A detailed heterogeneous agent model for a single asset financial market with trading via an order book

Roberto Mota N and Hernán Larralde

Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México, Cuernavaca, Morelos, C.P. 62210, México

E-mail: mvr@fis.unam.mx

Abstract. We present an agent based model of a single asset financial market that is capable of replicating several non-trivial statistical properties observed in real financial markets, generically referred to as stylized facts. While previous models reported in the literature are also capable of replicating some of these statistical properties, in general, they tend to oversimplify either the trading mechanisms or the behavior of the agents. In our model, we strived to capture the most important characteristics of both aspects to create agents that employ strategies inspired on those used in real markets, and, at the same time, a more realistic trade mechanism based on a double auction order book. We study the role of the distinct types of trader on the return statistics: specifically, correlation properties (or lack thereof), volatilty clustering, heavy tails, and the degree to which the distribution can be described by a log-normal. Further, by introducing the practice of "profit taking", our model is also capable of replicating the stylized fact related to an asymmetry in the distribution of losses and gains.

1. Introduction

In the past five decades a great number of time series of prices of various financial markets have become publicly available and been subjected to analysis to characterize their statistical properties [1, 2, 3, 4, 5]. From the study of these time series, a set of properties common to many different markets, time periods and instruments, have been identified. The universality of these properties is of interest because events that may affect the changes of price (returns) in a certain market need not be the same that affect the changes in another market. Nevertheless, the results of these investigations show that the variations in prices share some non trivial statistical properties which have been called *stylized facts*. In this work we present and study a model of a financial market and its participants which reproduces these stylized facts. Among the key results found in our model is that the emergence of asymmetry in the distribution of price changes is linked to a trading practice employed in real financial markets called *profit taking*.

The majority of approaches used today to model financial markets fall into one of two categories: statistical models adjusted with to fit the history of past prices and the so called Dynamic Stochastic General Equilibrium (DSGE) models. The first kind of models are able to produce reasonable representations and volatility forecasts of financial systems[6] as long as the statistical properties of the prices with which they were calibrated do not change by a large margin. The second kind of models assume a "representative agent" for each of the participant sectors in the financial system, with the goal of these representative agents being the maximization of their utility[7]. To avoid creating deterministic dynamics without periods of depression or growth, DSGE models use exogenous stochastic terms which are supposed to mimic the varying conditions of the market, such as sudden peaks in the demand of a certain financial instrument or changes in the pricing of a commodity.

Despite of the fact that these models are capable of providing some explanations of the phenomena observed in financial markets, the premises over which they are built are crude approximations of reality[8, 9] and as a such they are not useful to gain insight into statistical phenomena as rich at that observed in the empirical analysis of financial time series.

This situation has given rise to the exploration of financial systems as "complex systems", [10] considering financial markets as something closer to what they actually are: systems where great number of different components interact amongst each other in a way that gives rise spontaneously to the observed macroscopic statistical properties.

Among the many types of models which consider the complexity of financial markets, there is a particular kind called Agent Based Models which employ a bottom-up approach and allow the modeler to trace back the emergence of the macroscopic statistical properties of the system as a consequence of the microscopic behavioral traits of its constituent agents[11]. Many models have been created that are capable of reproducing several stylized facts and giving microscopic explanations of their origins. These models have been constructed, in general, in one of two ways: there are models in which the agents do not use a particular set of strategies and instead participate in the market in a random fashion, and models in which the agents follow different specific strategies inspired in actual strategies used by participants of real markets. The first type of models usually make use of market trading structures similar to those used in real markets, such a double auction order books, and as a consequence the price formation is directly driven by the offers (to buy and sell) supplied by the agents [12, 13, 14, 15, 16], while the latter type of models usually have prices adjusted in a stochastic manner[17, 18, 19, 20]. Our aim in this paper is to present a model in which trading is mediated through an order book with agents employing realistic non-random strategies, which reproduces most of the main stylized facts observed in financial data.

1.1. Stylized facts

Among the stylized facts reported in the literature, the most widespread, which we study in this paper, are the following:

Absence of auto-correlations

It is a well known fact that auto-correlation function of the returns $R(t)$ is essentially zero for any value of the lag τ (except at very short time in which there is a negative correlation "bounce" [2]) This characteristic has been used as support for the efficient market hypothesis [21] since, by having zero correlation, the changes in the sign of price changes are unpredictable. As a consequence of this unpredictability there is it is not possible to incur in the practice of arbitrage. This has led people to also name this stylized fact as *absence of arbitrage*[22].

Volatility Clustering

The absence of correlations presented by the returns has been used as evidence that supports a series of models in which, for example, it is assumed that the prices behave like random walks with independent and identically distributed returns [23]. Nevertheless, in addition to the absence of auto-correlations in the raw returns series, it has been observed that some non lineal functions of returns do exhibit auto-correlations that remain positive for relatively long times. This behavior implies that the returns have a tendency to "agglomerate" in time in groups of similar magnitude but unpredictable sign. In the words of Benoit Mandelbrot, who was the first to report this behavior: "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes" [4].

Heavy tailed distribution of returns

Empirical studies have shown that the distributions of price changes do not have a normal distribution [4, 24, 25], thus implying that prices do not evolve in time as a Brownian random walk. Studies of the complementary cumulative distribution, defined as:

$$F(x) = 1 - Prob(X < x),$$

have shown that it behaves approximately as a power law $F(x) \sim x^{-\beta}$ with an exponent $\beta \in [2, 4]$ [22, 24]. This means that it is more likely to observe large price fluctuations than it would be if the prices followed a Brownian random walk. One way to quantify how much a distribution deviates from a normal distribution is to measure its excess kurtosis κ[26], which should have a value of $\kappa = 0$ for a normal distribution and a positive value for a distribution with heavy tails. Measurements of this quantity have found values of κ far from the normal regime; for instance, the kurtosis for the Standard & Poor's index measured over time intervals of 5 minutes has been reported to have a value of $\kappa \approx 16$[27].

Asymmetry in the distribution of returns

In addition to the heavy tailed returns distribution, it has also been observed that in many markets, large negative returns are present but there are not equally large positive returns. This characteristic is related to a negative skewness in the returns distribution, which has been reported in empirical studies[2].

Log-normal distribution of volatilities

The probability distribution of the volatility of individual firm shares and of indexes, defined as the average of the absolute returns over a time window, is well approximated by a log-normal distribution in its central part, while its tail is well adjusted by a power law with exponent μ satisfying $\mu \approx 3$[28].

In the next section we present a detailed description of the agent based model we propose: We begin by describing the double auction order book mechanism for buying and selling, followed by the trading rules employed by each type of agent: technical and fundamental. Our technical agents follow a "Moving average oscilator" strategy [29], which is commonly used by real technical traders. These traders also incur in profit taking if the price of the asset exceeds a certain threshold. Heterogeneity among technical agents is achieved by assigning different strategy parameters ("personalities") to different subsets of the technical agent population. The fundamental agents in our model "choose" a fundamental price, and change it according to the influx of news as well as distance to the positions of the rest of the agents in the market. The fundamental prices chosen by these agents, and their reaction to the incoming news, differ between each of the fundamental agents, as happens in real life.

