Implementing Brouwer's database of strongly regular graphs

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Abstract

Andries Brouwer maintains a public database of existence results for strongly regular graphs on $n \leq 1300$ vertices. We implemented most of the infinite families of graphs listed there in the open-source software Sagemath [15], as well as provided constructions of the "sporadic" cases, to obtain a graph for each set of parameters with known examples. Besides providing a convenient way to verify these existence results from the actual graphs, it also extends the database to higher values of n.

Keywords— 05E30: strongly regular graphs, association schemes, 68-04: explicit machine computation and programs

1 Introduction

Many researchers in algebraic combinatorics or an adjacent field at some point want to get their hands on a list of feasible parameters of strongly regular graphs, and on actual examples of graphs. These graphs are studied and/or used in hundreds of articles; recent highlights in using strongly regular graphs include A. Bondarenko's [2] and an improvement of the latter by T. Jenrich and A.E. Brouwer [34]. While parameters are available from A.E. Brouwer's online database [4], actually constructing an example can easily take a lot of time and effort. The project described here aims at making these tasks almost trivial by providing the necessary graph constructions, and a way to obtain a strongly regular graph from a set of parameters, in the computer algebra system Sagemath [15] (also known as Sage).

While exhaustive tables of non-isomorphic strongly regular graphs with small number n of vertices are available (see T. Spence [45] for $n \leq 64$), the sheer number of non-isomorphic examples (see e.g. D. Fon-Der-Flaass [18] or M. Muzychuk [40]) makes it hard to expect to be able to generate all of them in reasonable time. Thus we opted for a minimalist approach: for each set of parameters we are able to generate an example, provided that one is known. We

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note, however, that some of constructions implemented allow to generate many examples with the same parameters; e.g. we have implemented the construction to generate the point graph of the generalised quadrangle $T_2^*(\mathcal{O})$ (see [42]) from any hyperoval $\mathcal{O} \subset PG(2, 2^k)$. As well, many entries can be obtained by different implemented constructions, sometimes leading to isomorphic graphs.

Our desire to take on this project was motivated by the following considerations.

- One wants to double-check that the constructions are correct and their descriptions are complete; indeed, a program is more trustworthy than a proof in some situations, and coding a construction is a good test for completeness of the description provided.
- We wanted to see that the Sage combinatorial, graph-theoretic, and group-theoretic primitives to deal with such constructions are mature and versatile, so that coding of constructions is relatively easy and quick.
- One learns a lot while working on such a project, both the underlying mathematics, and how the toolset can be improved. In particular, one might come along simplifications of constructions, and this actually happened on couple of occasions.
- As time goes by, possible gaps in constructions are harder and harder to fill in. Reconstructing omitted proof details becomes a tricky and time-consuming task.

In particular, as far as the latter item is concerned, we seem to have uncovered at least one substantial gap in constructions (see Sect. 7). Furthermore, a number of constructions needed a feedback from their authors – sometimes quite substantial – to code them.

A large part of the constructions use in an nontrivial way another combinatorial or algebraic object: block design, Hadamard matrix, two-graph, two-distance code, finite group, etc. In particular, at the start of the projects some of these were lacking in Sagemath, we needed to implement constructions of certain block designs, regular symmetric Hadamard matrices with constant diagonal (where the gap just mentioned was uncovered), skew-Hadamard matrices, and two-graphs. As well, a small database of two-distance codes was created (see Sect. 4.2).

The remainder of the paper consists of a short introduction to strongly regular graphs, pointing out particular relevant Sagemath features, and a description of our implementations. In the remainder we list constructions implemented, and discuss few gaps we discovered in the literature.

2 Strongly regular graphs and related objects

An undirected regular degree k graph Γ on n vertices (with $0 < k < n-1$) is called *strongly regular* if the vertices u and v of any edge have λ common neighbours, and the vertices u and v of any non-edge have μ common neighbours. One says that Γ has *parameters* (n, k, λ, μ). Note that the *complement* of Γ, i.e. the graph with the same set of vertices and edges being precisely the non-edges of Γ, is also a strongly regular whose parameters are related by a simple formula to these of Γ (see e.g. A.E.Brouwer and W.Haemers [8] for details).

