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Extending the MSSM with singlet higgs and right handed neutrino for the self-interacting Dark Matter

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Abstract

In order to meet the requirement of BBN, the right handed neutrino are added to the singlet higgs sector in the GNMSSM. The spectrum and feynman rules are calculated. the DM pheonomenology are also studied. In case of $\lambda \sim 0$, the singlet sector can give perfect explanation of relic abundance of dark matter and small cosmological structure simulations. The BBN constraints on the light mediator can be easily solved by decaying to the right handed neutrino. when the λ_N is at the order of $\mathcal{O}(0.1)$, the mass of the mediator can be constrained to several MeV.

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I. INTRODUCTION

Supersymmetry (SUSY) [1, 2] gives a natural solution to the hierarchy problem suffered by the Standard Model (SM). It provides a good dark matter (DM) candidate and realizes the gauge coupling unification. Among the SUSY models, the Minimal Supersymmetric Standard Model (MSSM) [3] has been intensively studied. However, the MSSM suffers from the μ-problem [4]. In order to solve such problem, a singlet field S is always introduced to replace the μ term of the MSSM, resulting model is the Next-to-Minimal Supersymmetric Standard Model(NMSSM).[5, 6] The effective μ-term is generated by the dynamical generation of the μ-term through the coupling SH_uH_d when S develops a vacuum expectation value (VEV), Nevertheless, since the higgs and dark matter have the singlet components, many other problems in of the MSSM can also be relaxed. For example, the little hierarchy problem of the MSSM is relaxed by the generation of an extra tree-level mass term for the SM-like higgs boson.

Since the gauge interaction of singlet dominant higgs almost vanishes, its mass can be tuned to any value. This is a special feature of the NMSSM, because of that the mass of higgs can be much smaller than the electro-weak (EW) scale, The interchange of the light higgs gives a conceivably self-interaction of the singlet sector. In the paper[7] we studied the possibility of the DM self-interaction to solve the small cosmological scale anomalies in the singlet extensions of the MSSM.[8] We found that the correlation between the DM annihilation rate and DM-nucleon spin independent (SI) cross section strongly constrains this model so that it cannot realize the DM self-interacting scenario in the NMSSM. After that, we found that DM self-interacting scenario can be realized by extending the singlet the most generally. (denoted as GNMSSM) In case of $\lambda \sim 0$, the singlet forms a dark sector. DM is singlino, light CP even higgs is the mediator. Enough large parameter space survives and the Sommerfeld enhancement factor can be realized too.

As pointed in [9], the singlet CP even higgs can not be exactly dark, it must decays before the start of the Big Bang Nucleosynthesis (BBN) (~ 1 sec) so the decay products will not affect BBN. This constraint sets a minimum interaction coupling between the Standard Model and the dark sector in order to facilitate the fast decay of the singlet higgs before the BBN era. But if the singlet higgs couples to the SM particles, the direct detection rate of DM will be enhanced greatly by the light mediator,[7] thus it will be excluded by measurement of the SI cross section. This seems to be an obstacle of the model.

One way to solve such obstacle it to add a right handed neutrino to the GNMSSM, the singlet higgs decay into right handed sterile neutrino, and it has nothing to do with the quarks, escaping from the the detection rate. In fact, there should be such a singlet in particle zoo, the oscillation of neutrino implies that the neutrino have mass and right handed neutrino should exist. A realistic SUSY model must involve in the massive neutrinos, too. The most economical introduction should be replace the singlet higgs with singlet right-handed neutrino superfield ($\widehat{N}$).[10] However, such model violates the R-parity, giving no DM candidate. In order to preserving R-parity, $\widehat{N}$ must be an additional supermultiplate. A TeV-scale Majorana mass for the right-handed neutrino is dynamically generated through the $\widehat{S}\widehat{N}\widehat{N}$ coupling when S develops a VEV, (note that such a TeV-scale majorana mass is too low for the see-saw mechanism and thus the neutrino Yukawa couplings H_uLN must be very small). In the same way, a TeV-scale mass for the right-handed sneutrino can also be generated, which can serve as a good dark matter candidate [11]. In paper [12], we find the right handed neutrino can enhance the mass of SM higgs several GeV.

