Long-run evolution of the global economy – Part 2: Hindcasts of innovation and growth

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Abstract

Long-range climate forecasts use integrated assessment models to link the global economy to greenhouse gas emissions. This paper evaluates an alternative economic framework outlined in part 1 of this study (*Garrett*, 2014) that approaches the global economy using purely physical principles rather than explicitly resolved societal dynamics. If this model is initialized with economic data from the 1950s, it yields hindcasts for how fast global economic production and energy consumption grew between 2000 and 2010 with skill scores $> 90\%$ relative to a model of persistence in trends. The model appears to attain high skill partly because there was a strong impulse of discovery of fossil fuel energy reserves in the mid-twentieth century that helped civilization to grow rapidly as a deterministic physical response. Forecasting the coming century may prove more of a challenge because the effect of the energy impulse appears to have nearly run its course. Nonetheless, an understanding of the external forces that drive civilization may help development of constrained futures for the coupled evolution of civilization and climate during the Anthropocene.

1 Introduction

Climate simulations require as input future scenarios for greenhouse gas emissions from integrated assessment models (IAMs). IAMs are designed to explore how best to optimize societal well-being while mitigating climate change. The calculations of human behaviors are made on a regional and sectoral basis and can be quite complex, possibly with hundreds of equations to account for the interplay between human decisions, technological change, and economic growth (*Moss et al.*, 2010; *IPCC*, 2014).

Periodically, model scenarios are updated to account for observed emissions trajectories. For example, it has been noted that the global carbon dioxide (CO_2) emission rate has not only grown along a "business-as-usual" (BAU) trajectory but has in fact slightly exceeded it (*Raupach et al.*, 2007; *Peters et al.*, 2013), in spite of a series of international accords aimed at achieving the opposite (*Nordhaus*, 2010).

What stability in emissions growth might suggest is that the human system has inertia, much like physical systems. Current variability reflects an accumulation of prior events, so persistent forces from the past tend to have the greatest influence on the present. Such large-scale trends tend to continue to persist into the future because they are the least responsive to current small-scale rapid forces that become diluted in the history of actions that preceded them (*Hasselman*, 1976). It may be that it is difficult to wean ourselves from fossil fuels today because we have spent at least a century accumulating a large global infrastructure for their consumption. It is not that current efforts to move civilization towards renewables cannot change this trajectory of carbon dependency but rather that it will take considerable effort and time.

Inertia offers plausibility to a BAU trajectory, particularly for something as highly integrated in the space and time as CO_2 emissions by civilization as a whole. Still, assuming persistence in trends is something that should only be taken so far. By analogy to meteorological forecasts, it is reasonable to assume that clearing skies will lead to a sunny day. However, prognostic weather models are based on fundamental physical principles that tell us that it cannot keep getting sunnier. Even a very simple set of equations dictates that at some point a front will pass, clouds will form, and a high-pressure system will decay. It is by getting the underlying physics right that we are able to achieve some level of *positive skill* in any forecast attempt (Fig. 1).

The macroeconomic components of IAMs do not offer true forecasts that can be assigned a skill score. Rather they reflect expert opinions (*Moss et al.*, 2010) and are mostly unconstrained by external physical forces since they are policy-driven and mathematically constructed so as to allow for an extremely broad range of possible futures (*Pindyck*, 2013). They consider labor, physical capital, and human inspired technological change to be the motive forces for economic production and growth. The focus is on individuals, nations, and economic sectors. The model equations describe how physical capital and human prosperity grow with time, and how energy choices tie in with greenhouse gas emissions (*Solow*, 1956; *Nordhaus and Sztorc*, 2013).

Part 1 of this study (*Garrett*, 2014, hereafter referred to as Part 1) described a second, more deterministic approach. CO_2 is considered to be long-lived and well mixed in the atmosphere, so the magnitude of greenhouse forcing is almost entirely unrelated to the national origin of anthropogenic emissions. Then, civilization can be described as a whole, one where small-scale details at personal, regional, or sectoral levels are not treated explicitly. The only quantity in the model that needs to be resolved is an aggregated global economy that is inclusive of all civilization elements, including human and physical capital combined.

As an alternative to IAMs, this new approach offers a means for integrating human systems and physical systems under a common framework, one where the governing equations are consistently derived from first thermodynamic principles. Much like the primitive equations of a prognostic weather model, global economic growth is expressed as a non-equilibrium response to external gradients driving energy dissipation and material flows. There is no explicit role for human decisions; physics does not readily allow for mathematical expressions of policy. Rather, economic innovation and growth is treated primarily as a geophysical phenomenon, in other words the totality of civilization is expressed as an emergent response to available reserves of raw materials and energy supplies.

Whatever the approach that is applied, it is important that any societal model be evaluated for performance. Weather, climate, and financial models are regularly evaluated through hindcasts or backtesting. Economic models that simulate the long-run development of humanity need not be an exception. A good model, even one that includes policy, should be able to reproduce current events with positive skill starting at a point some decades in the past. To beat the zero-skill hindcast of persistence, the model would invoke fits to concurrent trends to the minimum extent possible.

In the period following World War II, an economic "front" passed that propelled civilization towards unprecedented levels of prosperity and, by proxy, greenhouse gas emissions. This paper examines whether the theoretical model introduced in Part 1 can explain the evolution of this front. Section 2 outlines the philosophical and thermodynamic basis for describing economic evolution with physics. Section 3 evaluates this model from hindcasts. Sections 4 and 5 discuss and summarize the results.

2 Forces for economic growth

2.1 The relationship of energy dissipation to human wealth

A formal framework for the non-equilibrium thermodynamics of civilization growth was laid out in Part 1 (*Garrett*, 2014). The basis for a model linking economics to physics is a fundamental identity that relates a monetary expression for wealth to the rate at which civilization powers itself with primary energy sources (*Garrett*, 2011). We all have some sense of the difference between civilization and its uncivilized surroundings since we have farms, buildings, human population, vehicles, and communication networks. As shown in Fig. 2 and discussed in Appendix A, this difference implies the existence of a gradient between civilization and its environment. Gradients allow for irreversible thermodynamic flows. A consumption and dissipation of potential energy by civilization sustains internal reversible circulations within civilization that characterize all its activities.

The hypothesis that was made in Part 1 is that civilization is effectively a heat engine whose power can be represented in more human terms as economic wealth. Absent any energy consumption, civilization would necessarily decay towards an uncivilized, worthless equilibrium where the gradient ceased to exist and all internal circulations stopped. Wealth is able to grow only when net work is done to grow into the environment. Real economic production occurs when raw materials can be incorporated into civilization's structure faster than civilization decays (Fig. 2). Growth at a net positive rate expands civilization's interface with reserves of energy. It enlarges civilization, creating new wealth and a greater overall capacity of civilization to consume and sustain internal circulations. Expressed as an integral, current economic wealth is the net accumulation of past net physical work and real economic production.

In effect, there is no intrinsic wealth in and of itself. Rather, as discussed in Part 1, wealth is built from a network of connections, from the product of a material length density and an energetic potential. Connections are what enable
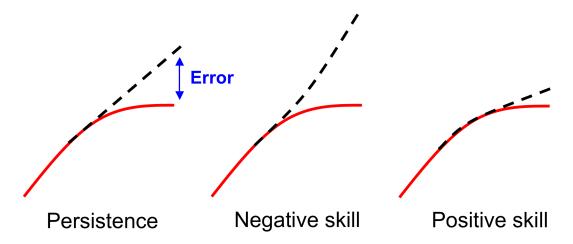
Figure 1: Positive skill in forecasts (black line) requires doing better than persistence in predicting future evolution of a quantity (red line).

dissipative flows insofar as there exist potential energy gradients to drive the flows. For civilization as a whole, wealth is sustained by primary power consumption through the connections we have to reserves of fossil, nuclear, and renewable energy sources. Within civilization, the interpretation is that wealth is due to the connections between and among ourselves and our "physical capital", and from the circulations along transportation, telecommunications, and social networks. All aspects of civilization, whether social or material, compete for globally available potential energy. Financial expressions of any element's value reflect the relative extent to which its connections enable civilization to irreversibly consume potential energy in order to sustain the reversible circulations of the global economy.

