Interacting Induced Dark Energy Model

Amir F. Bahrehbakhsh*
Department of Physics, Faculty of Science, Payam-e-Noor University, Iran

January 1, 2016

Abstract

Similar to the idea of the brane world scenarios, but based on the approach of the induced matter theory, for a non–vacuum five–dimensional version of general relativity, we propose a model in which the conventional matter sources considered as all kind of the matter (the baryonic and dark) and the induced terms emerging from the extra dimension supposed to be as dark energy. Then we investigate the FLRW type cosmological equations and illustrate that the model is capable to explain respectively the deceleration and then acceleration eras of the universe expansion with an interacting term between the matter and dark energy.

PACS number: $04.50.-h$; $04.50.Kd$; $04.20.Cv$; $04.90.+e$; $98.80.Jk$
Keywords: Induced–Matter Theory; FLRW Cosmology; Dark Energy.

1 Introduction

Observations of the brightness of distant type Ia–supernovas indicate that the expansion of the universe is presently accelerating [1]– [5]. Hence, in a school of thought, the universe should mainly be filled with what usually has been called dark energy [6, 7]. Therefore, a considerable amount of researches has been performed in the literature to explain the accelerated expansion of the universe. Most of the models involving dark energy, such as the quintessence [8]– [11], the k-essence [14, 15] and the chaplygin gas [12, 13] models, presuppose minimally coupled scalar fields with different potentials, which have been added in *priori* by hand, and therefore, their origins are not clearly known. Though, recently some efforts based on the Brans–Dicke theory in which the scalar field is non–minimally coupled to curvature, has been performed to explain this acceleration [16]– [21]. Nonetheless, in the recent two decades, explaining the accelerated expansion of the universe through the fundamental theories has been a great challenge.

On the other hand, in search of an improved theory of gravitation, attempts for a geometrical unification of gravity with other interactions have begun by using higher dimensions beyond our conventional four–dimensional ($4D$) space–time. After Nordstrøm [22], who was the first established a unified theory based on extra dimensions, Kaluza [23] and Klein [24] built a five–dimensional ($5D$) version of general relativity (GR) in which electrodynamics rises from an extra fifth dimension. After that, an intensive amount of works have been focused on this regard either via different mechanism for compactification of extra dimension or generalizing it to non–compact scenarios [25] such as the space–time–matter (STM) or induced–matter (IM) theories [26, 27] and the Brane World theories [28]. The significant of the IM theories is that inducing $5D$ field equations without matter sources leads to the $4D$ field equations with matter sources. In another word, the matter sources of $4D$ space–times can be viewed as a manifestation of extra dimensions.

*email: amirfarshad@gmail.com

In this work, similar to the idea of the brane world scenarios, but based on the approach of the IM theory, for a non–vacuum $5D$ version of GR, we propose a model in which the conventional matter sources to be considered as all kind of the matter (the baryonic and dark) and the induced terms emerging from the extra dimension supposed to be as dark energy. We employ a generalized $5D$ Friedmann–Lemaître–Robertson–Walker (FLRW) type metric for the model and investigate its cosmological implications.

In the next section, we give a brief review of the $5D$ version of GR and induce the non–vacuum $5D$ field equations in a $4D$ hypersurface and then collect the extra terms emerging from the scale factor of the fifth dimension as dark energy component of the energy–momentum tensor in addition to the common energy–momentum tensor of the matter (the baryonic and dark). In section 3, we derive the FLRW cosmological equations by considering a generalized FLRW metric in a $5D$ space–time. Then, we obtain the total energy conservation equation and separate it into two interacting energy conservation equations one for all the matter and the other for dark energy. After that, by some manipulation we find the interacting term in the energy conservation equations and show that the model can explain respectively the deceleration and then acceleration phases of the universe expansion. Finally, in the last section conclusion is presented.