Example 2.1 *Let Γ be the graph with vertices being k-subsets of an m-set, with $k \leq \lfloor m/2 \rfloor$; two vertices are adjacent if the corresponding k-subsets intersect in a $(k-1)$-subset. Such graphs are called* Johnson graphs *and denoted by $J(m,k)$ (in Sagemath, $J(m,k)$ can be constructed by calling the function* `graphs.JohnsonGraph()`*). Then $J(m,2)$ is strongly regular graph, with parameters $(\binom{m}{2}, 2(m-2), m-2, 4)$.*

Some sources further require that both Γ and its complement are connected; in terms of parameters this means $0 < \mu < k$. This excludes the trivial case of Γ (or its complement) being disjoint union of complete graphs of the same size. Sagemath implementation does not impose this restriction.

A considerable number of techniques allowing to rule out existence of a strongly regular graph Γ with given parameters (n, k, λ, μ) is known, e.g. based on computing eigenvalues of the adjacency matrix A of Γ. As A generates a dimension 3 commutative subalgebra of $\mathbb{C}^{n \times n}$, one sees that there are just 3 distinct eigenvalues of A, and they are determined by the parameters (e.g. the largest eigenvalue is k). Sagemath implements parameter-based techniques to rule out sets of parameters from A.E.Brouwer and van J.Lint [10], and from A.E.Brouwer, A.M.Cohen, and A.Neumaier [7].

We use standard terminology for finite permutation groups, finite simple groups, and geometries over finite fields from [7, 8].

3 Structure of the implementation

The strongly regular graphs are split into two categories: the fixed-size graphs (see Sect.4) and the families of strongly regular graphs (see Sect.5). The parameters (n, k, λ, μ) of fixed-size graphs are hardcoded, while each family of strongly regular graphs has a helper function which takes (n, k, λ, μ) as an INPUT and answers whether the graph family is able to produce a graphs with the required parameters. Some families forward their queries to the databases of Balanced Incomplete Block Designs, of Orthogonal Arrays, of Hadamard matrices of various types, and of 2-weight codes.

With this design, it takes ≤ 3 seconds on a modern laptop to know which graphs on < 1300 vertices can be produced by the implemented constructions (i.e. as far as the online database goes).

4 Fixed-size constructions

4.1 "Sporadic" examples

Here we did not attempt to give an exhaustive list of references for each graph, for some of them have several papers devoted to them in one or another way.

Here we identify the corresponding graphs by their parameters, and provide references and some construction details for each of them.

(36, 14, 4, 6) Hubaut [29, S.9].
Subgraph of common neighbours of a triangle in Suzuki graph.

(50, 7, 0, 1) [8, Sect.9.1.7 (iv)]. The Hoffman-Singleton graph.

(56, 10, 0, 2) [8, Sect.9.1.7 (v)]. The Sims-Gewirtz graph.

(77, 16, 0, 4) [8, Sect.9.1.7 (vi)]. The M_{22}-graph.

(100, 22, 0, 6) [8, Sect.9.1.7 (vii)]. The Higman-Sims graph.

(100, 44, 18, 20) Jørgensen and Klin [35].
Built as a Cayley graph.

(100, 45, 20, 20) [35].
Built as a Cayley graph.

(105, 32, 4, 12) Goethals and Seidel [21], Coolsaet [12].

(120, 63, 30, 36) R.Mathon, cf. [10, Sect.6.A].
The distance-2 graph of $J(10, 3)$.

(120, 77, 52, 44) Unique by J. Degraer K. Coolsaet [14].
We first build a $2-(21, 7, 12)$ design, by removing two points from the Witt design on 23 points. We then build the intersection graph of blocks with intersection size 3.

(126, 25, 8, 4) R.Mathon, cf. [10, Sect.6.A].
The distance-(1 or 4) graph of $J(9, 4)$.

(126, 50, 13, 24) Goethals, cf. [10].

(144, 39, 6, 12) A.A. Ivanov, M.H. Klin, and I.A. Faradjev [32, Table 9].
An orbital of degree 39 (among 2 such orbitals) of the group $PSL_3(3)$ acting on the (right) cosets of a subgroup of order 39.

(162, 56, 10, 24) [29, S.12].
Subgraph induced on the neighbours of a vertex in the complement of McLaughlin graph.

(175, 72, 20, 36) [10, Sect.10.B (iv)].

Obtained from the line graph Λ of Hoffman-Singleton Graph, by setting two vertices to be adjacent if their distance in Λ is exactly 2. For more information, see `http://www.win.tue.nl/~aeb/graphs/McL.html`.

(176, 49, 12, 14) Brouwer [5].