After describing the model, the paper continues with a section in which we present the results obtained in simulations of the model and we focus on the stylized facts listed above, comparing the behavior of the model with representative empirical data. We also study the effect of varying the relative populations of agents as well as the parameters that control the practice of profit taking by the technical agents in the system.

We end with a section of concluding remarks.

2. Model

2.1. General Aspects

The model represents a financial market in wich a single asset, whose price is denoted by P_t is traded through a double auction order book by N agents. A double auction order book is the collection of all the *orders*, with an order being an offer to sell (ask) or to buy (bid) some amount of an asset. The orders are separated in two types, there are market orders and limit orders [30]. Market orders only specify the amount of units of the asset that its owner wants to buy or to sell (the volume), and the price at which these orders are executed is the best available price in the order book at that moment. The best prices in the book are called *best ask*, which corresponds to the order to sell at the lowest price, and the *best bid*, which corresponds to the order to buy at the highest price; the difference between the best ask and the best bid is called *spread*. Limit orders, on the other hand, do specify both a volume and a limit price which is the lowest or highest at which the owner of the order is willing to trade. If the owner of a limit order wishes to buy, then the limit price is the highest price the owner is willing to pay to buy the required volume specified in the order, similarly, if the owner of the order wishes to sell, the limit price is the lowest price the owner is willing to accept to sell the asset. When a limit order of a certain type, say a buy order (respectively, a sell order) enters the book with a price which is higher (lower) than the best ask (bid) price, then a trade is executed. Figure 1 shows the basic structure of an order book and Figure 2 shows the process of storage or elimination of limit orders.

Figure 1: Structure of an order book. Limit orders are stored using a price-time precedence rule: first they are stored in the level corresponding to their price and then they are ordered by their time of arrival. The difference between the price of the order to sell at the lowest price (best ask) and the price of the order to buy at the highest price (best bid) is called *spread*.

In the model, the population of agents is divided into two different sub-populations, with each sub-population employing one of two basic trading strategies: fundamental analysis or technical analysis.

Agents employing fundamental analysis believe that on the long run, the price of the asset P_t will reach its *fundamental price* p_f which represents the estimation of the "true" value of the asset based on the information that each agent possesses. In general, the fundamental prices of an asset are different for every fundamental agent. The strategy of a fundamental agent is then to try to take advantage of the deviations between asset's price P_t and the the agent's estimated fundamental price p_f, before the latter price is reached. Agents employing technical analysis, on the other hand, try to identify and exploit trends in the price time series.

These two types of strategies are representative of the main strategies used in real life trading and were first introduced in the Lux-Marchesi (LM) model [31]. The effects of these strategies on the dynamics of the price are opposed: while fundamental agents tend to stabilize the prices around the average value of their fundamental prices, technical agents tend to create periods of violent price changes.

Most of the parameters controlling the behavior of each agent are assigned at the beginning of each simulation, and even if two agents belong to the same group (fundamental or technical) the difference in the values of their controlling parameters

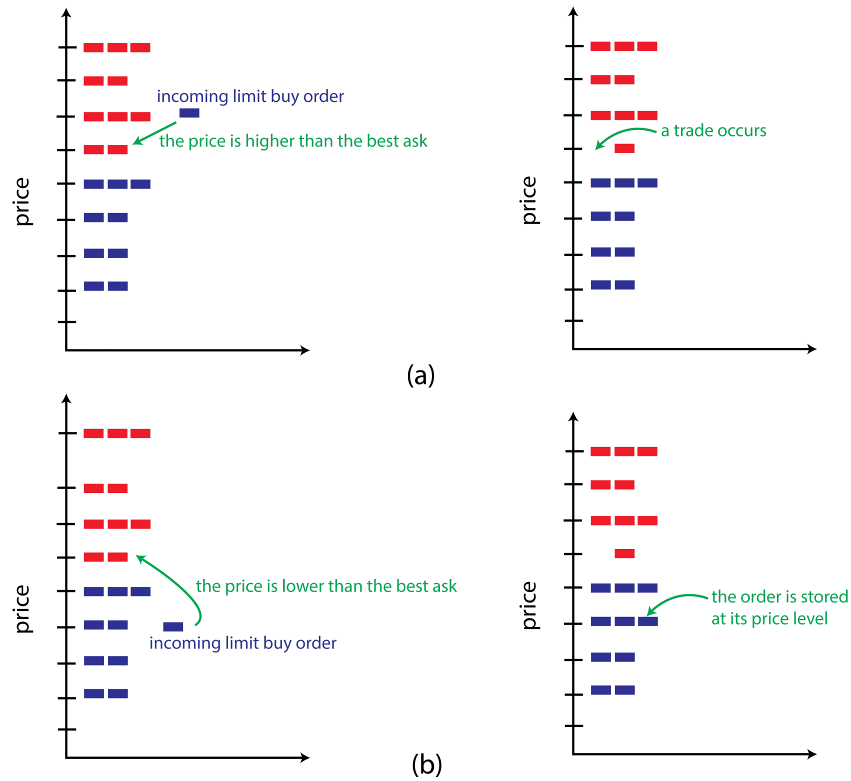


Figure 2: Dynamics of order storage and removal. When a new limit order arrives to the book, if its price is such that it crosses the price of the best order of the opposite type, a transaction occurs. If the price does not cross the opposite best price, the order is stored at its corresponding price level.

will generate different ”personalities”, in such a way that, for example, there can be agents willing to pay more than other agents to obtain a unit of the asset, or other agents may wait longer before placing an order.

We make time run in discrete units corresponding to simulation steps and on each simulation step, each fundamental agent will engage in trading with a probability p_{active} while technical agents will be active when they observe a favorable trend or when they can obtain a high immediate profit, as will be explained later.

In our system, every agent is assigned unlimited credit, and short selling is allowed. These two liberties are meant to ensure that an agent is able to engage in trading whenever it becomes active, thus providing the market with enough liquidity.

Although the model we propose includes the main components of the Lux-Marchesi model, there are important differences in the way in which we designed both the agents and the market environment. Of central importance is the fact that in our model the process of price formation is directly governed by the demand and supply provided by the agents and all the transactions are mediated through a double auction order book; as well as the fact that by assigning different parameters, leading to different “personalities”, we include heterogeneity within each type of trader. Further, in our model the only source of "exogenous" randomness, are the entry times of the fundamental agents and the time of arrival and nature of the news in the system. Other sources of randomness concern the reactions of the fundamental agents to the incoming news.

2.2. Types of Agents

2.2.1. Technical Agents As mentioned before, technical agents employ "technical analysis" in an attempt to predict the future behavior of the price time series with the purpose of exploiting the knowledge of that future behavior.

In our model, technical agents utilize a technique used in real life called Moving Average-Oscillator (MAO), which consists in a pair of moving averages with different window size: a long period average called the *slow moving average*, and a short period average aptly called the *fast moving average*. The fast moving average is intended to capture the tendency of the price movements in a short term and the slow moving average has the purpose of capturing the long term trend. Figure 3 shows an example of this technical indicator.

