

# Testing the gas mass density profile of galaxy clusters with distance duality relation

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## ABSTRACT

In this paper, assuming the validity of distance duality relation,  $\eta = D_L(z)(1+z)^{-2}/D_A(z) = 1$ , where  $D_A(z)$  and  $D_L(z)$  are the angular and the luminosity distance respectively, we explore two kinds of gas mass density profiles of clusters: the isothermal  $\beta$  model and the non-isothermal double- $\beta$  model. In our analysis, performed on 38 massive galaxy clusters observed by *Chandra* (within the redshift range of  $0.14 < z < 0.89$ ), we use two types of cluster gas mass fraction data corresponding to different mass density profiles fitted to the X-ray data. Using two general parameterizations of  $\eta(z)$  (phenomenologically allowing for distance duality violation), we find that the non-isothermal double- $\beta$  model agrees better with the distance duality relation, while the isothermal  $\beta$  model tends to be marginally incompatible with the Etherington theorem at 68.3% CL. However, current accuracy of the data does not allow to distinguish between the two models for the gas-density distribution at a significant level.

**Key words:** X-rays: galaxies: clusters -(cosmology:) distance scale - cosmology: miscellaneous

## 1 INTRODUCTION

The distance duality relation (DDR thereafter):  $D_L/D_A(1+z)^{-2} = 1$ , which relates the luminosity distance  $D_L(z)$  to the angular diameter distance  $D_A(z)$  at a given redshift  $z$ , is known to hold in any metric theory of gravity. Hence, it has extensively been applied in modern observational cosmology (Schneider et al. 1992; Bassett & Kunz 2004; Cunha et al. 2007; Zhu et al. 2008b; Mantz et al. 2014; Holanda, Lima & Ribeiro 2011; Cao & Liang 2011). Until now, different astronomical observations have been used to check the validity of this reciprocity relation using the following expression

$$\frac{D_L}{D_A}(1+z)^{-2} = \eta(z), \quad (1)$$

where  $\eta(z)$  could be a function of the redshift  $z$  (of course DDR corresponds to  $\eta(z) = \text{const.} = 1$ ). For instance, Uzan et al. (2004) used the  $D_A$  estimated from Sunyaev-Zeldovich effect (SZE) and X-ray surface brightness of 18 galaxy clusters (Reese et al. 2002), and found DDR consistent with the observations at  $1\sigma$  CL. The validity of the distance duality relation was further confirmed by De Bernardis et al. (2006), with a larger sample of angular diameter distances from 38 galaxy clusters (Bonamente et al. 2006). More recently, instead of testing the reciprocity relation itself, Holanda, Lima & Ribeiro

(2011) discussed the possibility of using DDR (i.e. assuming that it is true) to test the geometrical shape of galaxy clusters. They used two angular diameter distance  $D_A$  measurements based on two different cluster geometries: ellipsoidal  $\beta$  model underlying De Filippis et al. (2005) data and spherical  $\beta$  model assumed by (Bonamente et al. 2006). They found that ellipsoidal geometry was more consistent with the DDR, and concluded that it was a better model to describe galaxy clusters.

In this paper, we propose a new method to derive observed  $\eta_{obs}(z)$  parameter from the cluster gas mass fraction  $f_{gas} = M_{gas}/M_{tot}$  inferred from X-ray data, and we use the assumption that DDR should be valid (i.e.  $\eta(z) = 1$ ) to discuss appropriateness of two cluster gas mass density profiles assumed. More specifically, we use two data sets of cluster gas mass fraction derived from *Chandra* X-ray data (LaRoque et al. 2006) under two different assumptions about the gas mass density profiles: isothermal  $\beta$  model and non-isothermal double- $\beta$  model.