Built from the symmetric Higman-Sims design. There exists an involution σ exchanging the points and blocks of the Higman-Sims design, such that each point is mapped on a block that contains it (i.e. σ is a polarity with all universal points). The graph is then built by making two vertices u, v adjacent whenever $v \in \sigma(u)$.

(176, 85, 48, 34) W.Haemers, cf. [10, Sect.10.B.(vi)].

Obtained from the $(175, 72, 20, 36)$-graph by attaching a isolated vertex and doing *Seidel switching* (cf. [8, Sect.10.6.1]) with respect to the disjoint union of 18 maximum cliques.

(176, 105, 68, 54) [29, S.7]; (a rank 3 representation of M_{22}).

We first build a $2-(22, 7, 16)$ design, by removing one point from the Witt design on 23 points. We then build the intersection graph of blocks with intersection size 3.

(196, 91, 42, 42) Ionin and Shrikhande [31].

(210, 99, 48, 45) Klin *et al.* [36].

S_7 acts on the 210 digraphs isomorphic to the disjoint union of K_1 and the circulant 6-vertex digraph which one can obtain using Sagemath as
`digraphs.Circulant(6,[1,4])`. This action has 16 orbitals; the package [17] found a merging of them, explicitly described in [36], resulting in this graph.

(231, 30, 9, 3) Brouwer [6]. The Cameron graph.

(243, 110, 37, 60) Goethals and Seidel [22].

Consider the orthogonal complement of the ternary Golay code, which has 243 words. On them we define a graph, with two words adjacent if their Hamming distance is 9.

(253, 140, 87, 65) [29, S.6]; a rank 3 representation of M_{23}.

We first build the Witt design on 23 points which is a $2-(23, 7, 21)$ design. We then build the intersection graph of blocks with intersection size 3.

(275, 112, 30, 56) [29, S.13]. The McLaughlin graph.

(276, 140, 58, 84) Haemers and Tonchev [24].

The graph is built from from McLaughlinGraph, with an added isolated vertex. We then perform Seidel switching on a set of 28 disjoint 5-cliques.

(280, 117, 44, 52) Mathon and Rosa [39].

The vertices of the graph are all 280 partitions of a set of cardinality 9 into 3-sets, e.g. $\{\{a,b,c\},\{d,e,f\},\{g,h,i\}\}$. The cross-intersection of two partitions $P = \{P_1, P_2, P_3\}$ and $P' = \{P'_1, P'_2, P'_3\}$ being defined as $\{P_i \cap P'_j : 1 \leq i,j \leq 3\}$, two vertices of 'G' are set to be adjacent if the cross-intersection of their respective partitions does not contain exactly 7 nonempty sets.

(280, 135, 70, 60) [32, Table 9, p.51].

This graph is built from the rank 4 action of J_2 on the cosets of a subgroup $3.PGL(2,9)$.

(416, 100, 36, 20) [29, S.14]; (rank 3 representation of $G_2(4)$).

This graph is isomorphic to the subgraph of the Suzuki graph [29, S.15] induced on the neighbors of a vertex.

(560, 208, 72, 80) [32, Table 9, p.45].

Obtained as the union of 4 orbitals (among the 13 that exist) of the group $Sz(8)$ in its primitive action on 560 points.

(630, 85, 20, 10) W. Haemers [23], see also [10, Sect.10.B.(v)].

This graph is the line graph of a $pg(5,18,2)$; its point graph is the $(175,72,20,36)$-srg from this table. One then selects a subset of 630 maximum cliques in the latter to form the set of lines of the $pg(5,18,2)$.

(784, 243, 82, 72) R. Mathon, cf. [10, Sect.6.D].

This and the following two are Mathon's graphs from merging classes in the product of pseudo-cyclic association scheme for action of $O_3(8)$ on elliptic lines in $PG(2,8)$, studied by H.D.L. Hollmann [27].

(784, 270, 98, 90) R. Mathon, cf. [10, Sect.6.D].

(784, 297, 116, 110) R. Mathon, cf. [10, Sect.6.D].

(1288,792, 476, 504) Brouwer and van Eijl [9].

This graph is built on the words of weight 12 in the binary Golay code. Two of them are then made adjacent if their symmetric difference has weight 12.

(1782,416, 100, 96) [29, S.15]. Suzuki graph, rank 3 representation of Suz.

