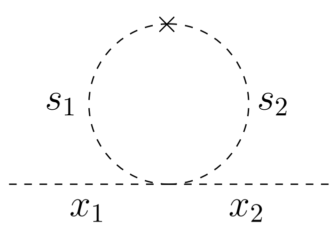

Figure 3: One-loop generation of $x_1 x_2$ term from $s_1 s_2$ term.

not quadratically divergent, it is still logarithmically divergent. This means a counterterm is required for $x_1^+ x_2^- + x_2^+ x_1^-$, thereby invalidating the Z_3 residual symmetry necessary to derive U_ω and thus Eq. (6).

In this proposal, the $A_4 \to Z_3$ breaking comes from $\bar{N}_L N_R$, with the Dirac fermions $N_{1,2,3}$ distinguished from one another by the residual Z_3 lepton triality through U_ω as shown in Eq. (1). The soft breaking of Z_3 by $s_1 s_2$ induces only a finite two-loop correction to the

$N_1 - N_2$ wavefunction mixing as shown in Fig. 4. Therefore this construction solves a long-

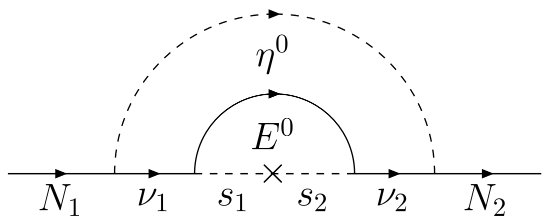

Figure 4: Two-loop $N_1 - N_2$ mixing from $s_1 s_2$ breaking of Z_3.

standing technical problem in renormalizable theories of A_4 flavor mixing. To summarize, (1) A_4 is respected by all dimension-four terms; (2) Z_3 is respected by all dimension-three terms; (3) Z_3 is broken arbitrarily by dimension-two terms to allow cobimaximal mixing according to Eq. (6); (4) the $s_i s_j$ terms generate very small finite radiative corrections to Z_3 breaking in the dimension-three terms, justifying the use of U_ω to obtain Eq. (6).

As for dark matter, there are in principle two stable components: the lightest N with $U(1)_D$ symmetry and the lightest s with Z_2 symmetry. Whereas N has only the allowed $\bar{N}_R(\nu_L \eta^0 - l_L \eta^+)$ interactions, s has others, i.e. $s^2 \Phi^\dagger \Phi$, $s^2 \eta^\dagger \eta$, $s^2 \chi^+ \chi^-$, as well as $s(\bar{\nu}_L E_R^0 + \bar{l}_L E_R^-)$. Their interplay to make up the total correct dark-matter relic abundance of the Universe and how they may be detected in underground direct-search experiments require further study.

An immediate consequence of radiative charged-lepton mass is that the Higgs Yukawa coupling $h\bar{l}l$ is no longer exactly $m_l/(246 \text{ GeV})$ as predicted by the standard model, as studied in detail already [27, 28]. Because of the Z_3 lepton triality, large anomalous muon magnetic moment may be accommodated while $\mu \to e\gamma$ is suppressed [28].

In conclusion, cobimaximal neutrino mixing ($\theta_{13} \neq 0, \theta_{23} = \pi/4, \delta_{CP} = \pm\pi/2$) is achieved rigorously in a renormalizable model of radiative charged-lepton and neutrino masses. The key is the soft breaking of A_4 to Z_3 by dimension-three terms, so that the subsequent breaking of Z_3 by dimension-two terms only introduces very small finite corrections to the U_ω transformation needed to obtain cobimaximal mixing as given by Eq. (6).

This work is supported in part by the U. S. Department of Energy under Grant No. de-sc0008541.

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