In our analysis we consider two particular parameterizations of phenomenological  $\eta(z)$  dependence: I.  $\eta = \eta_0 + \eta_{P1}z$ ; II.  $\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ . The first expression is a linear parametrization equivalent to the first order Taylor expansion in redshift. The second one is inspired by the commonly used CPL parametrization for dark energy equation of state (Linder 2003) and is equivalent to the first order expansion

in the scale factor  $a(t)$  which is the only gravitational degree of freedom in Freedman - Robertson - Walker cosmology. These two parametrizations have been extensively used to investigate the properties of dark energy in the literature (Cao et al. 2012; Cao & Zhu 2014, 2015). Assuming that the Etherington theorem is valid (which is quite a reasonable assumption), the best-fit value obtained with a given data set should be  $\eta_0 = 1$  and  $\eta_P = 0$ . Our results indicate that, the non-isothermal double- $\beta$  model tends to be more compatible with the reciprocity relation than the isothermal  $\beta$  model. This kind of result is an interesting example of how general principles (like DDR) could be used to assess the validity of assumptions concerning local physical conditions.

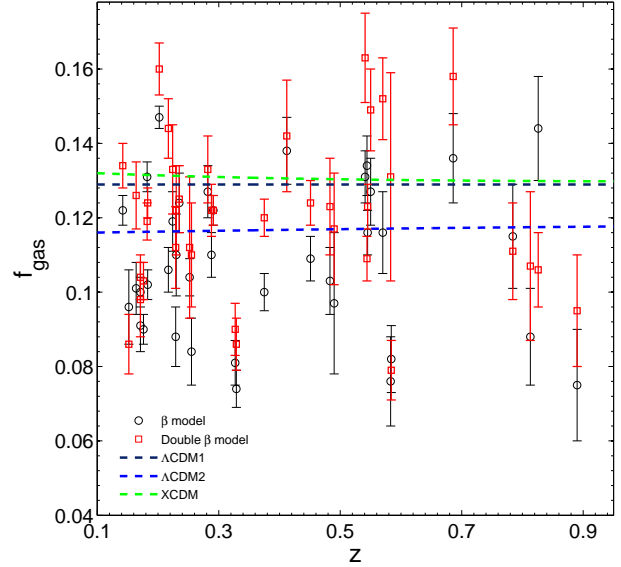
This paper is organized as follows. In Section 2 we present two samples of gas mass fraction data from 38 X-ray luminous galaxy clusters and their corresponding mass density profiles. Statistical method and constraint results on  $\eta(z)$  parameters are shown in Section 3. Finally, we summarize our main conclusions and make a discussion in Section 4.

## 2 GALAXY CLUSTER SAMPLES

The cluster gas mass fraction is defined as a ratio of the X-ray emitting gas mass to the total mass of a cluster, i.e.,  $f_{\text{gas}} = M_{\text{gas}}/M_{\text{tot}}$ . Gas mass  $M_{\text{gas}}$  is derived from the X-ray surface brightness while the total mass  $M_{\text{tot}}$  can be obtained by assuming that the gas is in hydrostatic equilibrium with the cluster NFW potential. In order to perform appropriate calculations, one needs the gas mass density profile. Therefore, in our analysis aimed at constraining free parameters ( $\eta_0, \eta_P$ ) in general expression for  $\eta(z)$ , we will use the gas mass fractions obtained with different gas mass density profiles.

In order to calculate  $\eta_{\text{obs}}$  from the data (according to Eq.(6) shown below) we use two sets of gas mass fraction  $f_{\text{gas}}$  both derived from a sample of 38 luminous X-ray clusters with temperatures  $T_{\text{gas}} > 5$  keV, observed by *Chandra* X-ray Observatory (LaRoque et al. 2006) located at redshifts from  $z = 0.14$  to  $z = 0.89$ . In fact, LaRoque et al. (2006) presented results of their analysis using both X-ray only data (for isothermal  $\beta$ -model) and a combination of *Chandra* X-ray data and BIMA/OVRO interferometric radio SZE data for the double- $\beta$  model. However, according to Holanda, Goncalves & Alcaniz (2012), the gas mass fraction measurement via SZE depends only on the angular diameter distance, so it is insensitive to the validity of the DDR. Therefore, only the high resolution *Chandra* X-ray data are considered in our analysis. The above mentioned two sets of  $f_{\text{gas}}$  were obtained within the most commonly used isothermal  $\beta$  model and under assumption of the non-isothermal double- $\beta$  model, respectively.