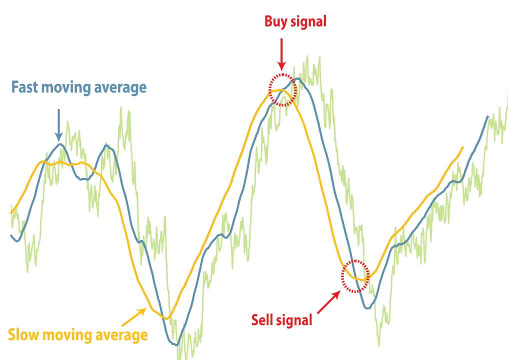


Figure 3: Moving average-oscillator (MAO). This is a common technical indicator which is formed by two moving averages of different window sizes that are constantly observed. The moving average with the largest window size is called *fast moving average* and the one with the smallest window is called *slow moving average*. When the fast moving average crosses the slow one from below, a signal to buy is generated; conversely, when the fast moving average crosses the slow from above, a signal to sell is generated.

When the fast average crosses the slow one from above, the MAO strategy suggests that this is a "signal to sell", since the prices show a short term tendency to fall below the long term trend captured by the slow moving average. Similarly, a "signal to buy" occurs when the fast moving average crosses the slow one from below, since this can be interpreted as the prices having a short time tendency to rise above the long term trend.

We employ the MAO indicator in our model because it is very simple and easy to implement, it is representative of the pletora of technical analysis tools and it is widely used in real life[32].

In our model, we use a variety of MAO indicators that differ by the window sizes of the two averages which compose them. For each of these indicators there is a population of technical agents following its evolution over time and engaging in trading as a result of the signals that the indicator generates. Further, when an indicator generates a signal (either a buy o sell signal) each technical agent following it chooses a particular waiting time t_{wait} before entering an order to the order book as suggested by the signal: if the indicator generated a buy signal, the agent will enter a buy market order, otherwise it will enter a sell market order. This waiting time between the moment in which the signal is generated and the moment in which an agent enters its order is meant to allow the price time series to move in the direction predicted by the indicator. If the agents were to immediately enter their orders after they received a signal, they would not take advantage of the rise or fall in prices that the trends point to. The waiting time t_{wait} of each technical agent is drawn from a uniform distribution in the interval $[0, t_{max}]$, and assigned to each agent from the beginning of the simulation.

A consequence of the way in which the MAO indicator is constructed, is that the technical agents should have perfectly alternating order flows, with a sell order following a previously entered buy order and vice-versa. This alternation arises from the fact that the MAO indicator generates signals when the two moving averages cross each other and for any of the two directions of crossing: the fast average crossing the slow one from below or from above, the next direction will be necessarily of the opposite kind.

There is, however, another mechanism which compels a technical agent to engage in trading, aside from following the technical indicator. This mechanism is called *profit taking* and it basically consists in selling the asset he owns when the price is sufficiently high with respect to the price at which the last unit was bought, irrespective of whether the MAO indicator generates a sell signal or not, thus providing the agent with an immediate profit. This is implemented as follows, when a technical agent enters an order to the book while following the indicator, that agent registers the price at which the order was executed in a variable called P_{signal}. If the price of the asset P_t deviates from P_{signal} by more than a factor γ, the agent will proceed to enter a new sell order; i.e. if after following a buy signal and entering the corresponding buy order to the order book the price of the asset is greater than $(1+\gamma)P_{signal}$, then the agent will place a sell market order, securing in this way an immediate profit. Figure 4 shows how profit taking is carried out in our model.

The profit taking mechanism is introduced in our model because it is a common practice in real financial markets and, as we will see, it turns out to have a strong effect on returns statistics.

2.2.2. Fundamental Agents A fundamental analysis strategy is based on two basic premises: the first one being that every asset has an intrinsic "fundamental price" p_f which corresponds to the asset's "true value", and the second one that in the short run, this fundamental price may be incorrectly estimated by the market participants but that in the long run, the market will correctly value the asset and its price will eventually reach the fundamental price p_f. An agent following a strategy of this

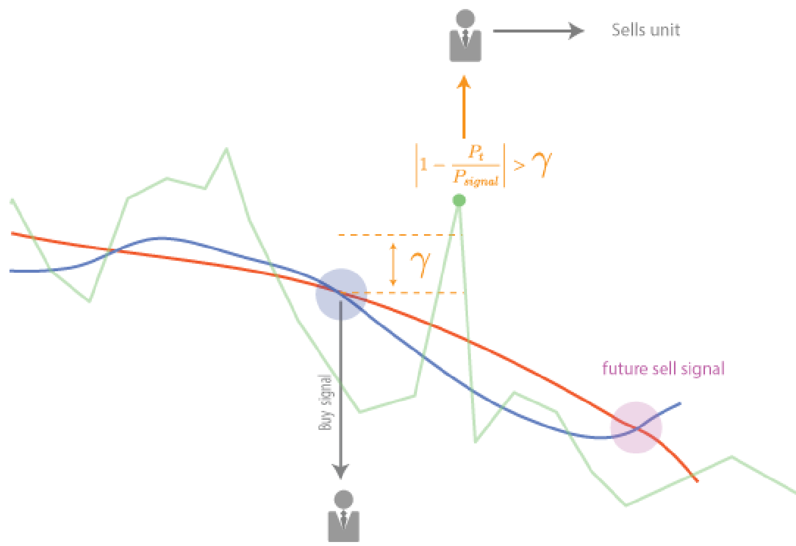


Figure 4: Profit taking mechanism: If after observing a signal to buy, the prices rise enough (in our case this is defined as the moment at which $\left|1 - \frac{P_t}{P_{signal}}\right|$ exceeds a parameter γ), the technical agent will proceed to enter a sell market order. This practice is commonly used by traders to insure an immediate profit.

kind will therefore buy an asset when the price at which it is being traded is below his estimation of its fundamental price p_f and will sell the asset when its price is above p_f. In this way a person following a fundamental strategy will take advantage of the differences between the prices at which the asset is traded over time and the fundamental price; until the asset finally reaches its fundamental price.

Each fundamental agent in our model has an individually assigned fundamental price, reflecting the differences in the estimation of the asset's value. Their strategy then is basically to buy a unit of the asset when there are other agents willing to sell for a lower price than his estimated p_f and to sell a unit when there are agents willing to buy at a higher price than his p_f. By doing this, a fundamental agent will buy the asset at lower prices and sell it at higher prices than its fundamental price, expecting in principle, to obtain profits when the price of the asset on the market P_t reaches p_f.

When a fundamental agent becomes active, there are three available actions that this agent can engage in : either to buy a unit of the asset, to sell it (even short sell) or to abstain from either. The decision of whether to buy, sell or abstain from participating will depend on the position of the agent's fundamental price p_f relative to the price of the nearest best order (best ask or best bid).