2 Induced Dark Energy Model

The Einstein field equations in five dimension can be written as

$$ {}^{(5)}G_{AB} = 8\pi G\, {}^{(5)}T_{AB}\,, \tag{1} $$

where $c = 1$, the capital Latin indices run from zero to four, ${}^{(5)}G_{AB}$ is $5D$ Einstein tensor and ${}^{(5)}T_{AB}$ is $5D$ energy–momentum tensor. As a possible assumption, we consider that ${}^{(5)}T_{\alpha\beta}$ represents the same baryonic and dark matter sources of a $4D$ hypersurface, i.e. $T^{(M)}_{\alpha\beta}$. Thus, in this case ${}^{(5)}T_{AB} = diag(\rho_{_M}, -p_{_M}, -p_{_M}, -p_{_M}, Q)$, where $\rho_{_M}$ and $p_{_M}$ are the energy density and the pressure of the matter, Q as we explain it later, contributes in the interaction between the matter and dark energy, and the Greek indices go from zero to three. Also, for cosmological purposes we restricts our attention to the $5D$ warped metrics of the form

$$ dS^2 = g_{_{AB}}(x^C)dx^A dx^B = {}^{(5)}g_{\mu\nu}(x^C)dx^\mu dx^\nu + g_{_{44}}(x^C)dy^2 \equiv {}^{(5)}g_{\mu\nu}(x^C)dx^\mu dx^\nu + \epsilon b^2(x^C)dy^2\,, \tag{2} $$

in local coordinates $x^A = (x^\mu, y)$, where y represents the fifth coordinate and $\epsilon^2 = 1$. By assuming the $5D$ space–time is foliated by a family of hypersurfaces, Σ, which are defined by fixed values of y, then, one can obtain the intrinsic metric of any typical hypersurface, e.g. $\Sigma_\circ(y = y_\circ)$, by restricting the line element (2) to displacements confined to it. Therefore, the induced metric on the hypersurface $\Sigma_\circ$ becomes

$$ ds^2 = {}^{(5)}g_{\mu\nu}(x^\alpha, y_\circ)dx^\mu dx^\nu \equiv g_{\mu\nu}dx^\mu dx^\nu\,, \tag{3} $$

thus the usual $4D$ space–time metric, $g_{\mu\nu}$, can be recovered. Therefore after some manipulations, equation (1) on the hypersurface $\Sigma_\circ$ can be written as

$$ G_{\alpha\beta} = 8\pi G(T^{(M)}_{\alpha\beta} + T^{(X)}_{\alpha\beta}), \tag{4} $$

where we consider $T^{(X)}_{\alpha\beta}$ as dark energy component of the energy–momentum tensor that analogous to the IM theory [20, 21] defined by

$$ T^{(X)}_{\alpha\beta} \equiv T^{(\mathrm{IM})}_{\alpha\beta} \equiv \frac{1}{8\pi G}\Bigg\{ \frac{b_{;\alpha\beta}}{b} - \frac{\Box b}{b} g_{\alpha\beta} - \frac{\epsilon}{2b^2}\Bigg[\frac{b'}{b} g'_{\alpha\beta} - g''_{\alpha\beta} + g^{\mu\nu}g'_{\alpha\mu}g'_{\beta\nu} - \frac{1}{2}g^{\mu\nu}g'_{\mu\nu}g'_{\alpha\beta} - g_{\alpha\beta}\left(\frac{b'}{b}g^{\mu\nu}g'_{\mu\nu} - g^{\mu\nu}g''_{\mu\nu} - \frac{1}{4}g^{\mu\nu}g^{\rho\sigma}g'_{\mu\nu}g'_{\rho\sigma} - \frac{3}{4}g'^{\mu\nu}g'_{\mu\nu} \right)\Bigg]\Bigg\}\,, \tag{5} $$

in which the prime denotes derivative with respect to the fifth coordinate.

In the following section, we consider a generalized FLRW metric in a $5D$ universe and investigate its cosmological properties.

3 Generalized FLRW Cosmology

For a $5D$ universe with an extra space–like dimension in addition to the three usual spatially homogenous and isotropic ones, metric (2), as a generalized FLRW solution, can be written as

$$dS^2 = -dt^2 + \tilde{a}^2(t,\tilde{y})\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] + \tilde{b}^2(t,\tilde{y})d\tilde{y}^2\,. \tag{6}$$

Generally, the scale factors $\tilde{a}$ and $\tilde{b}$ should be functions of cosmic time and extra dimension coordinate. However, for physical plausibility and simplicity, we assume that they are separable functions of time and extra coordinate. Besides, the functionality of the scale factor $\tilde{b}$ on $\tilde{y}$ could be eliminated by transforming to a new extra coordinate y, hence metric (6) can be rewritten as

$$dS^2 = -dt^2 + a^2(t)l^2(y)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] + b^2(t)dy^2\,. \tag{7}$$

