Constraints On Holographic Cosmological Models From Gamma Ray Bursts

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Abstract

We use Gamma Ray Bursts (GRBs) data to put additional constraints on a set of holographic dark energy models. GRBs are among the most complex, energetic and regular astrophysical events known universe providing us the opportunity to obtain information from the history of cosmic expansion up to about redshift of $z \sim 6$ offering us a complementary observational test to determine the nature of dark energy and they are also complementary to SNIa test. We found that the ΛCDM model is the best fit to the data, although a preliminary statistical analysis seems to indicate that the holographic models studied show interesting agreement with observations, except *Ricci Scale CPL* model. These results show the importance of GRBs measurements to provide additional observational constraints to alternative cosmological models, which are necessary to clarify the way in the paradigm of dark energy or potential alternatives.

Keywords: Gamma Ray Bursts, Holographic Principle, Dark Energy.
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1. Introduction

In order to explain the current acceleration of the universe, the fine-tuning problem of the value of Λ and the cosmic coincidence problem, different alternative models have been proposed. In framing the question of the nature of dark energy there are two generally direcctions. The firts is to assume a new type of component of energy density, Which can be a constant fluid density or dynamic density. The other direction is modify the Einstein's equations thinking that the metric is inappropriate or that gravity works differently on large scales. The observational tests are of great importance to discriminate between these scenarios [1]. The holographic dark energy is one dynamical DE model proposed in the context of quantum gravity, so called ***holographic principle***, the which arose from black hole and string theories [2]. The holographic principle states the number of degrees of freedom of a physical system, apart from being constrained by an infrared cutoff, it should be finite and it should scale with its bounding area rather than with its volume. Specifically, it is derived with the help of entropy-area relation of thermodynamics of black hole horizons in general relativity which is also known as the Bekenstein-Hawking entropy bound, i.e., $S \simeq M_p^2 L^2$, where S is the maximum entropy of the system of length L and $M_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass. This principle can be applied to the dynamics of the universe, where L may be associated with a cosmological scale and its energy density as:

$$\rho_H = \frac{3c^2 M_p^2}{L^2}. \quad (1)$$

If for example L is the Hubble's radius, which represents the current size of the universe, then the associated energy density represents the density of dark energy ρ_Λ.

2. Holographic Dark Energy Models

The Friedmann equations for a spatially flat universe can be written as:

$$3H = 8\pi G\left(\rho_m + \rho_H\right), \quad (2)$$

where ρ_m is the energy density of the matter component and ρ_H It is the holographic dark energy density. These components are related by an interaction Q term as:

$$\frac{d\rho_m}{dt} + 3H\rho_m = Q \qquad \frac{d\rho_H}{dt} + 3H(1+w_H)\rho_H = -Q, \quad (3)$$

where $w_H = p_H/\rho_H$ is the equation of state of holographic dark energy density. The rate of change of Hubble can be written as:

$$\frac{dH}{dt} = -\frac{3}{2}H^2\left(1 + w^{eff}\right), \quad (4)$$

where $w^{eff} = w/(1+r)$ is the effective equation of state of the cosmic fluid and $r = \rho_m/\rho_H$ is the ratio of energy densities, which is related to the saturation parameter c^2 as $c^2(1+r) = 1$, which establishes that energy in a box of size L should not exceed the energy of the black hole of the same size, under the condition $L^3\rho_H \leq M_p^2 L$. Different scales lead to different cosmological models [2].

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2.1. ΛCDM

We begin our analysis with the standard cosmological model. In this paradigm, the DE is provided by the cosmological constant Λ, with an EoS, such that $w = -1$. In this model the Friedmman equation $E^2(z,\Theta)$ for flat universe is given by

$$E^2(z,\Theta) = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda, \tag{5}$$

where Ω_m and Ω_Λ are the density parameters for matter and dark energy respectively and Ω_r is the radiation density parameter. The free parameters are h, Ω_m, Ω_Λ and the best fit is shown in Table 1.

Λ Cold Dark Matter model	
$h = 0.7009 \pm 0.0035$	$\Omega_\Lambda = 0.716 \pm 0.028$
$\Omega_m = 0.266 \pm 0.0042$	

Table 1: Best fit parameters with all data set to ΛCDM model.

2.2. Hubble Radius Scale

In this model $L = H^{-1}$ and his dark energy density $\rho_H = 3c^2M_p^2H^2$ and Friedmann equation can be written as:

$$E^2(z) = \left[(1-2q_0) + 2(1+q_0)(1+z)^{3n/2}\right]^{1/n} \left(\frac{1}{3}\right)^{1/n}, \tag{6}$$

where q_0 is the present value of the deceleration parameter. This model is similar to ΛCDM when n = 2. The free parameters are h, q_0 and n, whose best fit of parameters is shown in the Table 2.