Fig. 1 displays these data and their observational statistical uncertainties. We will also consider possible systematic uncertainties influencing the derived gas mass fraction following the discussion in (Bonamente et al. 2006) concerning their method of calculating angular diameter distances to clusters. The effect of these systematic uncertainties on the gas mass fractions is summarized in Table 3 of LaRoque et al. (2006). Note that the systematic difference between  $f_{\text{gas}}$  derived from the isothermal  $\beta$  model and the double- $\beta$  model still exists. From a purely statistical point



**Figure 1.** The cluster gas mass fraction data derived from a sample of 38 luminous X-ray clusters observed by *Chandra* X-ray Observatory (LaRoque et al. 2006). Black circles and red squares with the error bars correspond to the results obtained under the assumption of isothermal  $\beta$  model and non-isothermal double- $\beta$  model, respectively. The predictions of  $f_{\text{gas}}(z)$  under the assumption of DDR are also overplotted, for the three reference cosmologies (see Table 1) considered in this paper.

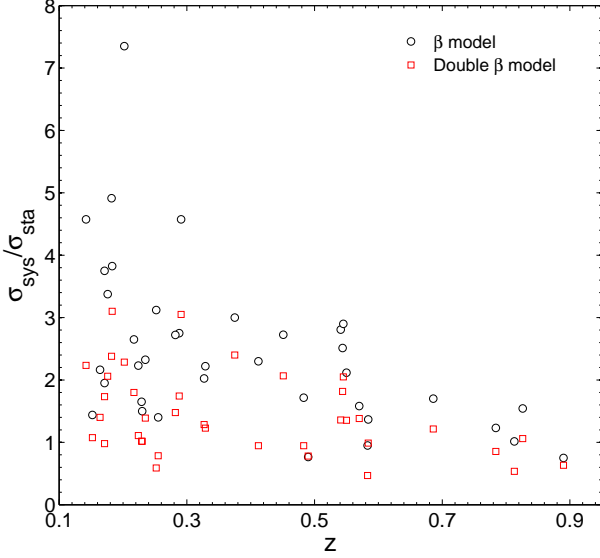
of view, non-parametric Wilcoxon signed rank test reveals that difference between these two datasets is significant at the level of  $p = 0.0001$ . More specifically, the assumption of isothermality can potentially affect the gas mass fraction measurements by about ( $-5\%$ ) through its effects on both gas mass and total mass. Therefore, an appropriate systematic uncertainty will be assigned to the gas mass fractions derived from the isothermal  $\beta$  model. In order to show the major sources of uncertainty in the present analysis, we display the ratio between systematic and statistical uncertainties for these data in Fig. 2.

In the frequently used isothermal  $\beta$  model, the 3-dimensional electron number density  $n_e$  can be written as (Cavaliere & Fusco-Femiano 1976, 1978; Grego et al. 2001; Reese et al. 2002; Ettori et al. 2004)

$$n_e(r) = n_{e0} \left( 1 + \frac{r^2}{r_c^2} \right)^{-3\beta/2}, \quad (2)$$

where  $n_{e0}$  is the central electron number density,  $r$  is the radius from the center of the cluster,  $r_c$  is the core radius of the intracluster medium (ICM), and  $\beta$  is a power law index. Although it has been widely used as a useful assumption in galaxy cluster studies, the isothermal  $\beta$  model still encounters some problems with describing the central emission excess seen in some clusters (Mohr et al. 1999; LaRoque et al. 2006) and the X-ray surface brightness at  $r > (1-1.5) r_{2500}$ , where  $r_{2500}$  is the radius at which the mean enclosed mass density is equal to  $2500\rho_{\text{crit}}$ . Such features, non-compatible with the isothermal  $\beta$  model have been observed in archival ROSAT PSPC data and numerical simulations (Mohr et al. 1999; Borgani et al. 2004).