By considering metric (7) and assuming $H \equiv \dot{a}/a$, $B \equiv \dot{b}/b$ and $L \equiv l'/l$, the Einstein equations (1) reduce as follows. The time component $A = 0 = B$ provides

$$H^2 = \frac{8\pi G}{3}\rho_{_M} - HB + \frac{1}{b^2}(L' + 2L^2) - \frac{k}{a^2l^2}\,, \tag{8}$$

the spatial components $A = B = 1, 2, 3$ give

$$\frac{\ddot{a}}{a} = -4\pi G p_{_M} - \frac{1}{2}H^2 - HB - \frac{1}{2}(\dot{B} + B^2) - \frac{1}{2b^2}(2L' + 3L^2) - \frac{k}{2a^2l^2}\,, \tag{9}$$

the $A = 4 = B$ component yields

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}Q - H^2 + \frac{1}{b^2}L^2 - \frac{k}{a^2l^2} \tag{10}$$

and finally $A = 0$ and $B = 4$ (and also $A = 4$ and $B = 0$) component becomes

$$L(H - B) = 0\,, \tag{11}$$

which obviously gives

$$B = H \qquad \text{and} \qquad b(t) = (b_\circ/a_\circ)a(t) \equiv \alpha a(t)\,. \tag{12}$$

By employing relation (4) we define energy density and pressure of dark energy as

$$\rho_{_X} \equiv T_{_{\circ\circ}} = \frac{3}{8\pi G}\Big[\frac{1}{\alpha^2a^2}(L' + 2L^2) - H^2\Big] \tag{13}$$

and

$$p_{_X} \equiv -T_{_{ii}} = \frac{1}{8\pi G}\Big[\dot{H} + 3H^2 - \frac{1}{\alpha^2a^2}(2L' + 3L^2)\Big]\,. \tag{14}$$

Hence, we can rewrite equations (8)–(10) by using these definitions and relation (12) as

$$H^2 = \frac{8\pi G}{3}\tilde{\rho} - \frac{k}{a^2l^2}\,, \tag{15}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\tilde{\rho} + 3\tilde{p}) \tag{16}$$

and

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}Q - H^2 - \frac{k}{a^2 l^2} + \frac{1}{\alpha^2 a^2}L^2 \,, \tag{17}$$

where $\tilde{\rho} \equiv \rho_{_M} + \rho_{_X}$ and $\tilde{p} \equiv p_{_M} + p_{_X}$.

To obtain energy conservation equation, one can take the time derivative of the equation (15) and substitute the equation (16) into it and get

$$\dot{\tilde{\rho}} + 3H(\tilde{\rho} + \tilde{p}) = 0 \,. \tag{18}$$

As the detailed coupling form among dark energy and matter is unclear, one can expect that their conservation equations may not to be independent. Hence, we assume equation (18) can plausibly separate into two distinguished equations for $\rho_{_X}$ and $\rho_{_M}$ as

$$\dot{\rho}_{_X} + 3H(\rho_{_X} + p_{_X}) = f(H) \tag{19}$$

and

$$\dot{\rho}_{_M} + 3H(\rho_{_M} + p_{_M}) = -f(H) \,, \tag{20}$$

where we assume $f(H)$ is a function of Hubble parameter and shows the interacting between dark energy and matter.

In order to find how expansion of the universe decelerate or accelerate, we study the acceleration equations (16) and (17). In this respect at the first, by considering relation (13) and equations (15) and (17) and assuming $p_{_M} = 0$, we get

$$\alpha^2 a^2 \Big[\dot{H} + \frac{8\pi G}{3}(\rho_{_M} - Q)\Big] = -(L' + L^2) = -C \,, \tag{21}$$

where C is a constant and obtain from the fact that the left hand side of the equation is a function of cosmic time but the right hand side of the equation is a function of extra dimension coordinate, thus both sides of the equation must be equal to a constant. Beside, substituting relations (13) and (14) into the equation (19), gets

$$\alpha^2 a^2 \Big(\dot{H} + \frac{8\pi G}{3}\frac{f(H)}{H}\Big) = -(L' + L^2) = -C \,. \tag{22}$$