4.2 Two-weight codes database

The rest of the fixed-size constructions of strongly regular graphs in the database originate from linear d-dimensional two-weight codes of length ℓ with weights w_1 and w_2 over $\mathbb{F}_q$. We use data shared by Eric Chen [11], data due to Axel Kohnert [37] and shared by Alfred Wassermann, data from I. Bouyukliev and J. Simonis [3, Theorem 4.1], and from L.A. Disset [16].

Graph parameters				Code parameters					Ref.
n	k	λ	μ	q	ℓ	d	w_1	w_2	
81	50	31	30	3	15	4	9	12	[11]
243	220	199	200	3	55	5	36	45	[11]
256	153	92	90	4	34	4	24	28	[11]
256	170	114	110	2	85	8	40	48	[11]
256	187	138	132	2	68	8	32	40	[11]
512	73	12	10	2	219	9	96	112	[11]
512	219	102	84	2	73	9	32	40	[11]
512	315	202	180	2	70	9	32	40	[37]
625	364	213	210	5	65	4	50	55	[11]
625	416	279	272	5	52	4	40	45	[11]
625	468	353	342	5	39	4	30	35	[3]
729	336	153	156	3	168	6	108	117	[16]
729	420	243	240	3	154	6	99	108	[11]
729	448	277	272	3	140	6	90	99	[37]
729	476	313	306	3	126	6	81	90	[11]
729	532	391	380	3	98	6	63	72	[11]
729	560	433	420	3	84	6	54	63	[11]
729	616	523	506	3	56	6	36	45	[11]
1024	363	122	132	4	121	5	88	96	[16]
1024	396	148	156	4	132	5	96	104	[16]
1024	429	176	182	4	143	5	104	112	[16]
1024	825	668	650	2	198	10	96	112	[11]

Note that some of these codes are members of infinite families; this will be explored and extended in forthcoming work.

5 Infinite families

These are roughly divided into two parts: graphs related to finite geometries over finite fields (in particular various classical geometries), and graphs obtained by combinatorial constructions.

5.1 Graphs from finite geometries

Here q denotes a prime power, and $\epsilon \in \{-,+\}$.

- Graphs arising from projective geometry designs are discussed in Sect. 5.2, along with other Steiner graphs.
- Paley graphs. The vertices are the elements of $\mathbb{F}_q$, with $q \equiv 1 \mod 4$; two vertices are adjacent if their difference is a nonzero square in $\mathbb{F}_q$; see [8, 9.1.2].

- Polar space graphs. These include polar spaces for orthogonal and unitary groups, see entries $O^{\epsilon}_{2d}(q)$, $O_{2d+1}(q)$, and $U_d(q)$ in [8, Table 9.9]. Sagemath also has an implementation of polar spaces for symplectic groups (entry $Sp_{2d}(q)$ in [loc.cit.]), but we do not use them in the database, as they have the same parameters as these for orthogonal groups.

- Generalised quadrangle graphs, $GQ(s,t)$ in [8, Table 9.9]. Apart from these appearing as polar space graphs, with $s = t = q$, $s^2 = t = q$, and $s = q^2$, $t = q^3$, we provide other examples, as follows.

  – Unitary dual polar graphs. This gives $s = q^3$, $t = q^2$.

  – $GQ(q-1, q+1)$-graphs for q odd are constructed following Ahrens and Szekeres, see [42, 3.1.5], and for q even we provide the $T^*_2(\mathcal{O})$ construction, see [42, 3.1.3], from a hyperoval $\mathcal{O}$ in $PG(2, q)$.

  – $GQ(q+1, q-1)$ are constructed as line graphs of $GQ(q-1, q+1)$.

- Affine polar graphs. These are the entry $VO^{\epsilon}_{2d}(q)$ in [8, Table 9.9].

- Graphs of non-degenerate hyperplanes of orthogonal polar spaces, with adjacency specified by degenerate intersection; see $NO^{\epsilon}_{2d+1}(q)$ in [8, Table 9.9]. These are constructions by Wilbrink, cf. [10, Sect.7.C]. The implementation in Sagemath simply takes the appropriate orbit and orbital of the orthogonal group acting on the hyperplanes using parameters of the graph, namely $v = q^d(q^d + \epsilon)/2$, $k = (q^d - \epsilon)(q^(d-1) + \epsilon)$.

- Graphs of non-isotropic points of polar spaces, with adjacency specified by orthogonality. These include a number of cases.

  – Non-isotropic points of orthogonal polar spaces over $\mathbb{F}_2$; see $NO^{\epsilon}_{2d}(2)$ in [8, Table 9.9].