While energy consumption is required to economically produce and grow, a generally much greater amount is required to sustain circulations within the networks of connections that have accumulated from prior production in the past. An analog is an adult human. People's bodies are also a network of connections that have grown through childhood and adolescence. Far more of current daily food consumption goes towards maintaining life than to any extra weight gain. Similarly, value added to civilization through construction of a house decades ago still contributes to value today, by being part of a larger network that supports the daily rhythms of its inhabitants.

The analytical formulation of this hypothesis is that instantaneous power dissipation, or the rate of primary energy consumption a by all of civilization (expressible in units of energy per time or power), is linked through a constant λ (expressible in units of power per currency) to civilization's inflation-adjusted economic value (or civilization wealth) C (expressible in units of currency). Wealth is defined as an accumulation of the gross world product (GWP) Y, adjusted for inflation at market exchange rates (MER) (*Garrett*, 2011). MER units are used rather than purchasing power parity units (PPP) since the focus is not on short-term inequalities between people and nations but rather the sum of all activities within the global economy with an eye to variability in the long run. Thus,

$$a = \lambda C = \lambda \int_0^t Y(t')\mathrm{d}t'. \tag{1}$$

Alternatively, and taking the derivative with respect to time, economic production is a representation of the growth of wealth:

$$\frac{\mathrm{d}C}{\mathrm{d}t} = Y, \tag{2}$$

where, since $a = \lambda C$, the production function is given by an increase in the capacity to consume energy:

$$Y = \frac{1}{\lambda}\frac{\mathrm{d}a}{\mathrm{d}t}. \tag{3}$$

Crucially, Eq. (1) is a hypothesis that can be tested using available data. As described in greater detail in the Supporting Information of Part 1, GWP estimates from *Maddison* (2003) and the United Nations (*Nations*, 2010) are
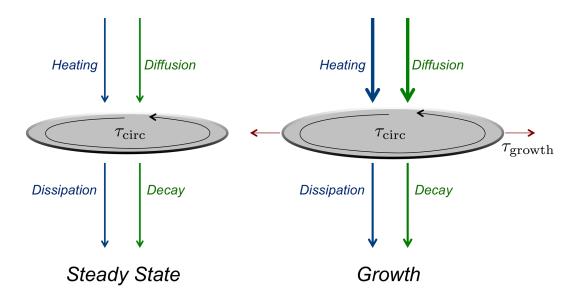
Figure 2: Thermodynamic representation of an open system. Reversible circulations within a system that lies along a constant potential have a characteristic time τ_{circ}. Circulations are sustained by a dissipation of a potential energy source that heats the system. The system maintains a steady state (left panel) because energetic (blue) and material (green) flows enter and leave the system at the same rate. Where there is a positive imbalance (right panel), the system grows irreversibly with timescale $\tau_{\mathrm{growth}} \gg \tau_{\mathrm{circ}}$. See Appendix A for details.

used for historical estimates of Y. Estimates of the global rate of primary energy consumption a are provided by the US Department of Energy (*DOE*, 2011). Expressing a in units of watts, and Y in units of 2005 MER US dollars per second, then wealth has units of 2005 MER US dollars and the constant λ has units of watts per 2005 MER US dollar. What was shown in Table S2 of *Garrett* (2014), and in graphical form in Fig. 3, is that, for the period 1970 to 2010 for which global statistics for power consumption are available, both a and $\int_0^t Y(t)\,\mathrm{d}t'$ have risen nearly in lockstep. The mean value of λ relating the two quantities is 7.1 mW per 2005 US dollar. Even though the GWP more than tripled over this time period, from year to year, the SD in the ratio $\lambda = a/C$ was just 1 %, implying an uncertainty in the mean at the 95 % confidence level of 0.1 mW per 2005 US dollar.

The constant λ is not derived from a correlation analysis (something that has been erroneously claimed by others, *Cullenward et al.*, 2011; *Scher and Koomey*, 2011), but instead it is obtained from the observation that the ratio of C to a has not changed from year to year even as C and a have. The observation is much like the basic expression of quantum mechanics where it was initially assumed, and then confirmed with measurements, that a photon's energy E and its frequency ν are linked through Planck's constant h. The empirical support for λ or h being effectively a constant stands on its own. But for the purposes of understanding the physics, the quantities they relate are not correlated but instead can be viewed as being interchangeable representations of the same thing.

The challenge might be to comprehend how a psychological construct like money could be tied to a thermodynamic construct like power through a constant. Economic value only goes so far as human judgement. Even with no one home and all the utilities turned off, a house still maintains some worth for as long as it can be perceived as being potentially useful by other active members of the global economy.

The interpretation might be that physical flows tie our brains to the global economy. Brains process a wealth of information from our environment using extraordinarily dense networks of axons and dendrites; patterns of oscillatory neuronal activity lead to the emergence of behavior and cognition; powering this brain activity requires approximately 20 % of the daily caloric input to the body as a whole (*Varela et al.*, 2001; *Lennie*, 2003; *Buzsáki and Draguhn*, 2004). Perhaps dissipative neuronal circulations along brain networks reflect our collective perception of real global economic wealth. They march to broader economic circulations along global civilization networks that are sustained by a dissipation of oil, coal, and other primary energy supplies. Eqs. (1) to (3) may seem unorthodox by traditional economic standards, but there may be some basis for interpreting λ as a type of psychological constant that links the physics of human perception to the thermodynamic flows that drive the global economy.
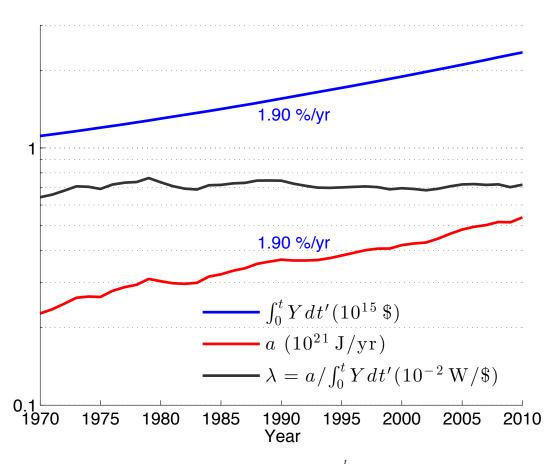
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Figure 3: Rates of global energy consumption a, global wealth $C = \int_0^t Y(t')\, dt'$, and the ratio $\lambda = a/C$ since 1970. The average rates of growth η for a and C in percent per year are shown for comparison. The average value of λ is 7.1 ± 0.1 mW per year 2005 USD. Note the y axis is a logarithmic scale.

As a point of comparison, traditional economic growth models are divorced from expressions of energy dissipation and physical flows, where wealth is expressed in terms of a physical capital, or as a stock that has an intrinsic value. New capital is produced using currently existing labor and capital, and production levels have no explicit dependence on external physical constraints (*Solow*, 1956). A more detailed outline and juxtaposition with the model here is described in Appendix B.

The field of macroeconomics is however making steps towards creating links with physics, pointing out that, along with labor and capital, energy must also be a factor of economic production (*Lotka*, 1922; *Soddy*, 1933; *Odum*, 1971; *Georgescu-Roegen*, 1993; *Hua and Bakshi*, 2004; *Annila and Salthe*, 2009). Quantified links between physical and financial quantities often rely upon a high observed correlation between national or sectoral economic production and energy consumption (*Costanza*, 1980; *Cleveland et al.*, 1984; *Brown et al.*, 2011). A few economic growth models use these data to partially substitute energy for labor and capital as a motive productive force (*Ayres and Warr*, 2009; *Kümmel*, 2011).