The second equalities of the equations (21) and (22) yield

$$\frac{l''}{l} \equiv L' + L^2 = C \tag{23}$$

and thus

$$l(y) = l_\circ e^{\pm\sqrt{C}(y - y_\circ)} \qquad \text{and} \qquad L = \sqrt{C} \,, \tag{24}$$

which immediately result $L' = 0$. As we have supposed that $l(y)$ and thereupon L are real functions, hence C is positive real constants. Also, eliminating $\dot{H}$ between the equations (21) and (22) gives the interacting term among the matter and dark energy as

$$f(H) = H(\rho_{_M} - Q) \,. \tag{25}$$

Finally, by some manipulation the equations (15)–(17) on the hypersurface $\Sigma_\circ(y = y_\circ)$ becomes

$$H^2 = \frac{8\pi G}{3}\tilde{\rho} - \frac{k}{a^2 l_\circ^2} = \frac{4\pi G}{3}\rho_{_M} + \frac{a_\circ^2 C}{b_\circ^2 a^2} - \frac{k}{2 l_\circ^2 a^2} \tag{26}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\tilde{\rho} + 3\tilde{p}) = \frac{8\pi G}{3}Q - \frac{4\pi G}{3}\rho_{_M} - \frac{k}{2 l_\circ^2 a^2} \,. \tag{27}$$

Generally, equation (27) indicates that the model can explain both the deceleration and acceleration phases. In the following we consider some simple cases and investigate their results.

A. Non–Interacting Case $f(H) = 0$

In the case $f(H) = 0$, from the equations (25) and (20) we have

$$Q = \rho_{_M} = \rho_{_{M\circ}} \left(\frac{a_\circ}{a}\right)^3 \tag{28}$$

and hence equation (27) becomes

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3} \rho_{_{M\circ}} \left(\frac{a_\circ}{a}\right)^3 - \frac{k}{2l_\circ^2 a^2} \,. \tag{29}$$

For spatially flat or open ($k = 0$ and -1) universe, equation (29) illustrates eternal accelerated expansion, but for a closed one, it suggests that the expansion of the universe first accelerate and then decelerate. Therefore, the non–interacting case for all kind of the geometry has results contrary to the observations and hence it is not a suitable case.

B. Interacting Case with $Q = 0$

In the interacting case with $Q = 0$, the equations (25) and (20) become

$$f(H) = H\rho_{_M} \tag{30}$$

and

$$\rho_{_M} = \rho_{_{M\circ}} \left(\frac{a_\circ}{a}\right)^4 . \tag{31}$$

Thus, the equation (27) yields

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho_{_{M\circ}} \left(\frac{a_\circ}{a}\right)^4 - \frac{k}{2l_\circ^2 a^2} \tag{32}$$

and it is clear that just for an open universe the model can be consistent with the observation of the universe expansion epochs, i.e. first deceleration and then acceleration.

C. Interacting Case with Q as a Constant

Assuming the interacting case with $Q = C'$, results

$$f(H) = H(\rho_{_M} - C') \,, \tag{33}$$

$$\dot{\rho}_{_M} + 4H\rho_{_M} = C'H \tag{34}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho_{_M} + \frac{8\pi G}{3} C' - \frac{k}{2l_\circ^2 a^2} \,. \tag{35}$$

The first and second terms in the right hand side of the acceleration equation (35) respectively make universe expands with deceleration and acceleration and the third term depend on the geometry of the universe, has a contribution in deceleration (with $k = 1$), or acceleration (with $k = -1$) or also acts neutral (with $k = 0$). Since the measurements of anisotropies in the cosmic microwave background suggest that k is very closed to zero [29]– [32], one can eliminate the last term in the equation (35) and also may interpret the second term as the cosmological constant.

4 Conclusions

The observations illustrate that the universe is presently in an accelerated expanding phase. Hence, the main content of the universe should be consisted of what commonly called dark energy. However, an enormous amount of work has been performed to explain this acceleration, but the origin and the nature of dark energy is unknown yet.

In this work, similar to the idea of the brane world scenarios, and following the approach of the induced matter theory, we have investigated the cosmological implications of a non–vacuum five–dimensional version of general relativity in order to explain both deceleration and acceleration eras of the universe expansion. In this respect, on a $4D$ hypersurface, we have classified the energy–momentum tensor into two parts. One part represents all kind of the matter (the baryonic and dark) and a contribution of interacting term, and the other one contains every extra terms emerging from the scale factor of the fifth dimension, have been considered as the energy–momentum tensor of dark energy. Then, by considering a generalized FLRW metric in a $5D$ space–time, we have derived the FLRW cosmological equations on the $4D$ hypersurface and separated the total energy conservation equation into the two equation, one for the matter and the other one for dark energy with interacting term between them.