In this work we will study the SUSY model of singlet higgs and right handed neutrino, and check that if the self-interacting DM scenario and BBN constraints can be realized. We will take into account of constraints from DM relic abundance. We organize the content as follows. In Sec. II, we will show the detail of the model and discuss the general interactions of DM. In Sec. III we show the numerical results, and conclusion is given in Sec. IV.

II. MODEL, SPECTRUM AND FEYNMAN RULES

As talked in the introduction, the μ term can not be replaced by singlet right handed neutrino $\widehat{N}$ in the GNMSSM, there must be a singlet higgs $\widehat{S}$. The superpotential is [13]

$$W = W_{\text{Yukawa}} + \eta\widehat{S} + \frac{1}{2}\mu_s\widehat{S}^2 + \frac{1}{3}\kappa\widehat{S}^3 + \frac{1}{2}\mu_N\widehat{N}^2 + \lambda_N\widehat{S}\widehat{N}^2 \ , \tag{1}$$

where

$$W_{\text{Yukawa}} = Y_u\widehat{H}_u\cdot\widehat{Q}\widehat{u}_R - Y_d\widehat{H}_d\cdot\widehat{Q}\widehat{d}_R - Y_e\widehat{H}_d\cdot\widehat{L}\widehat{e}_R + Y_\nu\widehat{H}_u\cdot\widehat{L}\widehat{N} + \lambda\widehat{S}\widehat{H}_u\cdot\widehat{H}_d. \tag{2}$$

Note that here we impose R-parity and thus the linear and triple terms of $\widehat{N}$ are forbidden. As a result, there is no VEV for the right-handed sneutrino (we will show how to get the global minimum in the following). We set $\lambda \sim 0$ so that the singlet sector decouples from the SM sector. In the following discussion, we will concentrate on the singlet sector. The soft SUSY breaking terms take the form

$$\begin{aligned} -\mathcal{L}_{\rm soft} &= m_s^2|S|^2 + \left(C_\eta \eta S + \frac{1}{2}\, B_s\mu_s S^2 + \frac{1}{3}\,\kappa A_\kappa S^3 + {\rm h.c.}\right) \\ &+ m_{\tilde{N}}^2|\tilde{N}|^2 + \left(\frac{1}{2}\, B_N\mu_N \tilde{N}^2 + \lambda_N A_N S\tilde{N}^2 + {\rm h.c.}\right). \end{aligned} \tag{3}$$

The first line is the soft lagrangian of the singlet higgs, while the second line is of the right handed sneutrino. The main difference between the NMSSM and GNMSSM is reflected in their higgs sectors which contain different singlet higgs mass matrices and self-interactions. The difference mainly comes from the F-term potential:

$$\begin{aligned} V_{F_{\rm S}} &= |\eta + \mu_s S + \kappa S^2 + \lambda_N \tilde{N}^2|^2 \\ &= |\kappa S^2|^2 + \eta^2 + \mu_s^2|S|^2 + \left(\eta\mu_s S + \kappa\eta S^2 + \kappa\mu_s S^2 S^* + {\rm h.c.}\right) \end{aligned} \tag{4}$$

$$V_{F_{\rm N}} = |2\lambda_N S + \mu_N|^2|\tilde{N}|^2. \tag{5}$$

Since η^2 is a constant, the $\mu_s^2|S|^2, \eta\mu_s S, \kappa\eta S^2$ terms can be absorbed by the redefinition of the soft SUSY breaking parameters $m_s^2|S|^2,\ C_\eta\eta S, \frac{1}{2}\, B_s\mu_s S^2$. Then, the singlet potential is

$$\begin{aligned} V &= V_F + V_{\rm soft} \\ &= m_s^2|S|^2 + \left(C_\eta\eta S + \frac{1}{2}\, B_s\mu_s S^2 + \frac{1}{3}\,\kappa A_\kappa\ S^3 + \kappa\mu_s S^2 S^* + {\rm h.c.}\right). \end{aligned} \tag{6}$$

$$+\ |2\lambda_N S + \mu_N|^2|\tilde{N}|^2 + m_N^2|\tilde{N}|^2 + \left(\frac{1}{2}\, B_N\mu_N\tilde{N}^2 + \lambda_N A_N S\tilde{N}^2 + {\rm h.c.}\right) \tag{7}$$