Therefore, in order to overcome the limitations of the



**Figure 2.** The ratio between systematic and statistical uncertainties for the cluster gas mass fraction data. Black circles and red squares with the error bars correspond to the results obtained under the assumption of isothermal  $\beta$  model and non-isothermal double- $\beta$  model, respectively.

simplest isothermal  $\beta$  model we have also considered a more sophisticated cluster plasma model, the non-isothermal double- $\beta$  model. More specifically, its density profile is assumed to be of the following form:

$$n_e(r) = n_{e0} \left[ f \left( 1 + \frac{r^2}{r_{c1}^2} \right)^{-3\beta/2} + (1-f) \left( 1 + \frac{r^2}{r_{c2}^2} \right)^{-3\beta/2} \right] \quad (3)$$

where  $r_{c1}$  is the core radius representing the narrow, peaked central density component contributing to the central density  $n_{e0}$  by a factor  $0 \leq f \leq 1$ , and  $r_{c2}$  is the other core radius describing the broad, shallow outer density profile.

Let us now describe how we extracted the observed DDR parameter  $\eta_{obs}$  from the cluster gas mass fraction data. Massive galaxy clusters are the largest known bound systems in the Universe and are expected to provide a unique information of the matter content of the Universe. The baryonic-to-total mass ratio of clusters should closely match the ratio of  $\Omega_b/\Omega_m$ , where  $\Omega_b$  and  $\Omega_m$  are the present dimensionless density parameter of the baryonic matter and dust matter, respectively. Since the reconstructed  $f_{gas}$  depends on the angular diameter distances at different redshifts,  $f_{gas} \propto D_A^{1.5}(z)$ , a number of studies have used it to probe the acceleration of the Universe and thus the properties of dark energy (Allen et al. 2004, 2008; Ettori et al. 2009; Cao & Zhu 2014).

Our starting point is the following general expression for the gas mass fraction (see e.g. Allen et al. (2008); Holanda, Lima & Ribeiro (2011)):

$$\begin{aligned} f_{gas}(z) &= K A \Upsilon \left( \frac{\Omega_b}{\Omega_m} \right) \left( \frac{D_L^{ref}(z)}{D_L(z)} \right) \left( \frac{D_A^{ref}(z)}{D_A(z)} \right)^{1/2} \\ &= K A \Upsilon \left( \frac{\Omega_b}{\Omega_m} \right) \eta_{obs}^{-3/2} \left( \frac{D_A^{ref}(z)}{D_A(z)} \right)^{3/2} \end{aligned} \quad (4)$$

Here,  $\eta_{obs}$  is the value of the DDR parameter implied by observations.  $K$  is a calibration constant characterizing the systematic uncertainty on the overall normalization (Mantz et al. 2014). We use  $K = 0.94 \pm 0.09$  based on recent results from weak lensing mass measurements (Applegate et al. 2014; Allen et al. 2013).  $A$  is an angular correction factor quantifying the shift of the angular scale  $\theta_{2500}$  of the  $r_{2500}$  radius from  $\theta_{2500}^{ref}$  obtained in the reference cosmology assumed to be  $\Lambda$ CDM (Mantz et al. 2014). So, the  $A$  factor reads:

$$A = \left( \frac{\theta_{2500}^{ref}}{\theta_{2500}} \right)^\varepsilon \sim \left( \frac{H(z) D_A(z)}{H^{ref}(z) D_A^{ref}(z)} \right)^\varepsilon \quad (5)$$

where the exponent  $\varepsilon$  assessed for the  $0.8-1.2 \theta_{2500}$  shell (Mantz et al. 2014; Allen et al. 2013) is  $\varepsilon = 0.442 \pm 0.035$ .  $\Upsilon$  is the gas depletion parameter related to thermodynamic history of X-ray emitting gas in the course of cluster formation. More specifically, this factor is modeled as  $\Upsilon_{2500} = \Upsilon_0(1 + \alpha_\Upsilon z)$ . We adopt uniform priors,  $\Upsilon_0 = 0.845 \pm 0.042$  and  $\alpha_\Upsilon = 0.00 \pm 0.05$  in our paper. This is motivated by the recent hydrodynamic simulations of the hottest clusters within  $0.8-1.2 \theta_{2500}$  shells (Battaglia et al. 2012; Planelles et al. 2013).  $\Omega_b$  is fixed at the best-fit value  $\Omega_b h^2 = 0.02205 \pm 0.00028$  suggested by Planck observations (Ade et al. 2014).  $D_A(z)$  denotes the true angular diameter distance and  $D_A^{ref}(z)$  is the angular diameter distance calculated in the reference cosmological model, which is a flat  $\Lambda$ CDM model with  $\Omega_m = 0.30$ ,  $h = 0.70$  (standard concordance model).