The model presented here differs, foremost because civilization is examined only as a whole, as an evolving organism whose growth is a response only to its changing ability to access external resources (*Gowdy and Krall*, 2013; *Herrmann-Pillath*, 2015). Nothing is said about internal trade between countries. Neither is a distinction made between human and physical capital: the capacity to consume external reserves of energy is considered a complete substitute for both these quantities at global scales. This enables the model to be strictly thermodynamic, with no requirement for dimensionally inconsistent fits to prior economic data that are dependent on the time and place that is considered. The model's validity as an economic tool rests only on the observation of a fixed ratio between energy consumption a and the time integral of inflation-adjusted economic production C (Eq. 1). The theoretical interpretation is that current energy consumption and dissipation sustains all of civilization's circulations, even human perceptions,

insofar as they have accumulated through prior economic production.

2.2 Past economic innovation as the engine for current economic growth

The most direct implication of the existence of a constant value for λ is that economic wealth cannot be decoupled from energy consumption. For the past, reconstructions of global rates of energy consumption going back 2000 years are provided in Table S3 of *Garrett* (2014). For the future, CO_2 emissions will be inextricably linked to global prosperity for as long as the economy relies on fossil fuels (*Garrett*, 2011). Increasing energy efficiency may be a commonly supposed mechanism for reducing energy consumption while maintaining wealth. However, as elaborated in Appendix B, this does not appear to be the case.

From Eq. (1), the relative growth rate of civilization wealth C and its rate of energy consumption a are equivalent:

$$\text{rate of return} = \eta = \frac{\mathrm{d}\ln a}{\mathrm{d}t} = \frac{\mathrm{d}\ln C}{\mathrm{d}t}. \tag{4}$$

Effectively, like interest on money in the bank, the parameter η represents the "rate of return" that civilization enjoys on its current wealth C, and that it sustains by consuming ever more power.

Since 1970, rates of return for a and C have varied, but both have averaged $\sim 1.90\,\%$ per year (Fig. 3). Substituting Eq. (1) into Eq. (4) yields a relationship between the rate of return and the inflation-adjusted GWP:

$$Y = \eta C = \eta \int_0^t Y(t')\mathrm{d}t', \tag{5}$$

or

$$\eta = \frac{Y}{\int_0^t Y(t')\mathrm{d}t'}. \tag{6}$$

So, the current rate of return has inertia since it is tied to the past. It expresses the ratio of current real production to the historical accumulation of past real production.

The rate of change of civilization's rate of return can be referred to as an "innovation rate":

$$\text{innovation rate} = \mathrm{d}\ln\eta / \mathrm{d}t. \tag{7}$$

Referring to an acceleration term $\mathrm{d}\ln\eta / \mathrm{d}t$ as an innovation might seem a bit arbitrary. However, in Appendix B it is shown that it corresponds directly to more traditional economic descriptions of innovation such as increases in the "total factor productivity" or the "production efficiency". For example, it is easy to show from Eqs. (1) and (5) that $\eta = \lambda Y / a$. Since λ is a constant, it follows that increases in the production efficiency (or inverse energy intensity) Y/a are equivalent to the expression for innovation $\mathrm{d}\ln\eta / \mathrm{d}t$. Innovation is a driving force for economic growth and energy consumption since it follows directly from Eqs. (4) and (5) that the real GWP relative growth rate is governed by the relationship

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}t} = \eta + \frac{\mathrm{d}\ln\eta}{\mathrm{d}t} \tag{8}$$

GWP growth = rate of return + innovation rate.

The rate of return η is equivalent to the time integral of past innovations through $\eta = \int_0^t \mathrm{d}\eta / \mathrm{d}t'\, \mathrm{d}t'$, and so Eq. (8) can be expressed as

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}t} = \int_0^t (\mathrm{d}\eta / \mathrm{d}t')\mathrm{d}t' + \frac{\mathrm{d}\eta / \mathrm{d}t}{\int_0^t (\mathrm{d}\eta / \mathrm{d}t')\mathrm{d}t'}. \tag{9}$$

The implication here is that current rates of GWP growth can be considered to be a consequence of past innovations (the first term) and current innovations insofar as they are not diluted by past innovations (the second term). The first
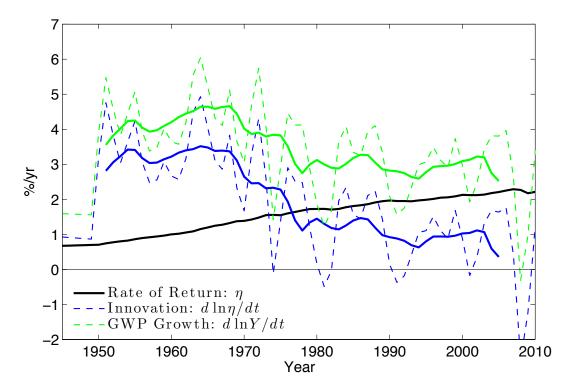
Figure 4: Time series of the rate of return, innovation, and the GWP growth rate, evaluated at global scales and expressed in percent per year. Solid lines represent a running decadal mean (see *Garrett*, 2014, for methods).

term implies that current GWP growth rates will tend to persist because past innovation is carried to the present; the second term implies that new technological advances will always struggle to replace older advances that are already in place (*Haff*, 2014). Placing an internal combustion engine on a carriage was revolutionary for its time, but only a series of more incremental changes have been made to the concept since. Any new dramatic change has to compete with the large vehicular infrastructure that has already been put in place.

Figure 4 shows how rates of return, innovation rates, and GWP growth have been changing in recent decades based on the data sets provided in the Supporting Information of Part 1. The rate of return η generally has had an upward trend. In 2008, the rate of return on global wealth reached an all time historical high of 2.24 % per year, up from 1.93 % per year in 1990 and 0.71 % per year in 1950.

Meanwhile, innovation rates have declined. The rate of growth of the rate of return, or $d\ln\eta/dt$, has dropped from around 4 % per year in 1950 to near stagnation today. Unprecedented gains in production efficiency that were obtained in the two decades after World War II appear to have since given way to much more incremental innovation.

From Eq. (8), GWP growth is the sum of these two pressures. On the one hand, positive innovation has had a lasting positive impact on the GWP since it has led to an ever increasing rate of return η. On the other hand, innovation rates have declined. Figure 4 shows that, between 1950 and 1970, GWP growth rates were between 4 and 5 % per year. Since 1980, they have been closer to 3 % per year. Increasingly, the long-term increase in civilization's rate of return η has been offset by the long-term decrease in innovation $d\ln\eta/dt$. The turning point was in the late 1970s, when, as shown in Fig. 4, innovation rates dipped below rates of return. Between 1950 and 1975, current innovation was the largest contributor to current GWP growth rates. Since then, continued GWP growth has relied increasingly on innovations made in the first two decades since the end of World War II (Eq. 9).

2.3 Physical forces for innovation

For the special case that there is positive net convergence of matter in a system, the system grows (see Fig. 2 and Appendix A). It extends its interface with accessible reserves of energy and matter. An enlarged interface allows for faster rates of consumption. The result is a positive feedback that allows growth to accelerate. This is a basic recipe for emergent or exponential growth. One important aspect of this feedback, however, is that rates of exponential growth

are never constant. Rather, they increase when new reserves of energy or matter are discovered and they decrease when there is accelerated decay.

The thermodynamics of this recipe (*Garrett*, 2012a) were applied in Part 1 to the emergent growth of civilization and its rates of return on wealth (*Garrett*, 2014). It was shown that the rate of return η can be broken down into the proportionality

$$\eta \propto (1-\delta)\frac{\Delta H_R}{N_S^{2/3} e_S^{\text{tot}}}. \tag{10}$$

Here, N_S represents the amount of matter or mass within civilization. N_S grows from a positive imbalance between civilization's incorporation of raw materials from the environment and civilization's material decay. δ is the decay parameter that accounts for how rapidly N_S falls apart due to natural causes. ΔH_R represents the size of the energy reserves that are available to be consumed by civilization. The term e_S^{tot} represents how much of this energy must be consumed by civilization in order that raw materials can be added to civilization's fabric, thereby adding to N_S. The exponent $2/3$ arises from how flows are down a gradient and across an interface.