  – One class of non-isotropic points of orthogonal polar spaces over $\mathbb{F}_3$; see $NO^{\epsilon}_{2d}(3)$ in [8, Table 9.9].

  – One class of non-isotropic points of orthogonal polar spaces (specified by a non-degenerate quadratic form F) over $\mathbb{F}_5$; see $NO^{\epsilon\perp}_{2d+1}(5)$ in [8, Table 9.9]. This is a construction by Wilbrink, cf. [10, Sect.7.D], where the class of points p is described in terms of the type of the quadric specified by $p^{\perp} \cap Q$, where Q is the set of isotropic points of the space, i.e. $Q := \{x \in PG(2d, 5) \mid F(x) = 0\}$, and $p^{\perp} := \{x \in Q \mid F(p + x) = F(p)\}$. The implementation in Sagemath takes $\{x \in PG(2d, 5) \mid F(x) = \pm 1\}$ for $\epsilon = +$, and the rest of non-isotropic points for $\epsilon = -$.

  – Non-isotropic points of unitary polar spaces; see $NUd(q)$ in [8, Table 9.9].

- Graphs of Taylor two-graphs, see [8, Table 9.9] and [10, Sect.7E]. Note that we implement an efficient construction that does not need all the triples of the corresponding two-graphs, by first directly constructing the descendant strongly regular graph on q^3 vertices, and a partition of its vertices into cliques. The latter provides a set to perform Seidel switching on the disjoint union with K_1, and obtain the strongly regular graph on q^3+1 vertices. See Sagemath documentation for `graphs.TaylorTwographSRG` for details.

5.2 Graphs from combinatorics

- Johnson Graphs $J(m, 2)$, see Example 2.1.
- Orthogonal Array block graphs $OA(k, n)$ – thanks to a massive amount of work in 2013/2014 and to the collaboration of Julian R. Abel and Vincent Delecroix, Sage is able to build a very large collection of state-of-the-art orthogonal arrays. No new constructions of Orthogonal Arrays were needed for the present work, and the link between Sage's database of Orthogonal Arrays and the database of Strongly Regular Graphs filled three new entries in Andries Brouwer's database.
- Steiner Graphs (intersection graphs of BIBD) – Sage can already build several families of Balanced Incomplete Block Designs (when $k \le 5$, or projective planes, or other recursive constructions and fixed-size instances). Several new constructions from the Handbook of Combinatorial Designs were added to Sage while working on this project.
- Goethals-Seidel graphs, see [21].
- Haemers graphs, see [10, Sect.8.A].
- RSHCD – a (n, ϵ)-regular symmetric Hadamard matrix M with constant diagonal is an $n \times n$ symmetric ± 1-matrix such that: 1) $MM^T = nI$; 2) its rows sums are all equal to $\delta\epsilon\sqrt{n}$, where $\epsilon \in \{-1, +1\}$ and δ is the (constant) diagonal value of M. These matrices yield regular two-graphs.
- Two-graph descendants. Each *regular two-graph* (a certain class of 3-uniform v-vertex hypergraphs having 2μ three-edges on each pair of points, cf. e.g. [8, Chap.10]) gives rise to a strongly regular graph with parameters $(v-1, 2\mu, 3\mu - v/2, \mu)$ obtained by descendant construction, see e.g. [8, Sect.10.3].
- Switch OA Graphs – these strongly regular graphs are obtained from Orthogonal Array block graphs (see above). From such a graph G obtained from an $OA(k, n)$, the procedure is to (1) add a new isolated vertex v; (2) perform Seidel switching on the union of $\{v\}$ and several disjoint n-cocliques of G. Note that a n-coclique in G corresponds to a parallel class of the $OA(k, n)$, and that those are easily obtained from an $OA(k+1, n)$ (i.e. a *resolvable* $OA(k, n)$).

- Polhill Graphs – In [43], Polhill produced 5 new strongly regular graphs on 1024 vertices as Cayley graphs. His construction is able to produce larger strongly regular graphs of order ≥ 4096, though the current implementation only covers the $n = 1024$ range.
- Mathon's pseudo-cyclic strongly regular graphs related to symmetric conference matrices, optionally parameterised by a strongly regular graph with parameters of a Paley graph, and a skew-symmetric Latin square [38, 44].
- Pseudo-Paley and Pasechnik graphs from skew-Hadamard matrices. These are constructions due to Goethals-Seidel [10] and Pasechnik [41], constructing graphs on $(4m-1)^2$ vertices from skew-Hadamard matrices of order $4m$. Sage builds the corresponding skew-Hadamard matrices from a small database featuring classical constructions of skew-Hadamard matrices from [26] and small examples from (anti)-circulant matrices [20, 47].