Hubble Radius Scale	
$h = 0.7004 \pm 0.0038$	$n = 1.71 \pm 0.20$
$q_0 = 0.569 \pm 0.047$	

Table 2: Best fit parameters with all data set to *HRS* model.

2.3. Future Event Horizon $\xi = 1$

With $L = R_E$ the holographic DE density $\rho_H = 3c^2M_p^2R_E^{-2}$ where R_E is the future event horizon. The Friedmann equation is given by:

$$E^2(z) = (1+z)^{3/2-1/c}\sqrt{\frac{1+r_0(1+z)}{r_0+1}} \left[\frac{\sqrt{r_0(1+z)+1}-1}{\sqrt{r_0+1}+1}\right]^{2/c}, \tag{7}$$

where $R_E = c\sqrt{(1+r)}H^{-1}$ and $r_0 = \Omega_0/(1-\Omega_0)$. The best fit of free parameters h, r_0 and c is given in the Table 3.

Future Event Horizon $\xi = 1$	
$h = 0.6799 \pm 0.0025$	$r_0 = 0.322 \pm 0.032$
$c = 1.046 \pm 0.017$	

Table 3: Best fit parameters with all data set to *FEH* model.

2.4. Ricci Scale CPL

The Ricci scalar $R = 6(2H^2+\dot{H})$ is relate to cutoff-scale through $L^2 = 6/R$ and energy density:

$$\rho_H = 3c^2M_p^2\frac{R}{6} = \alpha\left(2H^2+\dot{H}\right), \tag{8}$$

where $\alpha = 3c^2/8\pi G$. If we use the CPL parameterization $w(a) = w_0 + (1-a)w_1$, the Friedmann equation can be written as:

$$E^2(z,\Theta) = (1+z)^{\frac{3}{2}\frac{1+r_0+w_0+4w_1}{1+r_0+3w_1}} \left[\frac{1+r_0+3w_1\left(\frac{z}{1+z}\right)}{1+r_0}\right]^{-\frac{1}{2}\frac{1+r_0+3w_0}{1+r_0+3w_1}}. \tag{9}$$

The free parameters of this model are h, r_0, w_0 and w_1. The best fit is given in the Table 4.

Ricci Scale CPL	
$h = 0.6518 \pm 0.0021$	$r_0 = 9.39^{+0.51}_{-0.47}$
$w_0 = -2.64^{+0.49}_{-0.55}$	$w_1 = 0.46^{+0.34}_{-0.33}$

Table 4: Best fit parameters with all data set to *RSCPL* model.

2.5. Ricci Scale Q

If interaction term is given by $Q = 3H\beta\rho_H$, then

$$\beta = \frac{1}{1+r}\left[rw - \frac{\dot{w}}{H}\right] \tag{10}$$

and the EoS is given by:

$$w = -\frac{1}{6}\frac{u-s-(u+s)Aa^s}{1-Aa^s} \tag{11}$$

where $u \equiv r_0 - 3w_0 + 3\beta$, $v \equiv r_0 + 3w_0 + 3\beta$, $s \equiv (u^2 - 12\beta(1+r_0-3w_0))^{1/2}$ y $A \equiv (v-s)/(v+s)$. The Friedmann equation can be writen as:

$$E^2(z,\Theta) = \left[\frac{n(1+z)^{-s}-m}{n-m}\right]^{\frac{3}{2}\frac{lm-kn}{mns}} (1+z)^{\frac{3}{2}(1-\frac{k}{m})}, \tag{12}$$

such that $m \equiv 1 + r_0 - 1/2(v-s)$, $n \equiv [1 + r_0 - 1/2(v+s)]\,A$, $k \equiv 1/6(u-s)$ y $l \equiv 1/6(u+s)A$. The free parameters are h, r_0, w_0 and β, whose best fit is shown in the Table 5.

Ricci Scale Q	
$h = 0.6999 \pm 0.0038$	$r_0 = 0.201^{+0.025}_{-0.022}$
$w_0 = -0.842^{+0.055}_{-0.056}$	$\beta = -0.011^{+0.015}_{-0.019}$

Table 5: Best fit parameters with all data set to *RSCPL* model.