Building on the identity $a = \lambda C$, it was argued in *Garrett* (2014) that Eq. (10) implies that rates of economic innovation can be represented by

$$\frac{d\ln\eta}{dt} = -2\eta + \eta_{\text{tech}} \tag{11}$$

$$\tag{12}$$

The first term represents a drag on innovation due to a law of diminishing returns -2η. The second term expresses a rate of technological change η_{tech} due to changes in δ, ΔH_R, and e_S^{tot}. A distinction is made here between a technological advance and an innovation. Technological change only counts as an innovation if it overcomes diminishing returns to lead to a real increase in the rate of return η.

A law of diminishing returns is a characteristic feature of emergent systems. As indicated by Eq. (10), the exponential growth rates of larger, older objects with high values of N_S tend to be lower than for smaller, younger ones. In our case, our bodies are a complex network of nerves, neurons, veins, gastrointestinal tracts, and pulmonary tubes. We use this network so that we can interact with a network of electrical circuits, communication lines, plumbing, roads, shipping lanes, and aviation routes (*van Dijk*, 2012). Such networks have been built from a net accumulation of matter. So, as civilization grows, any given addition becomes increasingly incremental.

The implication of Eq. (11) is that, absent sufficiently rapid technological change, relative growth rates η will tend to decline, and innovation will turn negative. For example, from Eq. (11), innovation requires that $\eta_{\text{tech}} > 2\eta$. Or, by substituting Eq. (8) into Eq. (11), an expression for GWP growth is $d\ln Y/dt = -\eta + \eta_{\text{tech}}$, in which case maintenance of positive GWP growth requires that $\eta_{\text{tech}} > \eta$. That economic growth has been sustained over the past 150 years is a testament to the importance of technological change for overcoming diminishing returns.

The rate of technological change follows from the first derivate of Eq. (10):

$$\eta_{\text{tech}} = \frac{d\ln(1-\delta)}{dt} + \frac{d\ln\Delta H_R}{dt} - \frac{d\ln e_S^{\text{tot}}}{dt} \tag{13}$$

$$\eta_{\text{tech}} = \eta_\delta + \eta_R^{\text{net}} + \eta_e$$

$$\tag{14}$$

The first of these three forces is improved longevity. Suppose that civilization decays by losing matter at rate j_d. At the same time, it incorporates new matter at rate j_a, as discussed in Appendix A. A dimensionless decay parameter $\delta = j_d/j_a$ can be introduced that expresses the relative importance of material decay to material growth: if δ is zero, new material growth is not offset by decay. If δ declines then it is because new or existing matter lasts longer, representing an increase in civilization's longevity. For example, δ might decrease as civilization shifts from wood to steel as a construction material. Alternatively, it might increase due to more frequent natural disasters from climate change.

In Part 1, it was shown how the nominal GWP can be tied to the incorporation of new matter into civilization j_a; the real inflation-adjusted GWP can be tied to the net incorporation of new matter $j_a - j_d$. This yields the interesting result that physical decay is related to economic inflation. At domestic scales, the so-called "GDP deflator" is often used as an analog for the annual inflation rate $\langle i \rangle$ since it represents the fractional downward adjustment that is imposed on the

nominal GDP to obtain the real GDP. For civilization as a whole, the implication is that declining decay, or increased longevity, corresponds to a smaller GDP deflator, declining inflation, and faster real GWP growth, i.e.,

$$\eta_\delta \simeq -\frac{d\langle\delta\rangle}{dt} \simeq -\frac{d\langle i\rangle}{dt}. \tag{15}$$

The second force for technological change in Eq. (13) is discovery of new energy reserves. Where discovery exceeds reserve depletion, it accelerates economic innovation through an increase in the size of available energy reserves ΔH_R (*Smil*, 2006; *Ayres and Warr*, 2009). Energy reserves decline as they are consumed at rate a. Meanwhile, civilization discovers new reserves at rate D. The rate of net discovery is

$$\eta_R^{net} = \frac{D-a}{\Delta H_R}. \tag{16}$$

Provided that reserves expand faster than they are depleted, the rate η_R^{net} is positive. It represents a technological advance because there is reduced competition for available resources. From Eq. (10), larger reserves enable higher rates of return for the relative growth of wealth and energy consumption.

The specific enthalpy of civilization e_S^{tot} in Eq. (13) is an expression of the amount of power a that is required for civilization to extract raw materials and to incorporate them into civilization's fabric at rate j_a. If the ratio a/j_a declines, then civilization becomes more energy-efficient.

For example, mining and forestry is currently powered by large diesel engines rather than human and animal labor. Civilization is able to extract raw materials with comparative efficiency and lengthen civilization networks at a correspondingly greater rate. Using less energy, we are able to build more roads, lengthen communications networks, and even increase population, as we too are made of matter and are part of civilization's fabric. Where the extraction efficiency of raw materials improves, it is an effective force for technological change defined by

$$\eta_e = \frac{d\ln j_a}{dt} - \frac{d\ln(a)}{dt}. \tag{17}$$

2.4 Deterministic solutions for economic growth

Equation (11) for innovation is logistic in form. That is, it could be expressed as $d\eta/dt = \eta_{tech}\eta - 2\eta^2$ with a rate of exponential growth η_{tech} and a drag rate on growth of -2η. An initial exponential growth phase yields to diminishing returns where rates of return stabilize (Fig. 5). If η_{tech} is constant, then the solution for the rate of return η is

$$\eta(t) = \frac{\eta_{tech}/2}{1+(G-1)\exp(-\eta_{tech}t)}, \tag{18}$$

where

$$G = \frac{1}{2}\frac{\eta_{tech}}{\eta_0} \tag{19}$$

represents a "growth number" (*Garrett*, 2012a, 2014) and the subscript 0 indicates the initial observed value for η_0. The solution for η in Eq. (18) is sigmoidal. Provided G is greater than 1, rates of return initially increase exponentially and saturate at a rate of $\eta_{tech}/2$. So, for example, if η_{tech} is sustained at 5 % per year, then one would expect rates of return to grow sigmoidally towards 2.5 % per year. The characteristic time for the exponential growth phase would be $1/\eta_{tech}$, or 20 years.

From Eq. (8), the corresponding time-dependent solution for GWP growth assuming a fixed rate of technological change is

$$\frac{d\ln Y}{dt}(t) = \frac{\eta_{tech}}{2}\left[\frac{1+2(G-1)\exp(-\eta_{tech}t)}{1+(G-1)\exp(-\eta_{tech}t)}\right]. \tag{20}$$

Here, GWP growth rates also saturate at a value of $\eta_{tech}/2$, but if $G > 1$ then this is by way of decline rather than growth. Thus, rates of return on wealth (Eq. 18) and rates of GWP growth (Eq. 20) should have a tendency to converge with time. This is in fact precisely the behavior that has been observed in the past few decades. Figure 4 shows values of η and $d\ln Y/dt$ that differ by about a factor of 4 in 1950 but that are approaching from opposite directions towards a common value of about 2.5 % per year.
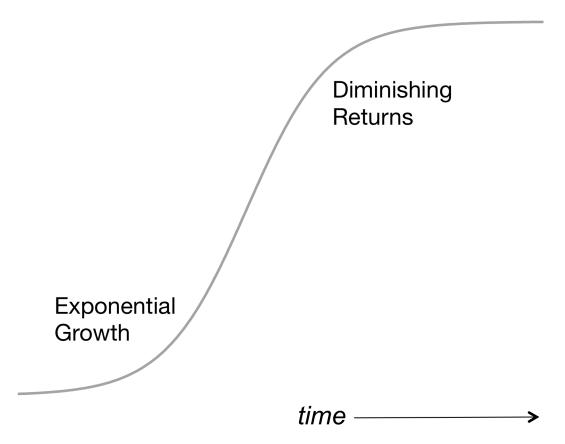
Figure 5: Illustration of the logistic curve.