The spectrum of the model is simple, The chiral supermultiplet of singlet higgs contains a complex scalar and a Majorana fermion χ. After the scalar component getting a VEV v_s, we can get one CP-even higgs h and one CP-odd higgs a. The mass spectrum and the relevant Feynman rules are

$$m_\chi = 2\kappa v_s + \mu_S, \tag{8}$$

$$m_h^2 = \kappa v_s s(4\kappa v_s + A_\kappa + 3\mu_S) - C_\eta\eta/v_s, \tag{9}$$

$$m_a^2 = -2B_s\mu_s - \kappa v_s(3A_\kappa + \mu_S) - C_\eta\eta/v_s, \tag{10}$$

As for the neutrino sector, a Majorana mass can be generated through the coupling SN^2 and $\mu_N N^2$. In this paper we set μ_N at TeV scale, thus the Majorana mass is too small for the conventional see-saw mechanism and the Yukawa coupling $y_\nu H_u L N$ should be very small ($y_N \ll 1$) and be neglected. Since there is no Dirac mass term here, the mass spectrum of the right-handed neutrino sector is also very simple. Denoting $\tilde{N} = R + iM$, there are only one CP-even right-handed sneutrino (denoted as R) and one CP-odd right-handed sneutrino (denoted as M). The right-handed neutrino is denoted as N. From Eq. (7), we can get the spectrum as

$$\begin{aligned} m_R^2 &= (2\lambda_N v_s + \mu_N)^2 + M_{\tilde{N}}^2 + 2\lambda_N v_s A_N + 2\lambda_N(\kappa v_s^2 - \lambda v_u v_d) \\ m_M^2 &= (2\lambda_N v_s + \mu_N)^2 + M_{\tilde{N}}^2 - 2\lambda_N v_s A_N - 2\lambda_N(\kappa v_s^2 - \lambda v_u v_d) \\ m_N &= \mu_N + 2\lambda_N v_s. \end{aligned} \tag{11}$$

Note that in our numerical study we require M_R^2 and M_M^2 be positive, and, as a result, the global minimum of the scalar potential locates at the zero point of the right-handed sneutrino field (the right-handed sneutrino has no VEV and R-parity is preserved).

With the above spectrum we can get the couplings between the higgs and the right-handed neutrino/sneutrino. We list the corresponding Feynman Rules for the following calculation:

$$V_{hhh} = -\sqrt{2}k(6\kappa s + A_\kappa + 3\mu_S) = -\sqrt{2}\kappa(3m_\chi + A_\kappa), \tag{12}$$

$$V_{haa} = -\sqrt{2}\kappa(2\kappa s - A_\kappa + \mu_S) = -\sqrt{2}\kappa(m_\chi - A_\kappa), \tag{13}$$

$$V_{h\chi\chi} = -\sqrt{2}\kappa, \tag{14}$$

$$V_{a\chi\chi} = \sqrt{2}i\kappa\gamma^5. \tag{15}$$

$$V_{hNN} = -\sqrt{2}\lambda_N \tag{16}$$

$$V_{aNN} = \sqrt{2}i\lambda_N\gamma^5 \tag{17}$$

$$V_{R\chi N} = -\frac{1}{\sqrt{2}}\lambda_N \tag{18}$$

$$V_{M\chi N} = \frac{i}{\sqrt{2}}\lambda_N\gamma^5 \tag{19}$$

Note that lighter right handed sneutino can be a DM candidate. In this work, we require that the LSP is singlino χ, As talked in the introduction, the predictions of the ΛCDM model on small cosmological scale structures seem not so successful and there are mainly three anomalies: *missing satellites*, *cusp vs core*, *too big to fail*. [14–16]. To solve these problems, they needs a proper self-interaction between dark matter that can gives the non-relativistic self-scattering cross section.[17] The scattering cross section between DM can be described by quantum mechanics. In our model interchange of h between singlino χ forms a Yukawa potential