If $p_f > B_{sell}$, where B_{sell} the price of the best ask, the agent will proceed to buy since there are agents willing to sell for a lower price than that which the agent considers to be the correct price. Similarly if $p_f < B_{buy}$, where B_{buy} is the price of the best buy, the agent will proceed to sell since there are agents willing to buy offering more than the correct price. If neither of these two conditions is fulfilled, i.e. if $B_{sell} > p_f > B_{buy}$ then there will be no competitive offers, since the lowest price at which the agent could buy a unit of the asset is higher than p_f, and the highest price at which it could sell a unit is lower than p_f. Thus, when this condition arises the agent will abstain from participating in the market.

When an agent decides to buy or sell, the decision to do it by entering a limit o a market order will depend on the distance between p_f and the price of the nearest best order. Specifically, if the agent decides to buy, it will do so by emitting a market buy order when its fundamental price is above the price of the best sell offer by more than a certain threshold χ_{market}, i.e. when $p_f > B_{sell}(1 + \chi_{market})$, and it will emit a limit buy order when p_f is below this threshold. Similarly, when the agent decides to sell, it will do so by emitting a market sell order if its p_f is below the best buy offer by more than the threshold χ_{market}, i.e. when $p_f < B_{sell}(1 - \chi_{market})$, otherwise it will emit a limit sell order. Just like every other parameter defining the behavior of a fundamental agents, every agent is assigned an individual threshold χ_{market} from the beginning. The figure 5 shows this decision making algorithm.

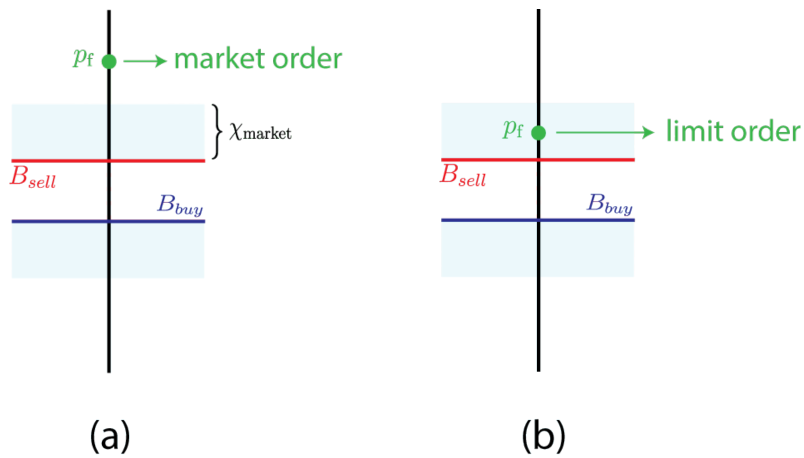


Figure 5: Order selection algorithm for fundamental agents. In (a) we show the conditions that lead to a fundamental agent to introduce a market order: if the fundamental price p_f is higher by more than a threshold χ_{market} (specific to each trader) with respect to the price of the nearest best order, the agent will proceed to enter a market order. Otherwise, the agent will proceed to enter a limit order (b). In the figure the orders would be "buy" oreders as the agent's fundamental price lies above the best ask.

On the occasions in which a fundamental agent decides to enter a limit order, the actual price of the order is extracted from a shifted symmetric exponential distribution of the form:

$$f(x; \lambda_{limit}, \mu_{spread}) = \lambda_{limit} e^{-|\lambda_{limit}(x - \mu_{spread})|}$$

where μ_{spread} is the average price of the best orders: $\mu_{spread} = \frac{1}{2}(B_{sell} + B_{buy})$. By assigning the prices of limit orders in this way, they will have a greater tendency to cluster around μ_{spread} which is a representative measure of the central price at which the market participants are valuing the asset. This behavior is intended to reflect the situation in which the prices are not good enough to enter a market order, so the fundamental agents will proceed to bargain with limit orders at prices that will be close to the central price in the market.

In real life, a great deal of the information about the financial health of a company used by agents following a fundamental strategy is taken from financial statements and other sources of information concerning the well being of said company. To include this feature of fundamental analysis in our model, we introduce a flow of news modeled as a sequence of IID random variables ζ_t taken from a normal distribution with mean μ_{news} and variance σ_{news}, and the time intervals betwen succesive news taken from a Poisson distribution. Here, ζ_t represents the mean value by which the news will change the fundamental prices of the asset, with the magnitude and sign of ζ_t corresponding directly to the nature of the news: a negative signal representing the notification of "bad news" and a positive one the notification of "good news". When, in the context of our model, news are issued at a given time t, every fundamental agent re-adjusts its fundamental price from $p_f(t)$ to $p_f(t) + \Delta p_f(t)$ where $\Delta p_f(t)$ is again extracted from a normal distribution with zero mean and variance $\sigma_{\Delta p_f}$ as illustrated in Figure 6. By changing their prices in this way, the majority of fundamental agents will change their prices accordingly with the sign of ζ_t, however, depending on the magnitude of the news, some agents may even extract a Δp_f with an opposite sign to ζ_t. This diversity of response to a news item attempts to reflect the possibilty of diverse interpretations of the information by the fundamental agents, even to the degree of choosing a change of fundamental price with the opposite sign of the news received. Of course, the probability of this diminishes drastically as the magnitud of the news ζ_t becomes larger. The fundamental price of each agent is chosen from a uniform distribution at the beginning of a simulation.

Finally, although a fundamental agent bases its trading strategy in the differences between its fundamental price and the prices at which the market values the asset, if too large a difference is present, the agent will try to get closer to the central market price μ_{spread}. This feature is meant to capture the attention that a fundamental agent pays to the opinions of the whole population of agents, which constitutes a mild manner of "herding behavior". If the valuation of the fundamental price that an agent has is too far from the price at which it is being traded, the agent will move its fundamental price closer to the central price μ_{spread}. This can be interpreted as a precautionary move by the agent since such a big difference between p_f and μ_{spread} could point to information that was not incorporated in the determination of his fundamental price, or that an ineffective incorporation of the available information was made.

To determine when the difference between p_f and μ_{spread} is "too big", each agent compares this difference with a threshold $\chi_{opinion}$, if at the time a fundamental agent becomes active, such agent observes that

$$\chi_{opinion} < \left| 1 - \frac{p_f}{\mu_{s_{spread}}} \right|$$

Then the agent will adjust its price to get closer to μ_{spread} in the following way:

$$p_f = \begin{cases} \mu_{spread}\left(1 + \chi_{opinion}\right), & if\ \ p_f \geq \mu_{spread} \\ \mu_{spread}(1 - \chi_{opinion}), & if\ \ p_f < \mu_{spread} \end{cases}$$

By doing this, the agent will get as close to μ_{spread} as the maximum tolerance ($\chi_{opinion}$) between its opinion and the opinion of the population (μ_{spread}) allows.