3   Model validation through hindcasts

Equation (11) is a new expression for the long-run evolution of the global economy and its resource consumption. Three approaches are now taken to test its validity.

3.1   The functional form relating innovation to growth

Figure 6 shows the relationship between innovation rates and rates of return over the past three centuries derived from GWP estimates from *Maddison* (2003) and the United Nations (*Nations*, 2010), using Eqs. (6) and (7) (see Sect. 2 and the Supporting Information of Part 1 for methods and associated statistics). Rapid innovation and accelerating rates of return characterized the industrial revolution and the late 1940s. Periods of subsiding innovation followed 1910 and 1950.

Equation (11) implies that, if rates of technological change η_{tech} are roughly a constant, innovation rates $\mathrm{d}\ln\eta/\mathrm{d}t$ should be related to rates of return η by a slope of about -2; the intercept should be equivalent to the rate of technological change η_{tech} given by Eq. (13). Focussing on the period since 1950, where statistical reconstructions of GWP are yearly and presumably most reliable (*Maddison*, 2003), Fig. 6 shows that the past 60 years have been characterized by a least-squares fit relationship between innovation rates $\mathrm{d}\ln\eta/\mathrm{d}t$ and rates of return η (with 95 % uncertainty bounds) given by

$$\frac{\mathrm{d}\ln\eta}{\mathrm{d}t} = -(2.54\pm0.54)\eta + (0.06\pm0.01). \tag{21}$$

Within the stated uncertainty, the observed slope relating innovation to rates of return is consistent with the theoretically expected value of -2 that comes from a law of diminishing returns. The implied rate of technological discovery for this time period η_{tech} is the intercept of the fit, or about 6 % per year. The magnitude of the difference of the fit
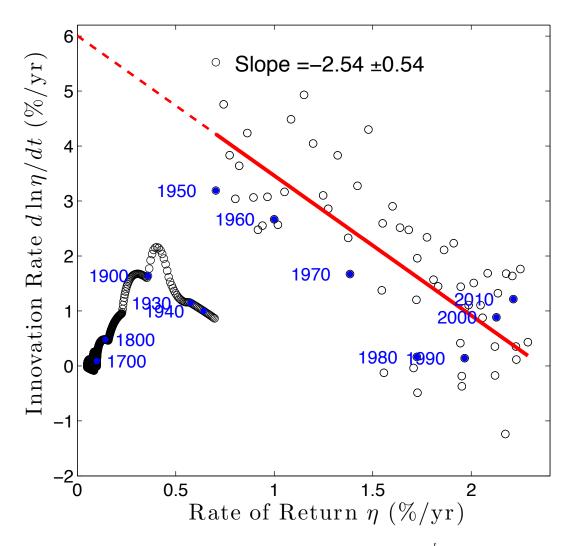
Figure 6: The global innovation rate $d\ln\eta/dt$ versus the global rate of return $\eta = Y / \int_0^t Y\, dt'$ (Eq. 4). Select years are shown for reference. Since 1950, innovation is related to growth through the functional relationship $d\ln\eta/dt = S\eta + b$, where the slope and intercept shown by the red line, with 95 % confidence limits, are $S = -2.54 \pm 0.54$ and $b = 0.06 \pm 0.01$.

from the anticipated slope might be an indication that η_{tech} has not in fact been a strict constant but rather has declined with slowly with time, as discussed below.

3.2 Hindcasts of long-run civilization growth

The second test is to approach the problem as a hindcast. A hypothetical economic forecaster in 1960 might have noted that the average values of η and $d\ln\eta/dt$ between 1950 and 1960 were 0.9 % and 3.3 % per year, respectively. From Eq. (11), this implies that η_{tech} was 5.1 % per year during this period. Applying Eqs. (18) and (20), and using an initial value for η_0 of 1.0 % per year in 1960, the forecaster could then have obtained the trajectories for economic innovation and growth that are shown in Fig. 7.

Fifty-year hindcasts are summarized in Table 1 along with skill scores defined relative to a reference model of persistence in trends (*Society*, 2014):

$$\text{Skill score} = 1 - \frac{|\text{error(hindcast)}|}{|\text{error(persistence)}|}. \tag{22}$$

Table 1: For key economic parameters, a comparison between observed annual growth rates and 50-year predictions made assuming either a reference model of persistence or a hindcast model given by Eq. (11). Persistence is derived from historical rates between 1950 and 1960. The "observed" time period is 2000 to 2010. The skill score is derived from $1 - \text{error(hindcast)}/\text{error(persistence)}$, where error is derived relative to observed rates. Data are shown in Figure 7.

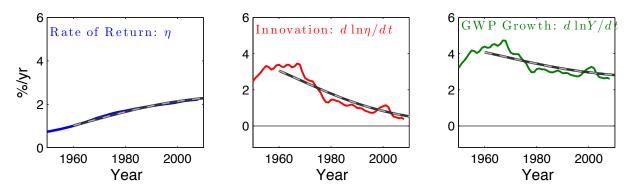|	Persistence ($\%\,\text{yr}^{-1}$)	Hindcast ($\%\,\text{yr}^{-1}$)	Observed ($\%\,\text{yr}^{-1}$)	Skill score (%)
Rate of return η ($\mathrm{d}\ln a/\mathrm{d}t$)	1.0	2.3	2.2 (2.4)	88 (96)
Innovation rate $\mathrm{d}\ln\eta/\mathrm{d}t$	3.3	0.4	0.4	100
GWP growth rate $\eta + \mathrm{d}\ln\eta/\mathrm{d}t$	4.0	2.8	2.6	91

Figure 7: Black, gray-dashed lines: hindcasts starting in 1960 of the global rate of return $\eta = Y/\int_0^t Y\,\mathrm{d}t'$, innovation rates $\mathrm{d}\ln\eta/\mathrm{d}t$, and the GWP growth rate $\mathrm{d}\ln Y/\mathrm{d}t = \eta + \mathrm{d}\ln\eta/\mathrm{d}t$. Hindcasts are derived from Eq. (18) assuming an average rate of technological change of $5.1\,\%\,\text{yr}^{-1}$ (dashed gray lines) derived from conditions observed in the 1950s. Solid colored lines: observed decadal running means. Hindcast values in 1960 represent persistence values shown in Table 1.

Skill scores are positive when the hindcast beats persistence in trends, and zero when they do not. For example, average rates of energy consumption growth in the past decade would have been forecast to be 2.3 % per year relative to an observed average of 2.4 % per year. Relative to a persistence prediction of 1.0 % per year, the skill score is 96 %. Alternatively, a forecast of the GWP growth rate for the first decade of this century would have been 2.8 % per year compared to the actual observed rate of 2.6 % per year. The persistence forecast based on the 1950 to 1960 period is 4.0 % per year, so the skill score is 91 %.

3.3 Observed magnitude of technological change

High skill scores suggest that it is possible to provide physically constrained scenarios for civilization evolution over the coming century using a simple logistic model given by Eq. (18). There do not appear to be other macroeconomic forecast models that are equally successful, and in any case, macroeconomic models are not normally evaluated through comparisons to multi-decadal historical data. If a comparison with data is made, it is not in the form of a true hindcast. The model is judged by the extent to which a sufficiently complex production function can be tuned to provide an accurate fit to prior observations (e.g., *Warr and Ayres*, 2006).

Using only a fixed value for η_{tech} as input to the model presented here appears to work very well, at least for global scales. Still, a more fully deterministic model would not rely on an assumed value for η_{tech}, even if it is a fit to data prior to the date of model initialization. It is reasonable to anticipate that future rates of resource discovery and material longevity will evolve with time. Accounting for such technological change might prove an important consideration for economic and climate forecasters over the coming century.