$$V(r) = \frac{\kappa}{2\pi} e^{-m_h r}. \tag{20}$$

Since mass of h can be tuned to any value, self interaction can be realized in case of without right handed neutrino. we will check the case of participation of right handed neutrino. Also we will set N lighter than $m_h/2$ so that h can decay to the right handed neutrino N to satisfy the BBN constraint. [20] The numerical results are shown in the following section.

III. REALIZING THE SELF-INTERACTING DARK MATTER

The numerical input for the simulation of small scales is the differential cross section $d\sigma/d\Omega$ as a function of the DM relative velocity v. The standard cross section $\sigma = \int d\Omega (d\sigma/d\Omega)$ receives a strong enhancement in the forward backward scattering limit ($\cos\theta \to \pm 1$), which does not change the DM particle trajectories. Thus two additional cross sections are defined to parameterize transport [21], the transfer cross section σ_T and the viscosity (or conductivity) cross section σ_V:[17–19]

$$\sigma_T = \int d\Omega \, (1-\cos\theta) \frac{d\sigma}{d\Omega}, \qquad \sigma_V = \int d\Omega \, \sin^2\theta \, \frac{d\sigma}{d\Omega}. \tag{21}$$

σ_T is for the estimation of the Dirac DM, while σ_T is for the Majorana DM. Since the singlino χ is a Majorana fermion, thus we should check the viscosity cross section which should be defined with two variables:

$$\frac{d\sigma_{VS}}{d\Omega} = |f(\theta) + f(\pi-\theta)|^2 = \frac{1}{k^2} \left| \sum_{\ell(\text{EVEN number})}^{\infty} (2\ell+1)(\exp(2i\delta_l)-1) P_\ell(\cos\theta) \right|^2 \tag{22}$$

$$\frac{d\sigma_{VA}}{d\Omega} = |f(\theta) - f(\pi-\theta)|^2 = \frac{1}{k^2} \left| \sum_{\ell(\text{ODD number})}^{\infty} (2\ell+1)(\exp(2i\delta_l)-1) P_\ell(\cos\theta) \right|^2 \tag{23}$$

Using the orthogonality relation for the Legendre polynomials, we can obtain

$$\frac{\sigma_{VS} k^2}{4\pi} = \sum_{\ell(\text{EVEN number})}^{\infty} 4\sin^2(\delta_{\ell+2}-\delta_\ell)(\ell+1)(\ell+2)/(2\ell+3), \tag{24}$$

$$\frac{\sigma_{VA} k^2}{4\pi} = \sum_{\ell(\text{ODD number})}^{\infty} 4\sin^2(\delta_{\ell+2}-\delta_\ell)(\ell+1)(\ell+2)/(2\ell+3). \tag{25}$$

The phase shift δ_ℓ must be computed by solving the Schrödinger equation

$$\frac{1}{r^2}\frac{d}{dr}\Big(r^2\frac{dR_\ell}{dr}\Big)+\Big(k^2-\frac{\ell(\ell+1)}{r^2}-2m_rV(r)\Big)R_\ell=0.$$

directly with partial wave expansion method. The total wave function of the spin-1/2 fermionic DM must be antisymmetric with respect to the interchange of two identical particles. Then the spatial wave function should be symmetric when the total spin is 0 (singlet) while the spatial wave function should be antisymmetric when the total spin is 1 (triplet). In our following analysis, we assume that the DM scatters with random orientations, thus the triplet is three times as likely as the singlet and the average cross section will be

$$\sigma_V=\frac{1}{4}\sigma_{VS}+\frac{3}{4}\sigma_{VA}\ . \tag{26}$$

Since $\lambda\sim 0$ we do not need to take care of the direct detection rate of DM, but the observed relic abundence of DM must be considered. In our model, χ can annihilate to singlet higgs and right handed neutrino, and the rate determines the relic density which give a stringent constraints on the model. The general form of the annihilation cross section is given by [22, 23]