By combining Eq.(4) and Eq.(5), we can obtain the observed value of  $\eta_{obs}$  as

$$\eta_{obs}^{3/2} = K \Upsilon_0 (1 + \alpha_\Upsilon z) \frac{\Omega_b}{\Omega_m} f_{gas}^{-1} \left( \frac{H(z)}{H^{ref}(z)} \right)^\varepsilon \left( \frac{D_A^{ref}(z)}{D_A(z)} \right)^{3/2-\varepsilon} \quad (6)$$

It is apparent that in our analysis,  $\eta(z)$  function includes two effects: possible DDR violation and our uncertainty with respect to the true cosmological model. Since our main goal is to test cluster gas mass density profiles based on the validity of distance duality relation,  $D_A(z)$  will be calculated in three ways. Firstly, by assuming the standard model with  $\Omega_m = 0.30$  and  $H_0 = 70.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , which is equivalent to the reference cosmology in cluster studies and very similar to the WMAP9 constraints (Komatsu et al. 2011). Secondly, by taking the best-fit matter density parameter and Hubble constant given by Planck Collaboration:  $\Omega_m = 0.315$  and  $H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Ade et al. 2014), in the framework of  $\Lambda$ CDM model. And thirdly, by considering the XCDM model, i.e. the one in which the equation of state  $p = w\rho$  for dark energy has constant  $w$  parameter. More precisely, we take the best fitted parameters from Planck+WMAP9 data:  $\Omega_m = 0.294$ ,  $w = -1.05$ , and  $H_0 = 70.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Cai et al. 2014). Summary of three cosmologies adopted here can be found in Table 1. For comparison, the predictions of  $f_{gas}(z)$  within these three cosmologies under the assumption of DDR and using the central values of four nuisance parameters are also shown in Fig 1.

### 3 ANALYSIS AND RESULTS

As we already mentioned,  $\eta(z)$  function captures phenomenologically our uncertainty about not only the DDR

**Table 1.** Parameters of three cosmologies considered: standard concordance  $\Lambda$ CDM model ( $\Lambda$ CDM1),  $\Lambda$ CDM best fitted to Planck data ( $\Lambda$ CDM1), XCDM model best fitted to Planck+WMAP9 data.

Cosmology	Cosmological parameters
$\Lambda$ CDM1	$\Omega_m = 0.30, H_0 = 70.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$
$\Lambda$ CDM2	$\Omega_m = 0.315, H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$
XCDM	$\Omega_m = 0.294, w = -1.05, H_0 = 70.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$

**Table 2.** Summary of the results for  $\eta(z) = \eta_0 + \eta_{P1}z$  and  $\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ , respectively, at  $1\sigma$  confidence level for the  $\beta$  model and double- $\beta$  model. The statistical and systematic uncertainties in  $\eta$  are shown separately. Two samples including the  $n = 38$  full sample and the  $n = 29$  sub-sample are used (see text for definitions). Source of cosmological priors are also given in brackets.