To this end, the third test is to try to quantify the thermodynamic forces outlined in Eq. (13) that would enable a more first-principles estimation of the value of η_{tech}. Methods and data sets for estimating a time series for the sizes of

Table 2: Components of technological change expressed as 20- and 60-year averages of growth rates. Bold numbers represent weighted averages. See Section 3.3 and Appendix B for details.

Mean growth rates (% yr^{-1})	1950–1970	1970–1990	1990–2010	1950–2010
Average raw materials per energy $\eta_{\mathbf{e}}$	**3.5**	**–0.7**	**0.7**	**1.3**
Cement and wood per energy	2.2	−0.8	−0.4	0.5
Iron and steel per energy	4.6	−1.4	1.4	1.7
Copper per energy	3.7	0.0	1.0	1.6
Total fossil reserves $\eta_{\mathbf{R}}^{\mathbf{net}}$	**3.6**	**1.3**	**0.7**	**2.0**
Oil reserves [production in EJ/year]	3.6 [59]	0.6 [133]	−0.7 [165]	1.1 [118]
Gas reserves [production in EJ/year]	8.2 [22]	2.4 [62]	0.6 [98]	3.7 [60]
Coal production [production in EJ/year]	2.2 [73]	1.9 [115]	2.3 [153]	2.2 [113]
Change in longevity η_{δ}	**–0.1**	**0.2**	**0.2**	**0.2**
Rate of technological change $\eta_{\mathbf{tech}} = \eta_{\mathbf{e}} + \eta_{\mathbf{R}}^{\mathbf{net}} + \eta_{\delta}$	**7.0**	**0.8**	**1.4**	**3.5**

Table 3: Twenty- and 60-year averages of rates of return (calculated using two independent techniques), innovation rates, and rates of technological change. Values are derived from Eqs. (11) and (13) and using data from Table 2.

Mean growth rates (% yr^{-1})	1950–1970	1970–1990	1990–2010	1950–2010
Observed rate of return $\eta = Y / \int_{0}^{t} Y \mathrm{d}t'\ (= (\mathrm{d}a/\mathrm{d}t)/a)$	1.0	1.7 (1.6)	2.1 (2.0)	1.6
Observed innovation rate $\mathrm{d}\ln\eta/\mathrm{d}t$	3.3	1.6	0.6	1.9
Calculated technological change $\eta_{\mathrm{tech}} = \mathrm{d}\ln\eta/\mathrm{d}t + 2\eta$	5.3	5.0	4.7	5.1
Observed technological change η_{tech}	7.1	0.8	1.2	3.5

energy reserves, the rate of energy consumption, the rate of raw material consumption, and economic inflation during the period between 1950 and 2010 are described in Appendix C and summarized in Table 2. Average rates are shown for three successive 20-year periods beginning in 1950, and for the 1950 to 2010 period as a whole.

What stands out in Table 2 is how there was unusually rapid technological change between 1950 and 1970. This period was characterized by rapidly growing access to reserves of oil, gas, and raw materials. It was followed by an abrupt slowdown in 1970 with no clear long-term recovery since. Summing over these forces, and averaged over the entire 1950 to 2010 period, rates of technological change η_{tech} are estimated to have been a respectable 3.5 % per year. Most of this growth took place in the first 20 years, when it achieved 7.0 % per year. The latest 20-year period averaged just 1.4 % per year.

Improved access to energy reserves and raw materials explains most of the variability in η_{tech}. Coal power production expanded steadily at a rate of about 2 % per year. Oil reserves, on the other hand, expanded at an average 3.6 % per year between 1950 and 1970 but shrunk at an average 0.7 % per year between 1990 and 2010. The amount of energy required to access key raw materials such as cement, wood, copper, and steel dropped by an average 3.5 % per year between 1950 and 1970, which implies rapid efficiency gains. Since 1970, energy consumption and raw material consumption have grown at nearly equivalent rates, implying no associated force for technological change.

As a check on the first-principles estimate that the average value of η_{tech} between 1950 and 2010 was 3.5 % per year, Table 3 shows the 20- and 60-year averages of η and $\mathrm{d}\ln\eta/\mathrm{d}t$, and it uses these to derive a rate of technological change η_{tech} from Eq. (11). (As a consequence of λ being a constant, calculated rates of return η are similar whether they are calculated from available energy statistics using Eq. (4) or from GWP statistics using Eq. (6). Both have averaged 1.6 % per year overall.)

Innovation rates $\mathrm{d}\ln\eta/\mathrm{d}t$ have been positive overall, meaning rising rates of return. Still, they declined from 3.3 % per year between 1950 and 1970 to just 0.6 % per year between 1970 and 1990. Thus, the estimated average rate of technological change derived from Eq. (11) (i.e., $\eta_{\mathrm{tech}} = \mathrm{d}\ln\eta/\mathrm{d}t + 2\eta$) is 5.1 % per year, similar to what was derived for the 1950 to 1960 time period as discussed in Sect. 3.2. In comparison, the rate of technological change estimated from the physical parameters described in Table 2 averages 3.5 % per year, or about one-third lower. Whether the residual 1.6 % per year is due to data uncertainties or theoretical considerations is unknown.

The hindcasts in Sect. 3.2 assumed a constant value for η_{tech}, whereas the observed rates summarized in Table 3 point towards much higher variability. Perhaps the reason a constant value nonetheless leads to hindcasts with high

skill scores is because there is a timescale of decades for externally forced technological change to diffuse throughout the global economy (e.g., *Rogers*, 2010). Assuming a fixed value for η_{tech} represents this timescale by smoothing the economy-wide impacts of a large impulse of innovative forces that occurred between 1950 and 1970.

4 Positive skill in economic forecasts

A logistic equation forms the basis of the prognostic model that provides hindcasts for civilization growth as a whole. At the level of empires, there have been similar waves of logistic or sigmoidal growth throughout history. An initial phase of exponential growth tends to be followed by slower rates of expansion. Ancient Rome's empire increased to cover $3\,500\,000\,\text{km}^2$ in its first 300 years, but only a further $1\,000\,000\,\text{km}^2$ in its second; the Mongol empire extended to $20\,000\,000\,\text{km}^2$ within 50 years, adding an additional $4\,000\,000\,\text{km}^2$ in the next (*Marchetti and Ausubel*, 2012). Growth at declining rates has also been noted in the adoption of new technologies (*Rogers*, 2010), the size of oil tankers (*Smil*, 2006), bacteria (*Zwietering et al.*, 1990), and snowflakes (*Pruppacher and Klett*, 1997).

In a very general way, these common emergent behaviors might be viewed as the response of a system to available reserves of potential energy and matter. Consumption of resources allows for expansion into more resources and hence more consumption. Eventually, new consumption becomes increasingly diluted by past consumption in which case growth slows. The mathematical expression of the dynamics is fairly simple (*Garrett*, 2014), and it has been shown here how it can serve as a foundation for making 50-year hindcasts of the global economy (Fig. 7).

The accuracy of the hindcasts shown here appears to be due in part to a remarkable burst of technological change that occurred between 1950 and 1970. Figure 8 encapsulates its magnitude. From available statistics, oil and gas reserves expanded faster than they were consumed. This changed around 1970. Reserves continued to be uncovered, but they only barely kept pace with increasing demand. Early innovation and growth began to act as a drag on future innovation.

How rapid resource discovery played out is captured mathematically by Eq. (11), at least assuming a fixed value for η_{tech} of about 5.1 % per year. Forecasting future scenarios may not be so easy because the evolutionary behavior is most clear when η_{tech} is large. Civilization growth rates η have nearly completed their adjustment to the asymptotic value of $\eta_{\text{tech}}/2$ that is predicted by Eq. (18). What this implies is that, because innovation appears to have dropped to relatively low levels in recent decades, there is no longer a clear past signal that can be relied upon to propel civilization forward in a prognostic model; the post-war impulse has largely run its course.