$$\sigma v=\frac{1}{4}\frac{\bar{\beta}_f}{8\pi sS}\left[|A(^1S_0)|^2+\frac{1}{3}\left(|A(^3P_0)|^2+|A(^3P_1)|^2\right)+|A(^3P_2)|^2\right], \tag{27}$$

where S is the symmetry factor, $A(^1S_0),\ A(^3P_0),\ A(^3P_1),\ A(^3P_2)$ are the contributions from different spin states of DM and β_f is given by

$$\bar{\beta}_f=\sqrt{1-2(m_X^2+m_Y^2)/s+(m_X^2+m_Y^2)^2/s^2}\ , \tag{28}$$

with X,Y being the final states. The feynman diagrams of the annihilation are shown in the Fig.1 and Fig.2. The Fig.1 shows the annihilation to the singlet higgs, in which ϕ can be h or a or both. The Fig.2 shows the annihilation to the right handed neutrino N. the analytical results of σv is shown in the appendix.

Next we do the numerical calculation to see if the model can survive under the constraints of relic density, scattering cross section and BBN. From the spectrum and Feynman rules above, we can see that the mass parameters of the six particles (h,a,χ,R,M,N) can be set freely because the relevant input soft parameters can take arbitrary values. This makes the following calculation much easier and we can choose the input parameters easily without fine-tuning. Then we have nine input parameters in our calculation, namely

$$m_\chi,\ m_h,\ m_a,\ m_R,\ m_M,\ m_N,\ A_\kappa,\ \kappa,\ \lambda_N. \tag{29}$$

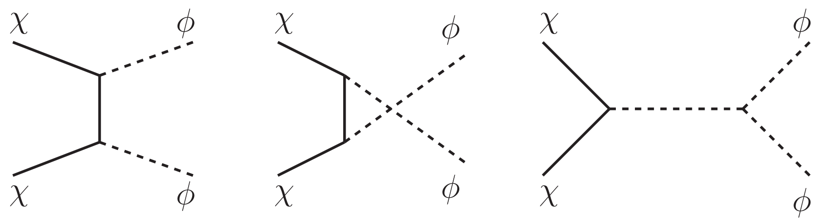

FIG. 1: Diagram of Dark matter annihilation to singlet higgs h, a.

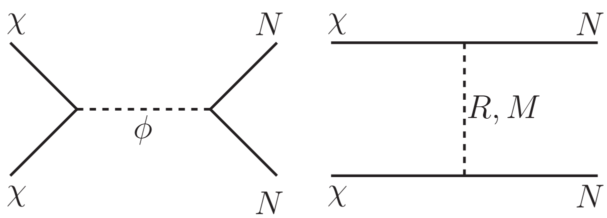

FIG. 2: Diagram of Dark matter annihilation to right handed neutrio N.

The first seven parameters are mass dimension, we scan them in the range of $(10^{-4},\ 10^{4})$ GeV, the last to parameters are dimensionless, we scan them in the range of $(10^{-11},\ 10^{1})$. Note that, in our scan we set $m_R, m_M > m_\chi$ and $m_N < m_h$. The numberical results are shown in Fig.3. which shows the survived points in $(m_\chi,\ m_h)$ space. The left plot shows the case of only one mediator scalar h, the middle plot shows the case of only singlet higgs in the GNMSSM, while the right plots shows the results of the model above which is GNMSSM plus a right handed neutrino.