$\beta$ model: Parameterizations (Sample/Cosmology)	$\eta_0$	$\eta_{P1}$
$\eta(z) = \eta_0 + \eta_{P1}z$ (Full sample/ $\Lambda$ CDM1)	$\eta_0 = 1.068 \pm 0.099(\text{stat}) \pm 0.034(\text{sys})$	$\eta_{P1} = -0.084 \pm 0.058(\text{stat}) \pm 0.129(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ (Full sample/ $\Lambda$ CDM1)	$\eta_0 = 1.080 \pm 0.104(\text{stat}) \pm 0.051(\text{sys})$	$\eta_{P2} = -0.148 \pm 0.117(\text{stat}) \pm 0.276(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P1}z$ (Sub-sample/ $\Lambda$ CDM1)	$\eta_0 = 1.108 \pm 0.115(\text{stat}) \pm 0.033(\text{sys})$	$\eta_{P1} = -0.144 \pm 0.075(\text{stat}) \pm 0.312(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P1}z$ (Sub-sample/ $\Lambda$ CDM2)	$\eta_0 = 1.028 \pm 0.104(\text{stat}) \pm 0.037(\text{sys})$	$\eta_{P1} = -0.127 \pm 0.072(\text{stat}) \pm 0.127(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ (Sub-sample/ $\Lambda$ CDM2)	$\eta_0 = 1.060 \pm 0.112(\text{stat}) \pm 0.047(\text{sys})$	$\eta_{P2} = -0.276 \pm 0.134(\text{stat}) \pm 0.282(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P1}z$ (Sub-sample/XCDM)	$\eta_0 = 1.242 \pm 0.127(\text{stat}) \pm 0.041(\text{sys})$	$\eta_{P1} = -0.265 \pm 0.087(\text{stat}) \pm 0.148(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ (Sub-sample/XCDM)	$\eta_0 = 1.285 \pm 0.135(\text{stat}) \pm 0.052(\text{sys})$	$\eta_{P2} = -0.529 \pm 0.166(\text{stat}) \pm 0.305(\text{sys})$
Double- $\beta$ model: Parameterizations (Sample/Cosmology)	$\eta_0$	$\eta_{P2}$
$\eta(z) = \eta_0 + \eta_{P1}z$ (Full sample/ $\Lambda$ CDM1)	$\eta_0 = 0.991 \pm 0.102(\text{stat}) \pm 0.015(\text{sys})$	$\eta_{P1} = -0.037 \pm 0.060(\text{stat}) \pm 0.087(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ (Full sample/ $\Lambda$ CDM1)	$\eta_0 = 1.002 \pm 0.104(\text{stat}) \pm 0.027(\text{sys})$	$\eta_{P2} = -0.092 \pm 0.114(\text{stat}) \pm 0.178(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P1}z$ (Sub-sample/ $\Lambda$ CDM1)	$\eta_0 = 0.981 \pm 0.104(\text{stat}) \pm 0.017(\text{sys})$	$\eta_{P1} = -0.023 \pm 0.069(\text{stat}) \pm 0.089(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ (Sub-sample/ $\Lambda$ CDM1)	$\eta_0 = 0.996 \pm 0.108(\text{stat}) \pm 0.030(\text{sys})$	$\eta_{P2} = -0.078 \pm 0.132(\text{stat}) \pm 0.184(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P1}z$ (Sub-sample/ $\Lambda$ CDM2)	$\eta_0 = 0.915 \pm 0.095(\text{stat}) \pm 0.018(\text{sys})$	$\eta_{P1} = -0.015 \pm 0.064(\text{stat}) \pm 0.082(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ (Sub-sample/ $\Lambda$ CDM2)	$\eta_0 = 0.924 \pm 0.102(\text{stat}) \pm 0.024(\text{sys})$	$\eta_{P2} = -0.049 \pm 0.121(\text{stat}) \pm 0.182(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P1}z$ (Sub-sample/XCDM)	$\eta_0 = 1.101 \pm 0.117(\text{stat}) \pm 0.024(\text{sys})$	$\eta_{P1} = -0.139 \pm 0.078(\text{stat}) \pm 0.08(\text{sys})$
$\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ (Sub-sample/XCDM)	$\eta_0 = 1.123 \pm 0.121(\text{stat}) \pm 0.027(\text{sys})$	$\eta_{P2} = -0.257 \pm 0.143(\text{stat}) \pm 0.196(\text{sys})$

but also about true cosmological model and possibly other systematics. Therefore it is reasonable to treat it as a function of redshift and start with two quite natural parameterizations:

$$\begin{cases} \eta(z) = \eta_0 + \eta_{P1}z, \\ \eta(z) = \eta_0 + \eta_{P2}z/(1+z). \end{cases} \quad (7)$$

where  $\eta_0$  and  $\eta_P$  parameters quantify the shift from the expected standard result ( $\eta_0 = 1, \eta_P = 0$ ).