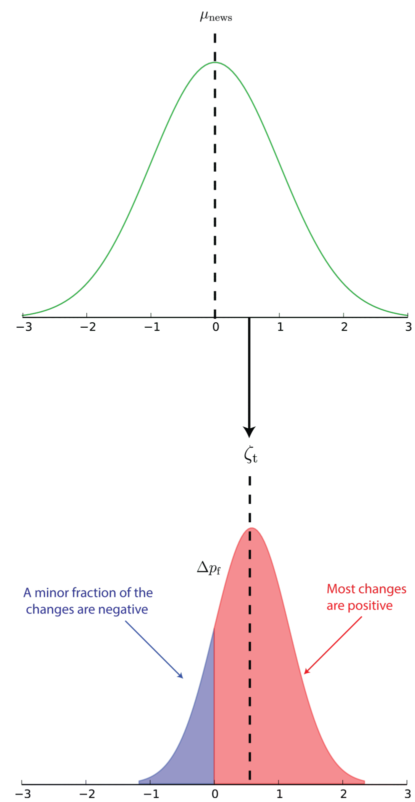


Figure 6: News and their effects on fundamental prices. We model news as a sequence of IID Gaussian random variables. When a realization of this sequence, representing news being issued, occurs, the fundamental prices of each agent are adjusted from p_f to $p_f + \Delta_{p_f}$ with Δ_{p_f} extracted from another normal distribution whose mean is equal to the value of the current news. In this way when highly positive news arrive, the majority of fundamental price changes will be positive; conversely, when highly negative news arrive, most price changes will also be negative.

3. Results

In this section we present the results obtained in various simulations. Although these results correspond to a particular set of values for the parameters, reasonable changes in the values of these parameters generate the same qualitative properties in the statistics of the model.

It is of critical importance for the stability of the system to have a flow of limit orders (liquidity) capable of filling the gaps that are created when market orders enter the order book. To achieve this, the parameters that govern the flow of limit and market orders emitted by the agents must not give rise to bursts of market orders with a volume so big that one side of the order book is emptied. It is in this sense that we speak above of reasonable changes in the values of the parameters. Thus, for example, if we were to allow greater volumes of market orders to be placed within shorter time windows, say, by including a larger number of technical agents in a simulation, then, the parameters that affect the input of limit orders must be changed accordingly, in such a way that the fundamental agents have enough time to restore the liquidity consumed by the increased number of market orders.

The following results were obtained with a population of 1000 fundamental agents and 1500 technical agents divided into two groups of 750 agents with technical indicators made of moving averages with window sizes of 4000 and 2000 time steps for one group and 2000 and 1000 time steps for the other. The other parameter values used for this run are shown in Table 1.

Parameter	Value
P_{active}	0.15
p_f	[20.0, 25.0]
χ_{market}	[0.005, 0.25]
$\chi_{opinion}$	[0.01, 0.1]
$\sigma_{\Delta_{p_f}}$	0.2
λ_{limit}	3
μ_{news}	0
σ_{news}	0.1
f_{news}	100
γ	0.01
t_{wait}	[0, 50]

Table 1: Values of the parameters corresponding to the presented results.

Time series corresponding to the prices and logarithmic returns are shown in figures 7 and 8a respectively, where we have defined the logarithmic returns as

$$r(t) = \log(P_t/P_{t-\tau})$$

for some lag τ. The blue bars in figure 8a signal the time steps in which technical agents were active. The bursts of greater volatility coincide with the activity of the technical agents while the times in which only fundamental agents were active (trading) present lower volatility.

In figure 9 we show the auto-correlation function of the returns, the blue line corresponds to the returns calculated time step by time step. In the inset we show the auto-correlation function for returns calculated every 50 steps, in both cases it can be seen that the auto-correlations is essentially zero for any value of the lag. It is interesting to note that the phenomenon know as "bid-ask bounce" can be observed in the returns generated by our simulations. This phenomenon consists in the presence of negative values of the auto-correlation function at very short lags and it is attributed to the fact that most transactions take place near the best ask or best bid and tend to bounce between these two values[2]. Since in our model the fundamental agents introduce their limit orders close to these two values, it is natural that we observe such negative auto-correlations.

In figure 10a we present the comparison between the auto-correlation of the returns (blue line) and the auto-correlation of the absolute value of the returns (red line). Here we observe that the auto-correlation function of the absolute returns remains positive over several a long time interval, and that it decays slowly to zero. Figure 10b illustrates the same auto-correlation functions for a representative company listed in the Standard & Poor's 500.

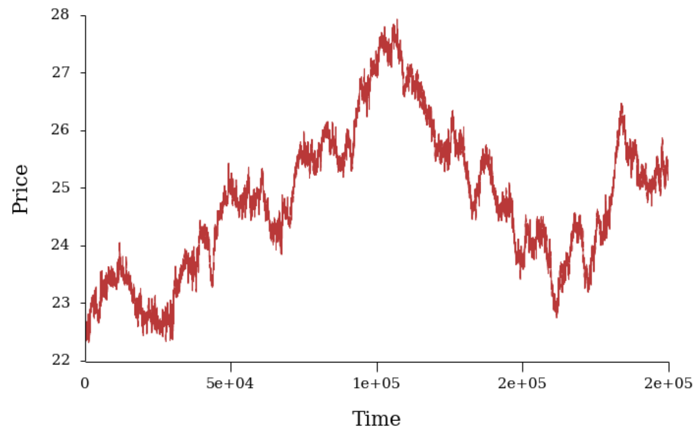


Figure 7: Representative time series of asset prices, determined as the last price the asset at each time step ("closing price"). The data was obtained from a full implementation of our model.

Figure 11a shows the distribution function of returns in our model. This distribution shows heavier tails than a normal distribution with the same mean and standard deviation and it is possible to observe that the left tail is heavier than the right one. The distribution presents a kurtosis of 3.6 and a skewness of -0.16 which is of the order of the mean skewness for the companies comprising the Standard & Poor's 500 index since 1998 which we measured at -0.33 from data made publicly available by QuantQuote. For comparison, figure 11b illustrates the distribution function of returns for a representative company listed in the Standard & Poor's 500.

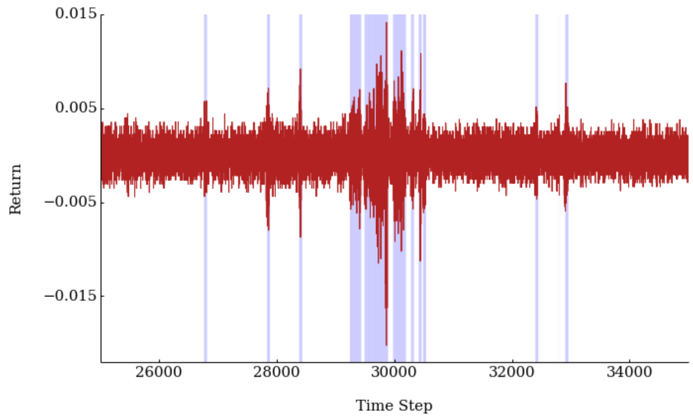
Figure 12a shows the cumulative complementary distribution of positive and negative returns, highlighting the asymmetry between losses and gains. The tail of the distribution of negative price changes is significantly heavier than the distribution of positive changes a fact that is consistent with the negative skewness displayed by the returns distribution. Figure 12b illustrates the corresponding distributions for a representative company listed in the Standard & Poor's 500.