6 Missing values

Among the 1150 parameters which are satisfiable according to Andries Brouwer's database, our implementation can realize 1122. Among the 28 that remain, many can be deduced from each other by taking complements, or are a two-graph descendant of another. The reduced list consists of 11 currently missing entries:

(196	90	40	42)	RSHCD$^-$ (may not exist, cf. Sect. 7)
(196	135	94	90)	Huang, Huang and Lin [28]
(324	152	70	72)	RSHCD$^-$ [25]
(324	153	72	72)	RSHCD+ [25]
(378	52	1	8)	Cossidente-Penttila hemisystem in $PG(3, 5^2)$ [13]
(378	116	34	36)	Muzychuk S6(n=3,d=3) [40]
(512	133	24	38)	Godsil (q=8,r=3) [19]
(676	325	156	156)	RSHCD$^+$ (may not exist, cf. Sect. 7)
(765	192	48	48)	Ionin-Kharaghani [30]
(900	435	210	210)	RSHCD$^+$ (may not exist, cf. Sect. 7)
(936	375	150	150)	Janko-Kharaghani [33]

Note that we already have a draft implementation of the construction from [13].

7 Incorrect RSHCD constructions ?

We were unable to reproduce several constructions of Regular Symmetric Hadamard matrices with Constant Diagonal (RSHCDs) and thus the corresponding strongly regular graphs (in the sense of [8, Sect.10.5]).

- In [8, Sect.10.5.1, (i)], the construction of RSHCD$(196, -)$ is attributed to [31], in which the existence of a $(4k^2, 2k^2 + k, k^2 + k)$-strongly regular graph, equivalent to a RSHCD$(196, -)$, is claimed in Theorem 8.2.26.(iii). The latter says that the RSHCD$(196, -)$ can be easily obtained from the RSHCD$(196, +)$ from [31, Theorem 8.2.26.(ii)]. While the construction of (ii) was successfully implemented in Sage, following the authors' instructions for (iii) did not lead us to the RSHCD$(196, -)$. Communication with the authors did not solve the issue, and we are not aware of any other proof of the existence of a $(196, 90, 40, 42)$-strongly regular graph.

- In [8, Sect.10.5.1, (iii)] one finds the following claim, attributed to [48, Corollary 5.12].

  If $n - 1$ and $n + 1$ are odd prime powers, there exists a RSHCD$(n^2, +)$.

  We implemented the construction provided in [48, Corollary 5.12], but that did not lead us to the expected strongly regular graph. We also note that while Corollary 5.12 does not claim that the provided matrices are regular, that claim appears in the theorem on which it relies. The author of [48, Corollary 5.12] did not answer our message, and we discarded this construction as broken in our work.

  This construction should have been able to produce a RSHCD$(676, +)$ and a RSHCD$(900, +)$.

- In [8, Sect.10.5.1, (iv)] one finds the following claim, attributed to [48, Corollary 5.16].

  If $a + 1$ is a prime power and there exists a conference matrix of order a, then there exists a RSHCD$(a^2, +)$.

  As with the previous result, following the instructions did not lead us to the expected result. This construction should have been able to produce a RSHCD$(676, +)$ and a RSHCD$(900, +)$.

8 Entries added to the database during this work

Through linking Sage's database of Orthogonal Arrays with its database of strongly regular graphs, we were able to update the following three values:

- $(196, 78, 32, 30)$ – can be obtained from an $OA(6, 14)$ [46]
- $(324, 102, 36, 30)$ – can be obtained from an $OA(6, 18)$ [1]
- $(324, 119, 46, 42)$ – can be obtained from an $OA(7, 18)$ [1]

This can be seen as a by-product of making two mathematical databases inter-operable, when they formerly only existed in printed form. In our implementation, any update of the combinatorial designs databases can benefit to the database of strongly regular graphs.

We obtained a $(1024, 462, 206, 210)$-strongly regular graph while going through the constructions from [43], though this value did not appear in the online database at that time.

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References

[1] R. J. R. Abel. Existence of five MOLS of orders 18 and 60. *J. Combin. Des.*, 23(4):135–139, 2015.