Using routines available within CosmoMC package (Lewis & Bridle 2002), we performed Monte Carlo simulations of the posterior likelihood  $\mathcal{L} \sim \exp(-\chi^2/2)$ , where

$$\chi^2 = \sum \frac{[\eta(z) - \eta_{obs}(z)]^2}{\sigma_{\eta_{obs}}^2}, \quad (8)$$

$\eta_{obs}(z)$  was calculated from the  $f_{gas}$  data via Eq.(6) and  $\sigma_{\eta_{obs}}$  was calculated according to the standard law of uncertainty propagation. Besides the statistical errors of X-ray observations, we have also considered systematic errors concerning instrument calibration  $\pm 6\%$ , X-ray background  $+2\%$ , hydrostatic equilibrium  $-10\%$ , and isother-

mal assumption  $-5\%$  (LaRoque et al. 2006). Combined in quadrature, they result in a typical relative error of 10% for the  $f_{gas}$  measurements with isothermal  $\beta$  model and 15% for the  $f_{gas}$  measurements with the non-isothermal double- $\beta$  model. We would stress however, that according to recent numerical simulations and comparisons between X-ray and lensing masses (Lau et al. 2009; Landry 2013; Giles et al. 2015), the hydrostatic mass underestimates the true mass especially at large radii (which causes the gas mass fraction to be overestimated), and the estimate of systematic uncertainties due to hydrostatic equilibrium can be larger than 10%. This effect still needs to be investigated with more available data. In our analysis, based on the results of Lau et al. (2009); Landry (2013), a systematic uncertainty of  $+10\%$  on the total mass  $M_{tot}$  is assessed for all clusters, which corresponds to a systematic error of  $-10\%$  on  $f_{gas}$ .

We performed a MCMC analysis and marginalized over the nuisance parameters ( $K, Y_0, \alpha_T, \varepsilon$ ) by multiplying the probability distribution functions and then integrating (Ganga et al. 1997). When  $D_A(z)$  is calculated within the fiducial model ( $\Omega_m = 1 - \Omega_\Lambda, H_0 = (0.30, 70 \text{ km s}^{-1} \text{ Mpc}^{-1})$ )

consistent with WMAP9 observations, we obtain the results shown in Table 2. Obviously both statistical and systematic uncertainties should be included in the analysis. Therefore they have been displayed explicitly in Table 2. In the case of the first parametrization  $\eta(z) = \eta_0 + \eta_{P1}z$ , the best-fit  $\eta$  parameters are  $\eta_0 = 1.068 \pm 0.133$ ,  $\eta_{P1} = -0.084 \pm 0.187$  (sta+sys) for the isothermal  $\beta$  model and  $\eta_0 = 0.991 \pm 0.117$ ,  $\eta_{P1} = -0.037 \pm 0.147$  (sta+sys) for the non-isothermal double- $\beta$  model. One can see that gas mass fractions obtained from the non-isothermal double beta model, are in better agreement with the reciprocity relation ( $\eta_0 = 1$ ,  $\eta_{P1} = 0$ ). However, from the statistical point of view this preference is marginal.