In figure 13a we present the distribution of volatilities measured as the average of the absolute value of returns $|r(t)|$ over a time window $T = n\Delta t$, i.e.

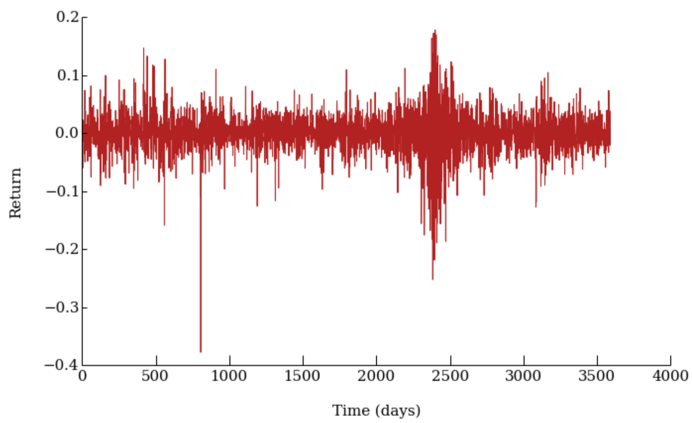
$$V_T(t) = \frac{1}{n} \sum_{t'=t}^{t+n-1} \left| r(t') \right|$$

For the present result we took values of $n = 30$ and $\Delta t = 1$ time steps. The distribution of volatilities is not well described by a log-normal distribution, however, the central part of the distribution may be approximated by one[28]. On the other hand, when we remove the technical agents from the simulation, the volatilities are remarkably well described by a log-normal distribution as shown in Figure 14, which corresponds to a run with the same parameter values described in Table 1 without technical agents.

This similarity in the central part of the volatility distributions in the scenarios with and without technical agents, along with a similar result obtained by Schmitt



(a) Returns corresponding to the simulation.



(b) Returns from Consol Energy Inc. Data obtained from QuantQuote[33].

Figure 8: Returns time series for the simulation (a) and comparison with empirical data from Consol Energy Inc (b). The blue shaded regions show the times in which technical agents were active, as can be seen, these times coincide with the periods with the largest changes of price.

et al[16] with their model in which the agents place orders with exponentially distributed volumes, is of interest since the flows of orders are very different in both cases (see figures 15a and 15a), yet, the majority of the volatilities can be described by log-normal distributions. This result may suggest that the order book mitigates in some sense, the variations in the shape of the incoming order "signal", in such a way that the variations in price (the volatilities) are not strongly affected by changes in the distribution of orders placed into the book.

In Figure 16 we plot the values of the average skewness of an ensemble of 50 simulations (for every point in the plot) as a function of the parameter γ. We analyze the behavior of the skewness as a function of γ because, as explained before, this parameter controls the minimum amount that a technical agent requires the price to rise above the price at which it bought the last unit of the asset before selling. That is,

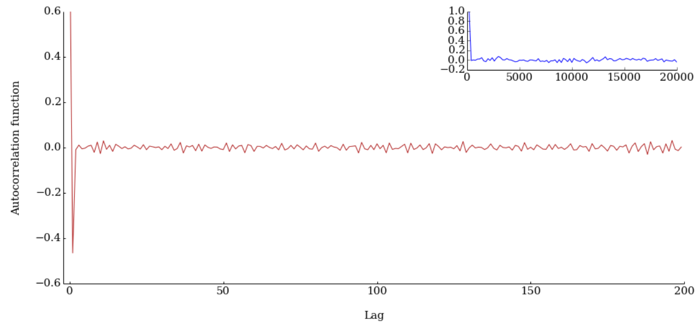
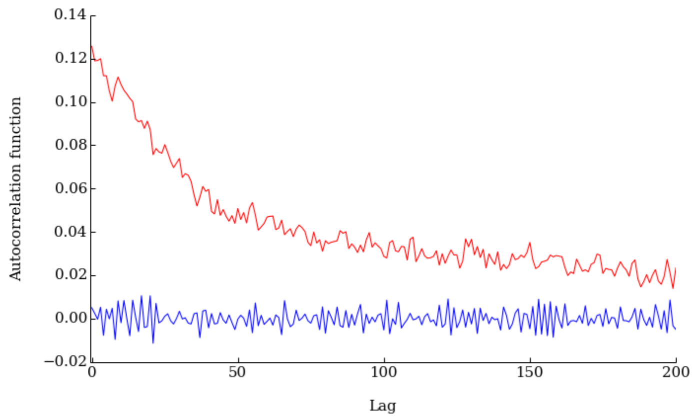


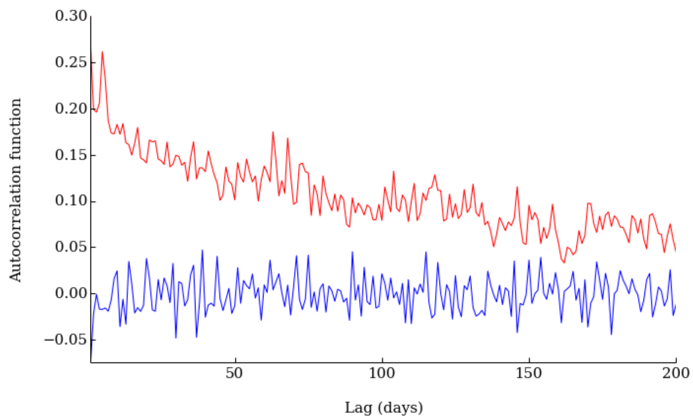
Figure 9: Auto-correlation functions of returns. There are essentially no correlations for any value of the lag, except for a negative correlation that lasts for a few steps at the beginning. This phenomenon is also observed in real returns series and has been called *bid-ask bounce*[2]. The reason behind this negative correlation is the fact that most transactions occur near the best ask or the best bid, and as such, the prices tend to "bounce" between these two values. The main figure corresponds to the autocorrelation function of the returns calculated every time step and the inset figure to the returns calculated every 200 steps.

it controls how often the population of technical agents will engage in profit taking. This is relevant because in the framework of our model, we propose this behavior as the cause of the negative asymmetry between losses and gains in the distribution of returns. This is because by engaging in profit taking, the population of technical agents creates large falls in the price of the asset and as a consequence it is more likely to observe large negative returns than large positive returns, which will in turn make the return distribution negatively skewed.

Similarly, in Figure 17 we plot the average kurtosis of an ensemble of 50 simulations as a function of the fraction of technical agents in the population. The kurtosis shows a monotonic increase with the number of technical agents, which strongly suggests that they are responsible for the deviations from normal behavior observed in the distribution of returns.