From Fig.3, we can see that the simulation of small structure gives a stringent constraints on the self interacting DM. Especially, when there is only one light mediator. However, there still are points left which satisfy all the requirements of Dwarf scale, Milky Way and cluster. GNMSSM without right handed neutrino has a larger parameter space to solve the anomalies of all three small scales The reason is that in the DM self-interaction model [18] DM can only annihilate into hh via t-channel and u-channel while in the GNMSSM DM can annihilate into hh, ha and aa via t-channel, u-channel and s-channel, as shown in Fig.1. The results of GNMSSM right handed neutrino are almost the same as that without it. The reason is that the BBN constraints in fact give a much lower bound on the coupling strength λ_N. The

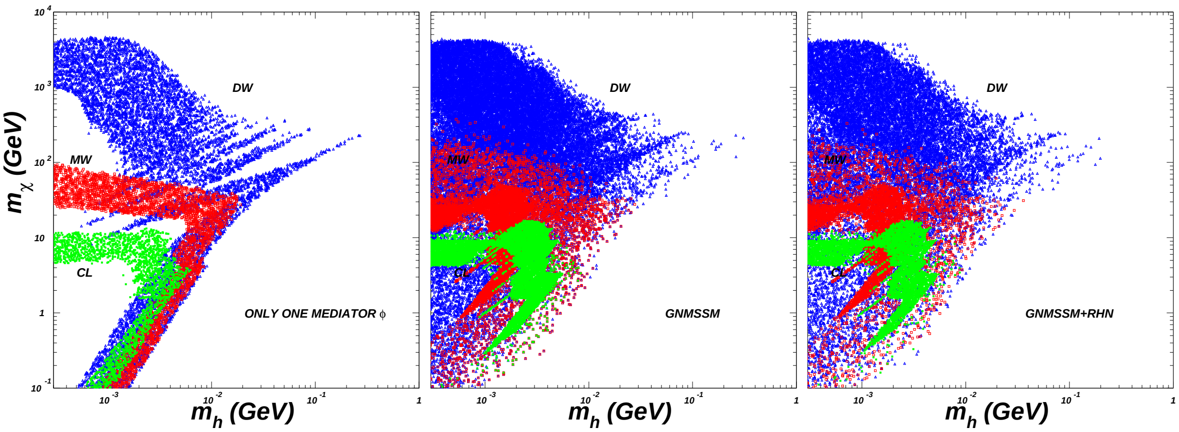

FIG. 3: The survived points in contraint of relic density, scattering cross section and BBN. The blue points are the points satisfy the simulation in the Dwarf scale ($\sigma/m_\chi \sim 0.1-10$ cm^2/g, the characteristic velocity is 10 km/s.) The red points are the points satisfy the simulation in the Milky Way ($\sigma/m_\chi \sim 0.1-1$ cm^2/g, the characteristic velocity is 200 km/s.) The green points are the points satisfy the simulation in the Milky Way ($\sigma/m_\chi \sim 0.1-1$ cm^2/g, the characteristic velocity is 1000 km/s.)

width of h in this case is

$$\Gamma_h = \lambda_N^2 \frac{\sqrt{m_h^2/4 - m_N^2}}{2\pi m_h^2}(m_h^2 - 4m_N^2) \tag{30}$$

$$\simeq \lambda_N^2 \frac{m_h}{4\pi}, \qquad \text{(in case of } m_h \gg m_N). \tag{31}$$

One can check that if λ_N is bigger than 10^{-10}, the mediator (10 MeV) can decay much earlier before 1 second, thus it is much easily to meet the BBN requirement.

Though BBN constraints on λ_N is loose, the survive parameter space of the model can be changed when λ_N is at order of $\mathcal{O}(0.1)$ when the annihilation to N become significant. This can been seen from the left plot of Fig. 4 that when λ_N increases up to $\mathcal{O}(0.1)$, κ begins to decrease to a small value. In this case, the annihilation channels to N (shown in Fig.2) give proper contribution to the relic density of DM. The right plot shows survived points in $(m_h,\ \lambda_N)$ space. We can see that as λ_N increases up, mass of mediator h are constraint to several MeV. Moreover an interaction strength $\lambda_N \sim \mathcal{O}(0.1)$ has been advocated a way of suppressing the standard active-to-sterile oscillation production process, easing the cosmological constraints from $N_{\rm eff}$.[24] In all, we conclude that adding right handed neutrino to the GNMSSM gives a satisfactory solution to the otherwise problematic decay of h.