This does not mean that the model described here lacks utility looking forward; rather, it implies that η_{tech} must be derived from something more than a fit to the past. To this end, three forces for technological change were identified (Eq. 13). One is how fast civilization networks fray from such externalities as natural disasters. The others address the accessibility of raw materials and how fast new energy reserves are discovered relative to their rates of depletion. Predictions of how these three factors combine may provide a basis for future scenarios for humanity, based more on external physical forces than internal human policies.

5 Conclusions

In Lewis Carroll's *Through the Looking Glass*, Alice was urged by the Red Queen to run with her ever faster. But, "however fast they went, they never seemed to pass anything". As the Red Queen put it, "Now, *here*, you see, it takes all the running you can do, to keep in the same place". In the 1950s and 1960s, civilization made exceptionally rapid gains in energy reserve discovery and resource extraction efficiency. This spurred a rapid acceleration of growth in global wealth that required an equal demand for energy. What followed post-1970 was more constrained growth because diminishing returns settles in for any large system and because fossil fuel resource discovery only just kept up with increasing demand (*Bardi and Lavacchi*, 2009; *Murray and King*, 2012).

Further along, we might anticipate that decay from natural disasters and environmental degradation will also play an important role in civilization's growth trajectory (*Arrow et al.*, 1995). Statistics presented here suggest that decay has thus far been a comparatively weak player. This may change if, as expected, atmospheric CO_2 concentrations reach "dangerous" levels and decay rates increase (*Hansen et al.*, 2007; *Matthews et al.*, 2009; *Garrett*, 2012b; *Mora et al.*, 2013).

Should diminishing returns, resource depletion, and decay combine to cause civilization growth to stall, then simulations described in Part 1 suggest that external forces may have the potential to push civilization into a phase of
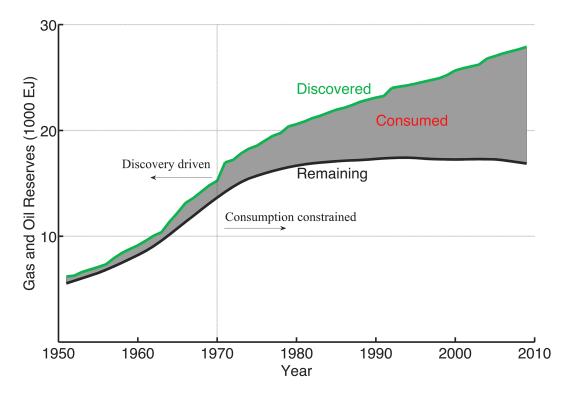
Figure 8: Discovered, consumed, and remaining global reserves of gas and oil since 1950 (source: *Energy*, 2011).

accelerating decline. Civilization lacks the extra energy required to compensate for continued natural disasters, much less grow, and so it tips towards collapse.

Contraction of wealth implies a rate of return η that is negative (Eq. 4). From Eq. (5), this suggests a global economy with a positive nominal GWP but, in effect, a negative real GWP. Fortunately, recent history does not provide a guide for such a global economic disaster. Still, one might imagine a scenario where historically accumulated global wealth shrinks because, at regional or sectoral levels, an ever smaller fraction of civilization remains involved in gross economic production. A nominal GWP remains to be tallied, but it is increasingly offset elsewhere by some combination of wars, a degrading environment, growing unemployment, inflation, death, and decay. Energy consumption is still required to support society – after all, we must always eat. But a diminishing portion of society is able to add net value calculated with monetary instruments that offer promises of future returns.

The silver lining of a contracting civilization might be slowing CO_2 emissions, and eventually slower climate change. The model introduced in Part 1 for making multi-decadal hindcasts of civilization evolution allows for both positive and negative feedbacks to be represented in the coupled evolution of the human–climate system. This paper shows that the economic side of this model is successful at reproducing the past 50 years of economic growth. The next step will be to use the model to provide a range of physically constrained forecasts for the evolution of civilization and the atmosphere for the remainder of this century.

A Reversible cycles and irreversible flows

An implicit consideration with the approach taken here is that it separates small, short-term, "micro-" economic behaviors from larger, longer-term, "macro-" economic evolution. From the perspective of thermodynamics, short-term equilibrium, reversible, cyclic behaviors that are not explicitly resolved are separated from longer-term non-equilibrium, irreversible dynamics that are resolved. This is a common strategy, one illustrated in Fig. 2, as familiar as the separation of the tachometer and speedometer in a car. One represents reversible engine cycles, whereas the other expresses the rate of irreversible travel down the road.

In general, reversible and irreversible processes are linked. This is because the second law of thermodynamics prescribes that all processes are irreversible. Introducing the concept of reversible circulations within a system is a

useful idealization. However, such circulations can only be sustained by an external, irreversible flow of energy and matter through the system. When open systems are near a balance or a steady state, then reversible circulations can be represented as a four-step Carnot cycle or heat engine whereby external heating raises the system potential so that raw materials diffuse from outside the system to inside the system (*Zemanksy and Dittman*, 1997). Waste heat is dissipated to the environment so that the system can relax to its ground potential state where it releases exhaust or undergoes decay. Averaged over time, the circulations within the system maintain a fixed amplitude and period τ_{circ}.

For example, the dynamic circulations of a hurricane are sustained by a inflow of oceanic heat and an outflow of thermal radiation to space (*Emanuel*, 1987). In the case of civilization, we consume energy in order to sustain circulations and extract raw materials from the environment, leaving behind material waste and radiated heat. Petroleum in a car propels our material selves to and from work, where we consume carbohydrates, proteins, and fats to propel electrical signals to and from our brains so that we can consume electricity from coal in order to propel charge along copper wires to and from our computers. Through radiation, frictional losses, and other inefficiencies, all potential energy is ultimately dissipated as waste heat to the atmosphere and ultimately through radiation to space at the mean planetary blackbody temperature of 255 K. Over short timescales, consumption approximately equals dissipation, and civilization circulations maintain a steady state.

Over longer timescales, any small imbalance between consumption and dissipation by civilization becomes magnified. Raw materials are slowly incorporated into civilization's fabric at rate $1/\tau_{\text{growth}}$ (Fig. 2). Further, at the same time that civilization grows, resources are discovered and depleted, and perhaps as a consequence of climate change, decay rates increase too. The focus shifts from the short-term reversible circulations associated with our daily lives to the longer-term timescales associated with the non-equilibrium, irreversible growth of civilization as a whole.

B   Comparisons with traditional economic frameworks

The definition for innovation $\mathrm{d}\ln\eta/\mathrm{d}t$ that has been introduced here is very similar to definitions that have been made elsewhere. Traditional neoclassical growth models calculate the nominal growth of "physical capital" K (in units of currency) from the difference between the portion s of production P that is a savings or investment and capital depreciation at rate δ

$$\frac{\mathrm{d}K}{\mathrm{d}t} = (Y - W) - \delta K = sY - \delta K, \tag{23}$$

where individual and government consumption is represented by $W = (1-s)Y$. What is not saved or invested in the future is consumed in the present.

Labor L (in units of worker hours) uses accumulated investments in physical capital to enable further production Y according to some functional form $f(K, L)$. A commonly used representation is the Cobb–Douglas production function

$$Y = AK^{\alpha}L^{1-\alpha}, \tag{24}$$

where A is a "total factor productivity" that accounts for any residual in the output Y that is not explained by the inputs K and L. The exponent α is empirically determined from a fit to past data and $\alpha \neq 1$. Unfortunately, this presents the drawback that the units for A are ill-defined and dependent on the scenario considered.