Afterwards, by investigating the cosmological equations we have shown that the model in general, is capable to explain respectively the deceleration and then after acceleration epochs of the universe expansion. It is notable that in this model, the geometry of the universe also can contribute in deceleration or acceleration. Finally, we have also studied three simple cases, one non–interacting case and two interacting cases. The non–interacting case has no results matched with observations but the interacting cases are consistent with deceleration and then acceleration phases of the universe expansion. However, one of the interacting cases suggests that the acceleration can occur only when the universe is open, the other one is capable to admit all kind of the geometry open, flat and closed ones.

Acknowledgements

We would like to thank Dr. S. Jalalzadeh for useful comments.

References

[1] S. Perlmutter *et al.*, *Nature* **391**, 51 (1998).

[2] A.G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998).

[3] S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999).

[4] P. de Bernardis *et al.*, *Astrophys. J.* **564**, 559 (2002).

[5] S.M. Carroll, "Why is the Universe Accelerating?", *astro–ph*/0310342.

[6] V. Sahni, *Lect. Notes Phys.* **653**, 141 (2004).

[7] M. Kamionkowski, "Dark Matter and Dark Energy", *astro–ph*/0706.2986.

[8] I. Zlatev, L. Wang and P.J. Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1999).

[9] A. Albrecht and C. Skordis, *Phys. Rev. Lett.* **84**, 2076 (2000).

[10] M.C. Bento, O. Bertolami and N.C. Santos, *Phys. Rev. D* **65**, 067301 (2002).

[11] D. Blais and D. Polarski, *Phys. Rev. D* **70**, 084008 (2004).

[12] A.Y. Kamenshchik, U. Moschella and V. Pasquier, *Phys. Lett. B* **511**, 265 (2001).

[13] M.C. Bento, O. Bertolami and A.A. Sen, *Phys. Rev. D* **66**, 043507 (2002).

[14] C. Armendariz–Picon, V. Mukhanov and P.J. Steinhardt, *Phys. Rev. Lett.* **85**, 4438 (2000).

[15] R.J. Scherrer, *Phys. Rev. Lett.* **93**, 011301 (2004).

[16] J.E.M. Aguilar, C. Romero and A. Barros, *Gen. Rel. Grav.* **40**, 117 (2008).

[17] J. Ponce de Leon, *Class. Quant. Grav.* **27**, 095002 (2010).

[18] J. Ponce de Leon, *JCAP* **03**, 030 (2010).

[19] S.M. M. Rasouli, M. Farhoudi and H.R. Sepangi, *Class. Quant. Grav.* **28**, 155004 (2011).

[20] A.F. Bahrehbakhsh, M. Farhoudi and H. Shojaie *Gen. Rel. Grav.* **43**, 847 (2011).

[21] A. F. Bahrehbakhsh, M. Farhoudi, and H. Vakili, *Int. J. Mod. Phys. D* **22**, 1350070 (2013).

[22] G. Nordstrøm, *Phys. Z.* **15**, 504 (1914).

[23] T. Kaluza, *Sitz. Preuss. Akad. Wiss.* **33**, 966 (1921).

[24] O. Klein, *Z. Phys.* **37**, 895 (1926).

[25] J.M. Overduin and P.S. Wesson, *Phys. Rep.* **283**, 303 (1997).

[26] P.S. Wesson, *Space–Time–Matter, Modern Kaluza–Klein Theory* (World Scientific, Singapore, 1999).

[27] P.S. Wesson, *Five–Dimensional Physics* (World Scientific, Singapore, 2006).

[28] M. Pavšič, "The Landscape of Theoretical Physics: A Global View from Point Particles to the Brane World and Beyond, in Search of a Unifying Principle" *gr–qc/0610061.*

[29] N.A. Bachcall *et al.*, *Science* **284**, 1481 (1999).

[30] P. de Bernardis *et al.*, *Nature* **404**, 955 (2000).

[31] S. Hanany *et al.*, *Astrophys. J.* **545**, L5 (2000).

[32] A. Balbi *et al.*, *Astrophys. J.* **545**, L1 (2000).