[2] A. Bondarenko. On Borsuk's conjecture for two-distance sets. *Discrete Comput. Geom.*, 51(3):509–515, 2014.

[3] I. Bouyukliev and J. Simonis. Some new results on optimal codes over $\mathbb{F}_5$. *Des. Codes Cryptogr.*, 30(1):97–111, 2003.

[4] A. E. Brouwer. Parameters of strongly regular graphs. `http://www.win.tue.nl/~aeb/graphs/srg/srgtab.html`.

[5] A. E. Brouwer. Polarities of G. Higman's symmetric design and a strongly regular graph on 176 vertices. *Aequationes Math.*, 25(1):77–82, 1982.

[6] A. E. Brouwer. Uniqueness and nonexistence of some graphs related to M_{22}. *Graphs Combin.*, 2:21–29, 1986.

[7] A. E. Brouwer, A. M. Cohen, and A. Neumaier. *Distance-regular graphs*, volume 18 of *Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)]*. Springer-Verlag, Berlin, 1989.

[8] A. E. Brouwer and W. H. Haemers. *Spectra of graphs*. Universitext. Springer, New York, 2012.

[9] A. E. Brouwer and C. A. van Eijl. On the p-rank of the adjacency matrices of strongly regular graphs. *J. Algebraic Combin.*, 1(4):329–346, 1992.

[10] A. E. Brouwer and J. H. van Lint. Strongly regular graphs and partial geometries. In *Enumeration and design (Waterloo, Ont., 1982)*, pages 85–122. Academic Press, Toronto, ON, 1984.

[11] E. Chen. *Online database of two-weight codes*, 2015. http://moodle.tec.hkr.se/~chen/research/2-weight-codes/search.php.

[12] K. Coolsaet. The uniqueness of the strongly regular graph srg$(105, 32, 4, 12)$. *Bull. Belg. Math. Soc. Simon Stevin*, 12(5):707–718, 2005.

[13] A. Cossidente and T. Penttila. Hemisystems on the Hermitian surface. *J. London Math. Soc. (2)*, 72(3):731–741, 2005.

[14] J. Degraer and K. Coolsaet. Classification of some strongly regular subgraphs of the mclaughlin graph. *Discr. Math.*, 308:395–400, 2008.

[15] T. S. Developers. *Sage Mathematics Software (Version 6.10)*, 2015. http://www.sagemath.org.

[16] L. A. Dissett. *Combinatorial and computational aspects of finite geometries.* ProQuest LLC, Ann Arbor, MI, 2000. Thesis (Ph.D.)–University of Toronto (Canada).

[17] I. A. Faradjev and M. H. Klin. Computer package for computations with coherent configurations. In S. M. Watt, editor, *Proceedings of the 1991 International Symposium on Symbolic and Algebraic Computation, ISSAC '91, Bonn, Germany, July 15-17, 1991*, pages 219–223. ACM, 1991. https://github.com/dimpase/coco.

[18] D. G. Fon-Der-Flaass. New prolific constructions of strongly regular graphs. *Adv. Geom.*, 2(3):301–306, 2002.

[19] C. D. Godsil. Krein covers of complete graphs. *Australas. J. Combin.*, 6:245–255, 1992.

[20] J.-M. Goethals and J. J. Seidel. A skew Hadamard matrix of order 36. *J. Austral. Math. Soc.*, 11:343–344, 1970.

[21] J.-M. Goethals and J. J. Seidel. Strongly regular graphs derived from combinatorial designs. *Canad. J. Math.*, 22:597–614, 1970.

[22] J.-M. Goethals and J. J. Seidel. The regular two-graph on 276 vertices. *Discrete Math.*, 12:143–158, 1975.

[23] W. Haemers. A new partial geometry constructed from the Hoffman-Singleton graph. In *Finite geometries and designs (Proc. Conf., Chelwood Gate, 1980)*, volume 49 of *London Math. Soc. Lecture Note Ser.*, pages 119–127. Cambridge Univ. Press, Cambridge-New York, 1981.

[24] W. H. Haemers and V. D. Tonchev. Spreads in strongly regular graphs. *Des. Codes Cryptogr.*, 8(1-2):145–157, 1996. Special issue dedicated to Hanfried Lenz.

[25] W. H. Haemers and Q. Xiang. Strongly regular graphs with parameters $(4m^4, 2m^4 + m^2, m^4 + m^2, m^4 + m^2)$ exist for all $m > 1$. *European J. Combin.*, 31(6):1553–1559, 2010.