The Solow growth model (*Solow*, 1957) expresses the prognostic form for Eq. (24) as

$$\frac{\mathrm{d}\ln Y}{\mathrm{d}t} = \frac{\mathrm{d}\ln A}{\mathrm{d}t} + \alpha \frac{\mathrm{d}\ln K}{\mathrm{d}t} + (1-\alpha)\frac{\mathrm{d}\ln L}{\mathrm{d}t}. \tag{25}$$

The term $\mathrm{d}\ln A/\mathrm{d}t$ has often been interpreted to represent technological progress. Such progress might be exogenous (*Solow*, 1957) or endogenous (*Grossman and Helpman*, 1990; *Romer*, 1994). If exogenous, then progress is considered to be due to an unknown external force. If endogenous, then it might come from targeted investments such as research and development.

In comparison, the alternative approach that has been presented here considers civilization as a whole. Labor is subsumed into total capital. The physics of energy dissipation suggest a focus on the connections between components within a global network rather than on the elements themselves. This adjustment requires only a slight, though important, modification to the Solow growth model in which $\alpha = 1$ and $Y = AK$, where A has fixed units of inverse

time. In this case, no fit to data is required to obtain α, and the units for A are consistent and physical. Equation (25) becomes equivalent to the expression $Y = \eta C$ in Eq. (5), where $\eta \equiv A$ and $C \equiv K$. Further, the expression $\mathrm{d}\ln A/\mathrm{d}t$ that is assumed to describe technological progress in neoclassical frameworks (Eq. 25) is then mathematically equivalent to the definition for innovation $\mathrm{d}\ln\eta/\mathrm{d}t$ in the thermodynamic framework (Eq. 7).

In energy economics, the term "production efficiency", or its inverse, the "energy intensity", is often used to relate the amount of economic output that society is able to obtain per unit of energy it consumes (*Sorrell*, 2007). More efficient, less energy-intense production is ascribed to technological change (e.g., *Pielke Jr et al.*, 2008).

The production efficiency can be defined mathematically as the ratio

$$f = Y/a. \tag{26}$$

From Eq. (26) and the expression $a = \lambda C$ (Eq. 1), where λ is a constant, the production efficiency can then be linked to wealth C through

$$f = \frac{1}{\lambda}\frac{Y}{C}. \tag{27}$$

With rearrangement, λf is then equivalent to the rate of return η in Eq. (5) that expresses how fast economic wealth C can be converted to economic production Y through $Y = \eta C$. More efficient production leads to faster growth of wealth through

$$\eta = \frac{\mathrm{d}\ln C}{\mathrm{d}t} = \lambda f. \tag{28}$$

It follows that

$$\frac{\mathrm{d}\ln\eta}{\mathrm{d}t} \equiv \frac{\mathrm{d}\ln f}{\mathrm{d}t}.$$

Increasing energy efficiency equates with innovation as defined by Eq. (7).

As a side note, since η is also equal to the rate of growth in energy consumption (Eq. 4), this yields the counterintuitive result that higher production efficiency accelerates growth in energy consumption. What is normally assumed is the reverse (*Pacala and Socolow*, 2004; *Raupach et al.*, 2007). While the concept of "backfire" has been reached within more traditional economic contexts (*Saunders*, 2000; *Alcott*, 2005), it is a conclusion that remains highly disputed, at least where economies are viewed at purely sectoral levels (*Sorrell*, 2007, 2014).

Here, increased production efficiency $f = Y/a$ leads to an acceleration of energy consumption at rate $\eta = \lambda f$ because it expands civilization's boundaries with new and existing energy reservoirs (*Garrett*, 2014). Energy reservoirs may eventually be depleted, but at any given point in time efficiency permits the positive feedback that leads to ever faster rates of consumption.

A simple example is to contrast a sick child with a healthy child. Without having to know the "sectoral" level details of cellular function, it is clear that a healthy child will grow fastest. Health here is an implicit representation of the child's ability to efficiently convert current food consumption to growth and increased future consumption. Food contains the energy and matter that the child requires to grow to adulthood. At this point, hopefully, a law of diminishing returns takes over so that weight is able to maintain a steady state.

C Estimated rates of technological change

Estimates of technological change rates η_{tech} require global-scale statistics for the size of energy reserves, the rate of energy consumption, the rate of raw material consumption, and economic inflation. A challenge is that the reliability and availability of statistics diminishes the further back one goes in time. Accurate record keeping can be a challenge even for the most developed nations, much less for every nation. While global statistics for inflation might be available since 1970, they are given for only for a few countries in the 1950s (United *Nations*, 2010).

It is also not obvious how to sensibly represent raw material consumption. Cement, steel, copper, and wood may be among the more obviously important components of the material flow to civilization, but their proportionate weights are far from clear. Steel is consumed in much greater volume than copper since it is a basic building material. But copper is an efficient conduit for electricity and equally important for civilization development.

With respect to energy reserves, the focus here is on fossil fuels since they remain the primary component of the global energy supply. Energy resources represent a total that may ultimately prove recoverable. Energy reserves represent the fraction of resources that is considered currently accessible given existing political and technological considerations. Unfortunately, there is no precise definition of what this means. Moreover, reserve and resource estimates are provided by countries and companies that may have political reasons to misrepresent the numbers (*Höök et al.*, 2010; *Sorrell et al.*, 2010).

The thermodynamic term ΔH_R in Eq. (16) represents the potential energy that is available to drive civilization flows. It would seem to be most obviously represented by reserves rather than resources since reserves are what are most accessible and they most directly exert an external pressure on civilization. A question that arises is how to provide some self-consistent way to add reserves of solid coal to reserves of natural gas and oil that diffuse to civilization as a fluid. Thermodynamically, any form of fossil fuel extraction requires some energy barrier to be crossed, or an amount of work that must be done, in order to make the potential energy immediately available so that it can diffuse to the economy. The rate of diffusion is proportional to a pressure gradient (in units energy density).

For example, well pressure forces a fluid fuel to the surface. Once the energy barrier of building the well is crossed, the magnitude of the pressure can be related to the well reserve size ΔH_R (*Höök et al.*, 2014). In contrast, coal reserves must be actively mined with a continuous energy expenditure. Even if the coal reserve is discovered, there remains a clear energetic cost in order to obtain an energetic return (*Murphy and Hall*, 2010; *Kiefer*, 2013). A hint at the importance of this energy barrier is that new fluid fuel reserves like oil and gas appear to affect economies much more rapidly than coal (*Bernanke et al.*, 1997; *Stijns*, 2005; *Höök et al.*, 2010, 2014), perhaps because they are more easily extracted and consumed.

In what is hopefully a defensible first step, the aforementioned concerns are addressed as follows for the purpose of calculating rates of technological change η_{tech}. Rates of growth of energy reserves (Eq. 16) are determined assuming that coal consumption is not reserve-constrained, and rather that the closest solid equivalent to reserves of oil and gas in terms of accessibility is coal-fired power plants. Like discovering and exploiting an oil well, a power plant must be constructed, and it is only at this point that the coal reserve can be accessed to power civilization. Total reserves are then the production weighted sum of the rates of growth of coal production capacity, oil reserves, and gas reserves (*Rutledge*, 2011; *Energy*, 2011).

Changes in civilization longevity are estimated using Eq. (15), which expresses decay in terms of inflation. Global inflation statistics since 1970 are readily available (United *Nations*, 2010). For the period before 1970, an estimate is an average is taken of the respective inflation rates from the USA, Great Britain, Japan, Germany, Italy, and France (*inflation.eu*, 2014).

Rates of change in the specific energy of raw material extraction $e_s^{\text{tot}} = a/j_a$ (Eq. 17) are derived from statistics for global rates of energy consumption from all sources a (*DOE*, 2011), and from statistics for the consumption of iron and steel, copper, wood (excluding fuelwood), and cement (*FAO*, 2012; *Boden et al.*, 2013; *Kelly and Matos*, 2014a,b). Wood and cement are treated as substitutable construction materials and are added according to their respective volumes. The total rate of change in j_a is then a simple average of the three rates of change: wood and cement, copper, and iron and steel.

Statistics for the components of technological change are provided in Table 2.

Acknowledgements

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