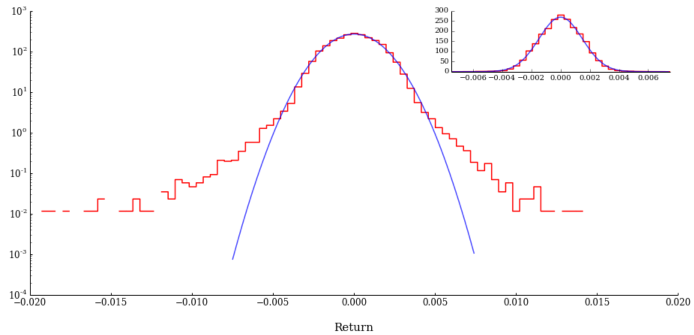


(a) Direct (blue) and absolute (red) auto-correlation functions of returns from our the simulation.

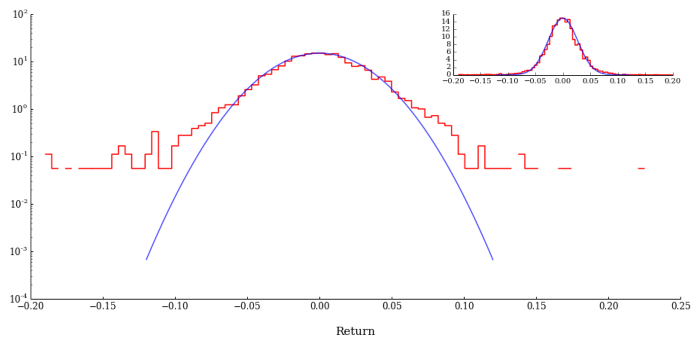


(b) Corresponding auto-correlation functions from Airgas Inc. Data obtained from QuantQuote[33].

Figure 10: Returns auto-correlation function for the simulation (a) and comparison with empirical data from Airgas Inc (b). While the auto-correlation of the direct returns (blue lines) is zero, the auto-correlation of the absolute value of the returns (red lines) remains positive for a long period of time, and decays slowly to zero.

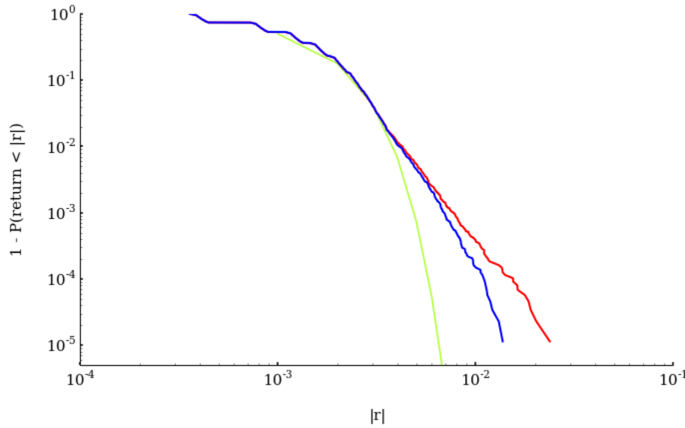


(a) Returns PDF corresponding from the simulation.

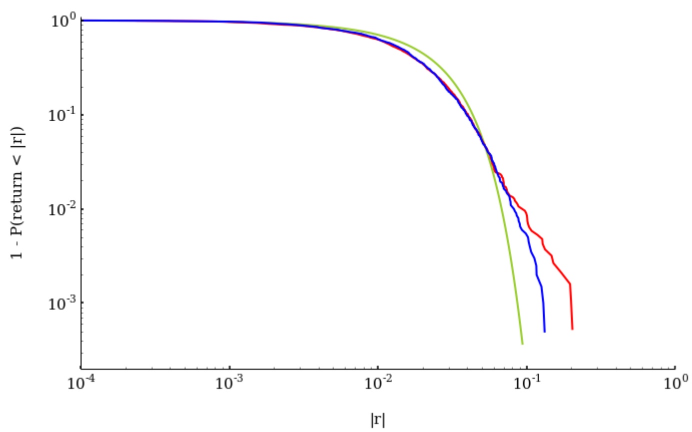


(b) Returns PDF from United States Steel Corporation. Data obtained from QuantQuote[33].

Figure 11: Returns PDF from the simulation (a) and comparison with empirical data from United States Steel Corporation, Inc. The tails of the distribution (red line) are clearly heavier than those of a normal distribution (blue line).

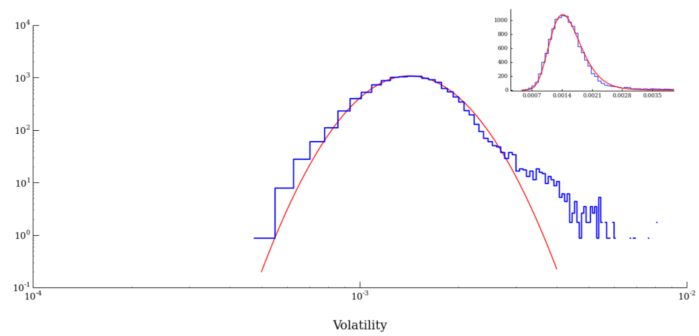


(a) Returns CDF corresponding to the simulation.

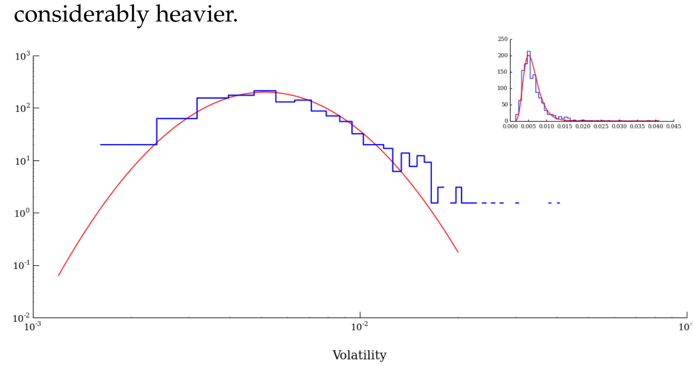


(b) Returns CDF from Anadarko Petroleum Corp. Data obtained from QuantQuote[33].

Figure 12: Comparison of the positive and negative returns CDF from the simulation (a) and from empirical data for Anadarko Petroleum Corp (b). It can be seen that the left tail of the distribution (red line), corresponding to the negative returns, is heavier than the right tail (blue line), corresponding to the positive returns. This is related to the negative skewness observed in the distribution.



(a) Volatilities PDF. It can be seen that, although the distribution of returns is not well described by a log-normal distribution: while the central region is qualitatively similar to one, the right tail is considerably heavier.



(b) Volatility distribution of Exelon Corp. Data obtained from QuantQuote[33].

Figure 13: Distribution of volatilities for a simulation with both fundamental agents and technical agents (a) and comparison with empirical data from the Standard & Poor's 500 (b). Data obtained from Yahoo Finance.