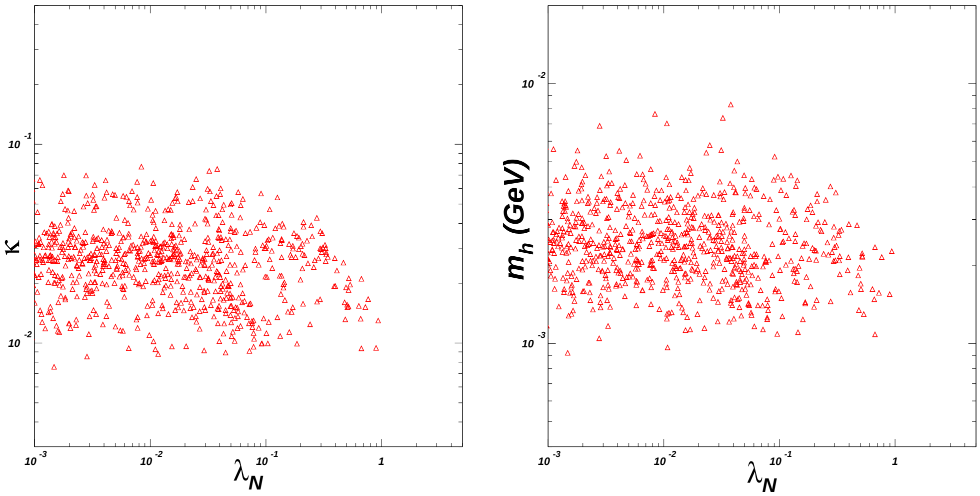

FIG. 4: Parameters m_h, κ versus λ_N of the survived points.

IV. CONCLUSIONS

In this work, in order to solve the BBN constraints on the singlet higgs we add the right handed neutrino to the GNMSSM. we calculated the spectrum and feynman rules. and studied the the DM phenomenalogy. In case the $\lambda \sim 0$ the singlet sector can give perfect explanation of relic abundance and small structure simulations. The BBN constraints on the light mediator h can be easily solved by decaying to the right handed neutrino. in case of the λ_N at the order of $\mathcal{O}(0.1)$, the mass of the mediator mass can be constrained to several MeV. Since we set $\lambda \sim 0$ the direct detection rate doesn't need to be considered. In case of $\lambda \neq 0$, the singlet higgs and right handed neutrino will interacts with the gauge sector, the resusts will appear in our further studies.

Acknowledgments

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Appendix

The amplitudes from different final states are given by

1. $\chi\chi \to hh$:

$$A(^3P_0) \;=\; 2\sqrt{6}v\kappa^2\left[\frac{R(3m_\chi + A_\kappa)}{4 - R(m_h)^2 + iG_h} - 2\frac{1 + R(m_\chi)}{P_\chi} + \frac{4}{3}\frac{\bar{\beta}_f^2}{P_\chi^2}\right], \tag{32}$$

$$A(^3P_2) \;=\; -(16/\sqrt{3})v\kappa^2\bar{\beta}_f^2/P_\chi^2\;. \tag{33}$$

2. $\chi\chi \to aa$:

$$A(^3P_0) \;=\; 2\sqrt{6}v\kappa^2\left[\frac{R(m_\chi - A_\kappa)}{4 - R(m_h)^2 + iG_h} - 2\frac{1 - R(m_\chi)}{P_\chi} + \frac{4}{3}\frac{\bar{\beta}_f^2}{P_\chi^2}\right], \tag{34}$$

$$A(^3P_2) \;=\; -(16/\sqrt{3})v\kappa^2\bar{\beta}_f^2/P_\chi^2\;. \tag{35}$$

3. $\chi\chi \to ha$:

$$\begin{aligned} A(^1S_0) \;=\;& -4\sqrt{2}\kappa^2\frac{R(m_\chi - A_\kappa)}{4 - R(m_a)^2}\left(1 + \frac{v^2}{8}\right) \\ & +8\sqrt{2}\kappa^2\frac{R(m_\chi)}{P_\chi}\left[1 + v^2\left(\frac{1}{8} - \frac{1}{2P_\chi} + \frac{\bar{\beta}_f^2}{3P_\chi^2}\right)\right] \\ & +2\sqrt{2}\kappa^2\left(R(m_a)^2 - R(m_h)^2\right)\left[1 + v^2\left(-\frac{1}{8} - \frac{1}{2P_\chi} + \frac{\bar{\beta}_f^2}{3P_\chi^2}\right)\right], \end{aligned} \tag{36}$$

$$A(^3P_1) \;=\; 8v\kappa^2\bar{\beta}_f^2/P_\chi^2\;. \tag{37}$$

4. $\chi\chi \to NN$:

$$\begin{aligned} A(^1S_0) \;=\;& \lambda_N^2\left[1 + v^2\left(-\frac{1}{2P_R} + \frac{\bar{\beta}_f^2}{3P_R^2}\right)\right]R(m_N)/P_R \\ & +\;\lambda_N^2\frac{1}{P_R}\left[1 + v^2\left(\frac{1}{4} - \frac{1}{2P_R} - \frac{\bar{\beta}_f^2}{6P_R} + \frac{\bar{\beta}_f^2}{3P_R^2}\right)\right] \\ & -\;\lambda_N^2\left[1 + v^2\left(-\frac{1}{2P_M} + \frac{\bar{\beta}_f^2}{3P_M^2}\right)\right]R(m_N)/P_M \\ & +\;\lambda_N^2\frac{1}{P_M}\left[1 + v^2\left(\frac{1}{4} - \frac{1}{2P_M} - \frac{\bar{\beta}_f^2}{6P_M} + \frac{\bar{\beta}_f^2}{3P_M^2}\right)\right] \\ & -\;8\kappa\lambda_N\frac{1}{4 - R(m_a)^2 + iG_a}\left(1 + \frac{v^2}{4}\right). \end{aligned} \tag{38}$$

$$\begin{aligned} A(^3P_{00}) \;=\;& \lambda_N^2\bar{\beta}_f\left[-\frac{1}{2}\left(\frac{1}{2P_R} - \frac{2}{3P_R^2}\right) + \frac{R(m_N)}{P_R^2} - \frac{1}{2}\left(\frac{1}{2P_M} - \frac{2}{3P_M^2}\right) - \frac{R(m_N)}{P_M^2}\right] \\ & -\;2\kappa\lambda_N\bar{\beta}_f\frac{1}{4 - R(m_h)^2 + iG_h} + 2\kappa\lambda_N\bar{\beta}_f\frac{1}{4 - R(m_a)^2 + iG_a} \end{aligned} \tag{39}$$

$$A(^3P_{10}) \ = \ 0. \tag{40}$$

$$A(^3P_{11})(\lambda') \ = \ \lambda_N^2\lambda'\bar{\beta}_f\left(-\frac{1}{P_R}+\frac{1}{P_R^2}+\frac{R(m_N)}{P_R^2}\right)-\lambda_N^2\lambda'\bar{\beta}_f\left(-\frac{1}{P_M}+\frac{1}{P_M^2}-\frac{R(m_N)}{P_M^2}\right). \tag{41}$$

$$A(^3P_{20}) \ = \ \lambda_N^2\bar{\beta}_f\left(\frac{R(m_N)+1}{P_R^2}-\frac{R(m_N)-1}{P_M^2}\right). \tag{42}$$

$$A(^3P_{21}) \ = \ \lambda_N^2\bar{\beta}_f\left(-\frac{1+R(m_N)}{P_R^2}+\frac{1-R(m_N)}{P_M^2}\right). \tag{43}$$

In the above formulas

$$R(m_X)=\frac{m_X}{m_\chi},\ P_j=1+R(m_j)^2-\frac{1}{2}(R(m_X)^2+R(m_Y)^2),\ G_i=\frac{\Gamma_i m_i}{m_\chi^2}. \tag{44}$$

and the λ' is the number of the final helicity state.

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