3. Gamma-Ray Bursts

3.1. GRBs Model

Gamma-ray bursts (GRBs) are the most luminous astrophysical events observable today, because they are at cosmological distances. The duration of a gamma-ray burst is typically a few seconds, but can range from a few milliseconds to several minutes. The initial burst at gammay-ray wavelengths is usually followed by a longer lived afterglow at longer wavelengths (X-ray, ultraviolet, optical, infrared, and radio). Gamma-ray bursts have been detected by orbiting satellites about two to three times per week. Most observed GRBs appear to be collimated emissions caused by the collapse of the core of a rapidly rotating, high-mass star into a black hole. At least once a day, a powerful source of gamma rays temporarily appears into the sky in an unpredictable location and later disappears, which last for milliseconds to minutes. In the location of the gamma ray event it is usually observed a dominant afterglow in X-rays, optical and radio after long decays.

3.2. GRBs Data

We use GRB data in the form of the model-independent distance from Wang (2008) [6], which were derived from the data of 69 GRBs with $0.17 \leq z \leq 6.6$ from Schaefer (2007). The GRB data are included in our analysis by adding the following term to the given model:

$$\chi^2_{GRB} = \left[\Delta\bar{r}_p(z_i)\right].(C^{-1}_{GRB})_{ij}.\left[\Delta\bar{r}_p(z_i)\right], \quad (13)$$

where $\Delta\bar{r}_p(z_i) = \bar{r}_p^{data}(z_i) - \bar{r}_p(z_i)$ and $\bar{r}_p(z_i)$ is given by

$$\bar{r}_p(z_i) = \frac{r_p(z)}{r_p(0.17)} \quad (14)$$

where

$$r_p(z) = \frac{(1+z)^{1/2}}{z}\frac{H_0}{ch}r(z) \quad (15)$$

and $r(z)$ is the comoving distance at z. The covariance matrix is given by:

$$C^{grb}_{ij} = \sigma(\bar{r}_p(z_i))\sigma(\bar{r}_p(z_j))\bar{C}^{grb}_{ij} \quad (16)$$

where $\bar{C}^{grb}_{ij}$ is the normalized covariance matrix:

$$\bar{C}^{grb}_{ij} = \begin{pmatrix} 1.0000 & & & & & \\ 0.7056 & 1.0000 & & & & \\ 0.7965 & 0.5653 & 1.0000 & & & \\ 0.6928 & 0.6449 & 0.5521 & 1.0000 & & \\ 0.5941 & 0.4601 & 0.5526 & 0.4271 & 1.0000 & \\ 0.5169 & 0.4376 & 0.4153 & 0.4242 & 0.2999 & 1.0000 \end{pmatrix} \quad (17)$$

Data point	(z)	$\bar{r}_p(z_i)^{dat}$	$\sigma(\bar{r}_p(z_i))^+$	$\sigma(\bar{r}_p(z_i))^-$
0	0.17	1.0000	-	-
1	1.036	0.9416	0.1688	0.1710
2	1.902	1.0011	0.1395	0.1409
3	2.768	0.9604	0.1801	0.1785
4	3.634	1.0598	0.1907	0.1882
5	4.500	1.0163	0.2555	0.2559
6	6.600	1.0862	0.3339	0.3434

Table 6: Distances independent model GRBs.

and

$$\sigma(\bar{r}_p) = \begin{cases} \sigma(\bar{r}_p(z_i))^+, if \bar{r}_p(z_i) \geq \bar{r}_p(z_i)^{dat} \\ \sigma(\bar{r}_p(z_i))^-, if \bar{r}_p(z_i) < \bar{r}_p(z_i)^{dat} \end{cases} \quad (18)$$

where $\sigma(\bar{r}_p(z_i))^+$ and $\sigma(\bar{r}_p(z_i))^-$, are 68% C.L errors given in the Table 6.
As complementary tests we use SNIa (580-Data point), CMB (1-Data point) and BAO (1-Data point) [4] (See Appendix).

4. Analysis and results

The maximum likelihood estimate for the best fit parameters is:

$$\mathcal{L}_{max} = exp\left[-\frac{1}{2}\chi^2_{min}\right] \quad (19)$$

If $\mathcal{L}_{max}$ has a Gaussian errors distribution, then $\chi^2_{min} = -2\ln\mathcal{L}_{max}$, So, for our analysis:

$$\chi^2_{min} = \chi^2_{GRBs} + \chi^2_{SNIa} + \chi^2_{CMB} + \chi^2_{BAO}. \quad (20)$$

The Figure 1 we shows the diagrams of statistical confidence at 1σ, 2σ and 3σ for different cosmological models and several parameter space, from a joint analysis of 69 GRBs (independent-model 6-Data point), SNIa (580-Data point), CMB (1-Data point), BAO (1-Data point) [4] [2].