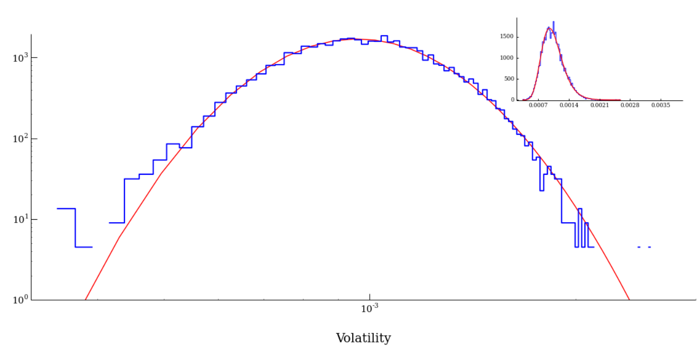
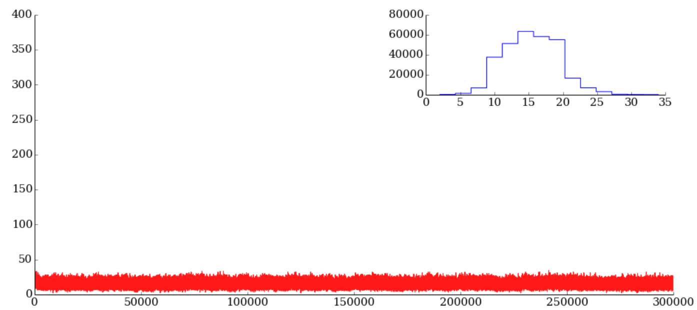
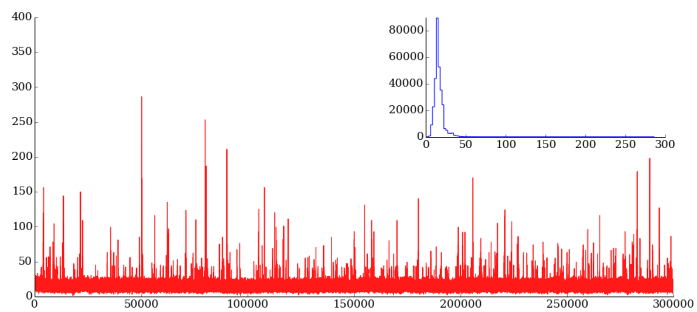


Figure 14: Volatilities of a simulation without technical agents. When only fundamental agents are used in a simulation, a log-normal distribution is a remarkably good description of the distribution of volatilities.



(a) Trading volumes for a simulation without technical agents. The volume forms a steady flow with little deviations from the mean volume.



(b) Trading volumes for a simulation with technical agents. There are big fluctuations in volume, rising above the "base line" created by the fundamental agents. These fluctuations are a consequence of the activity of technical agents.

Figure 15: Representative trading volumes for runs of the model without technical agents (a) and with technical agents (b). As can be seen, there are large fluctuations of the volume over time when technical agents are included in a simulation. The insets in each figure show the distribution of flows.

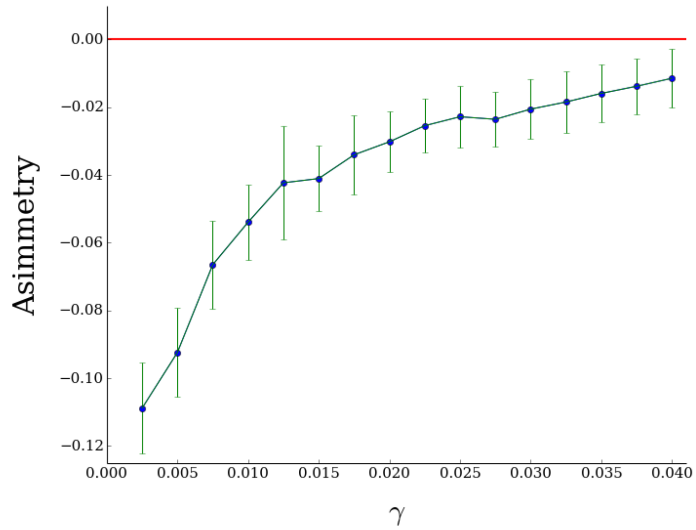


Figure 16: Mean skewness of the distributions of returns as a function of the profit taking threshold. As the profit taking threshold parameter γ becomes smaller, thus allowing the technical agents to engage incur in this practice more often, the mean skewness of the distributions of returns becomes more negative. This behavior points to a relation between the practice of profit taking and the loss-gain asymmetry stylized fact.

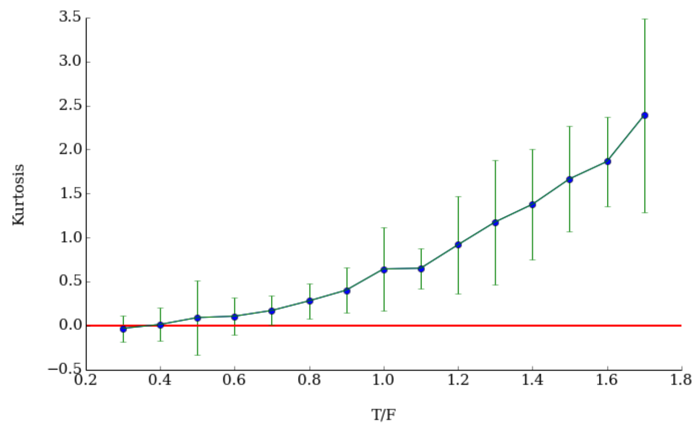


Figure 17: Mean kurtosis of the distributions of returns as a function of the technical to fundamental agents ratio. As the proportion of technical agents increases, so increases the mean kurtosis in the ensemble of simulations. This result suggests that the the presence of technical agents in the population is related to the stylized facts involving deviations from normality.

4. Conclusions

In this work we studied an agent based model of a single asset financial market which is capable of replicating several stylized facts previously reported on the literature. As in the LM model[31], we divided the population of agents into two groups according to the type of trading strategy they use: fundamental agents and technical agents. Our aim was to create a model whose agents behaved as realistically as in the LM model but with equally realistic market structures such as the inclusion of trading via a limit order book and heterogeneity in the values of the parameters that control the agents. We find, in accordance with previous models, that when the population of agents include technical agents, the returns present volatility clustering and a heavy tailed distribution. Further, we found that essentially no autocorrelation of the returns was present for any configuration of the populations. In addition to these main stylized facts, we find that when we allow the population of technical agents to incur in the practice of profit taking, the distribution of returns displays negative skewness and an asymmetry between losses and gains appears. By varying the frequency with which technical agents incur in profit taking, we can generate return distributions with varying degrees of separation in the tails. This dependence of the skewness over the frequency of profit taking suggests that this practice may be one of the causes of the appearance of the asymmetry in real financial markets.

Regarding the distribution of volatilities we find that only its central part is qualitatively similar to a lognormal distribution when technical agents are included in the population. If, on the other hand, we only include fundamental agents, the volatilities are remarkably well described by a lognormal distribution. The similarity of the volatility distributions in both scenarios, at least in the central part, suggests that its shape may not be strongly dependent on the detailed properties of the flow of incoming orders, since this flow varies significantly when technical agents are inserted in the population as compared with a population comprised entirely of fundamental agents.

We accompany our results with empirical data from real financial series chosen illustrate the various stylized facts reproduced by our model.

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