[26] M. Hall, Jr. *Combinatorial theory*. Wiley-Interscience Series in Discrete Mathematics. John Wiley & Sons, Inc., New York, second edition, 1986. A Wiley-Interscience Publication.

[27] H. D. L. Hollmann. Association schemes, 1982. MSc thesis, Eindhoven University of Technology.

[28] T. Huang, L. Huang, and M.-I. Lin. On a class of strongly regular designs and quasi-semisymmetric designs. In *Recent developments in algebra and related areas*, volume 8 of *Adv. Lect. Math. (ALM)*, pages 129–153. Int. Press, Somerville, MA, 2009.

[29] X. L. Hubaut. Strongly regular graphs. *Discrete Math.*, 13(4):357–381, 1975.

[30] Y. J. Ionin and H. Kharaghani. New families of strongly regular graphs. *J. Combin. Des.*, 11(3):208–217, 2003.

[31] Y. J. Ionin and M. S. Shrikhande. *Combinatorics of symmetric designs*, volume 5 of *New Mathematical Monographs*. Cambridge University Press, Cambridge, 2006.

[32] A. A. Ivanov, M. H. Klin, and I. A. Faradjev. The primitive representations of the non-abelian simple groups of order less than 10^6, part II. Preprint (Russian), 1984. The Insitute for System Studies (VNIISI), Moscow.

[33] Z. Janko and H. Kharaghani. A block negacyclic Bush-type Hadamard matrix and two strongly regular graphs. *J. Combin. Theory Ser. A*, 98(1):118–126, 2002.

[34] T. Jenrich and A. E. Brouwer. A 64-dimensional counterexample to Borsuk's conjecture. *Electron. J. Combin.*, 21(4):Paper 4.29, 3, 2014.

[35] L. K. Jørgensen and M. Klin. Switching of edges in strongly regular graphs. I. A family of partial difference sets on 100 vertices. *Electron. J. Combin.*, 10:Research Paper 17, 31, 2003.

[36] M. Klin, C. Pech, S. Reichard, A. Woldar, and M. Ziv-Av. Examples of computer experimentation in algebraic combinatorics. *Ars Math. Contemp.*, 3(2):237–258, 2010.

[37] A. Kohnert. Constructing two-weight codes with prescribed groups of automorphisms. *Discrete Appl. Math.*, 155(11):1451–1457, 2007.

[38] R. Mathon. Symmetric conference matrices of order $pq^2 + 1$. *Canad. J. Math.*, 30(2):321–331, 1978.

[39] R. Mathon and A. Rosa. A new strongly regular graph. *J. Combin. Theory Ser. A*, 38(1):84–86, 1985.

[40] M. Muzychuk. A generalization of Wallis-Fon-Der-Flaass construction of strongly regular graphs. *J. Algebraic Combin.*, 25(2):169–187, 2007.

[41] D. V. Pasechnik. Skew-symmetric association schemes with two classes and strongly regular graphs of type $L_{2n-1}(4n-1)$. *Acta Appl. Math.*, 29(1-2):129–138, 1992. Interactions between algebra and combinatorics.

[42] S. E. Payne and J. A. Thas. *Finite generalized quadrangles*. EMS Series of Lectures in Mathematics. European Mathematical Society (EMS), Zürich, second edition, 2009.

[43] J. Polhill. Negative Latin square type partial difference sets and amorphic association schemes with Galois rings. *J. Combin. Des.*, 17(3):266–282, 2009.

[44] J. J. Seidel and D. E. Taylor. Two-graphs, a second survey. In *Algebraic methods in graph theory, Vol. I, II (Szeged, 1978)*, volume 25 of *Colloq. Math. Soc. János Bolyai*, pages 689–711. North-Holland, Amsterdam-New York, 1981.

[45] T. Spence. Strongly regular graphs on at most 64 vertices. `http://www.maths.gla.ac.uk/~es/srgraphs.php`.

[46] D. T. Todorov. Four mutually orthogonal Latin squares of order 14. *J. Combin. Des.*, 20(8):363–367, 2012.

[47] J. Wallis. A skew-Hadamard matrix of order 92. *Bull. Austral. Math. Soc.*, 5:203–204, 1971.

[48] W. D. Wallis, A. P. Street, and J. S. Wallis. *Combinatorics: Room squares, sum-free sets, Hadamard matrices*. Lecture Notes in Mathematics, Vol. 292. Springer-Verlag, Berlin-New York, 1972.