In this paper we use the Akaike and Bayesian information criterion (AIC, BIC), which allow to compare cosmological models with different degrees of freedom, with respect to the observational evidence and the set of parameters [5]. The AIC and BIC can be calculated as:

$$AIC = -2\ln\mathcal{L}_{max} + 2k, \quad (21)$$

$$BIC = -2\ln\mathcal{L}_{max} + k\ln N, \quad (22)$$

where $\mathcal{L}_{max}$ is the maximum likelihood of the model under consideration, k is the number of parameters. BIC imposes a strict penalty against extra parameters for any set with N data. The prefered model is that which minimizes AIC and BIC, however, only the relative values between the different models is important [3]. The results are showes in the Table 7.

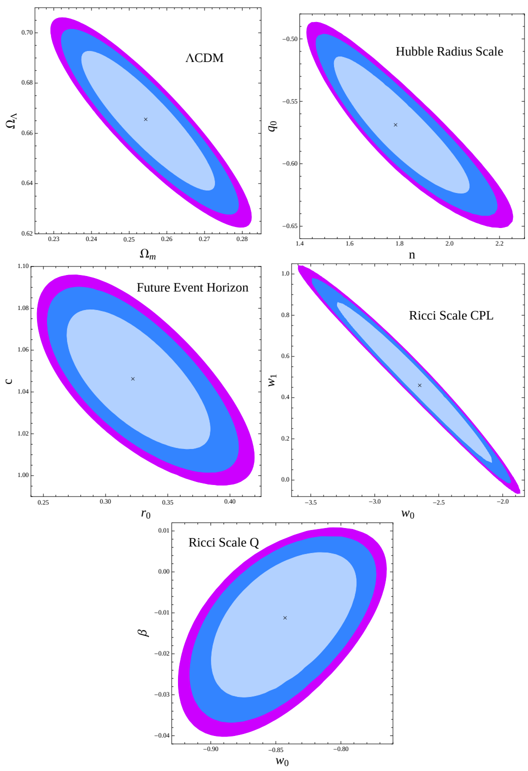

Figure 1: Diagrams of statistical confidence marginalizing different cosmological parameters at 1σ, 2σ and 3σ for the cosmological models.

Model	χ^2_{min}	AIC	BIC	ΔAIC	ΔBIC
ΛCDM	608.5	614.5	627.6	0.0	0.0
Hubble Radius S.	609.7	615.7	628.8	1.2	1.2
Future Event H.	657.3	663.3	676.4	48.8	48.8
Ricci scale CPL	917.3	925.3	924.8	310.8	315.2
Ricci scale Q	609.8	617.8	635.3	3.3	7.7

Table 7: AIC and BIC analysis to diferent dark energy models using all data sets.

5. Summary and discussion

We implement GRBs model-independent to complement SNIa Union2.1 sample to high redshift. We found that model-independent GRBs to provide additional observational constraints to constrain holographic dark energy models. Our analysis shows that ΛCDM model is preferred by ΔAIC and ΔBIC and the *Ricci Scale CPL* model can be ruled out for this analysis (Table 7).

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Appendix

Appendix .1. SNIa

Here, we use the Union 2.1 sample which contains 580 data. The SNIa data give the luminosity distance $d_L(z) = (1+z)r(z)$. We fit the SNIa with the cosmological model by minimizing the χ^2 value defined by

$$\chi^2_{SNIa} = \sum_{i=1}^{580} \frac{[\mu(z_i) - \mu_{obs}(z_i)]^2}{\sigma^2_{\mu_i}}, \quad (.1)$$

where $\mu(z) \equiv 5\log_{10}[d_L(z)/\mathtt{Mpc}] + 25$ is the theoretical value of the distance modulus, and μ_{obs} the corresponding observed.

Appendix .2. CMB

We also include CMB information by using the WMAP data. The χ^2_{cmb} for the CMB data is constructed as:

$$\chi^2_{cmb} = \frac{(1.7246 - R)^2}{0.03^2}. \quad (.2)$$

Here R is "shift parameter", defined as:

$$R = \frac{\sqrt{\Omega_m}}{c(1+z_*)} D_L(z). \quad (.3)$$

where $d_L(z) = D_L(z)/H_0$ and the redshift of decoupling z_* is $z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_m h^2)^{g_2}]$ and

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}}, g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}. \quad (.4)$$

Appendix .3. BAO

Similarly, for the DR7 BAO data, the χ^2 can be expressed as:

$$\chi^2_{6dFGS} = \left(\frac{d_z - 0.469}{0.017}\right)^2. \quad (.5)$$

where $d_z = r_s(z_d)/D_V(z)$ denotes the distance ratio. Here, $r_s(z_d)$ is the comoving sound horizon at the baryon drag epoch ($z_d = 0.35$) and $D_V(z)$ is the acoustic scale.