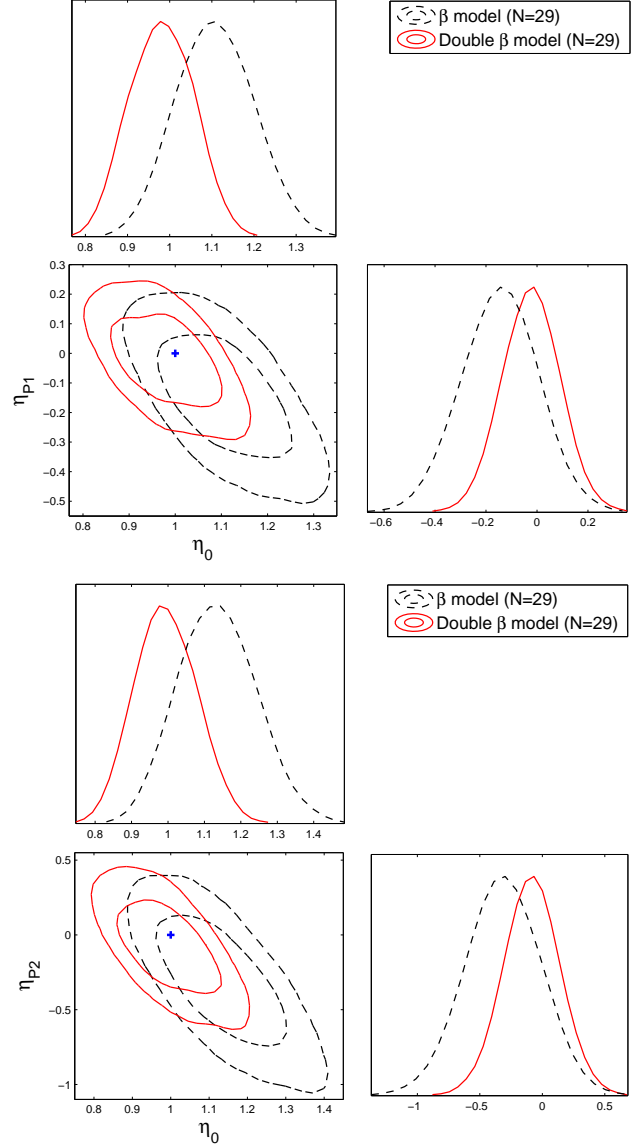
According to the findings of LaRoque et al. (2006), some objects: Abell 665, ZW 3146, RX J1347.5-1145, MS 1358.4 + 6245, Abell 1835, MACS J1423+2404, Abell 1914, Abell 2163, Abell 2204 have questionable reduced  $\chi^2$ . By excluding these objects from the full sample we obtained a sub-sample of 29 galaxy clusters, on which we performed a similar analysis. The results are displayed in Fig. 3. As compared with the previous case, one can see that the non-isothermal double- $\beta$  model is in even much better agreement with the DDR ( $\eta_0 = 0.981 \pm 0.121$ ,  $\eta_{P1} = -0.023 \pm 0.158$ ;  $\pm$  corresponds to 68.3% CL) than the isothermal  $\beta$  model ( $\eta_0 = 1.108 \pm 0.148$ ,  $\eta_{P1} = -0.144 \pm 0.387$ ). When compared with the previous analysis, the incompatibility of the isothermal  $\beta$ -model with the validity of DDR is clearly more evident at  $1\sigma$ , although the significance of this conclusion is not high enough from the statistical point of view.

In the case of second parametrization  $\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ , results for the  $n = 38$  full sample and  $n = 29$  sub-sample, are shown in Table 2. Respective confidence regions on the  $\eta_0 - \eta_{P2}$  plane and marginalized likelihood distributions for the parameters are shown in Fig. 3. As one can see the double- $\beta$  model again seems to be favored over the isothermal  $\beta$ -model.

We have also checked whether one can get tighter constraints on  $\eta$  (and hence gain more discriminative power concerning alternative density profiles) by fixing  $\eta_P = 0$ . The results are shown in Fig. 4. One can see that the two model density profiles are statistically compatible but again the double- $\beta$  model looks better with the mode of the likelihood coinciding with the DDR expectation  $\eta = 1$ .

As we already mentioned, one can contemplate other types of best fitted “true” cosmology instead of the standard concordance  $\Lambda$ CDM model. Therefore, we also considered the  $\Lambda$ CDM model but with parameters best fitted to the Planck data. In this case, we only analyzed the  $n = 29$  sub-sample and the results are presented on Table 2 and Fig. 5. Moreover, since  $\Lambda$ CDM while useful has its own conceptual problems and might not be the ultimate model of the Universe, we have also considered quintessential XCDM model. In particular, we have taken its parameters  $(w, \Omega_m = 1 - \Omega_\Lambda, H_0) = (-1.05, 0.294, 70.4 \text{ km s}^{-1} \text{ Mpc}^{-1})$  according to Cai et al. (2014) best-fit to Planck+WMAP9 data. The results obtained with  $n = 29$  sub-sample are presented in Table 2 and Fig. 6.

Even though, considering both statistical and systematic uncertainties, we find (at the level of best fitted values) that isothermal  $\beta$ -model is incompatible with the validity of DDR at  $1\sigma$ , it is difficult to distinguish these two density models by using  $f_{\text{gas}}$  measurements. As discussed in

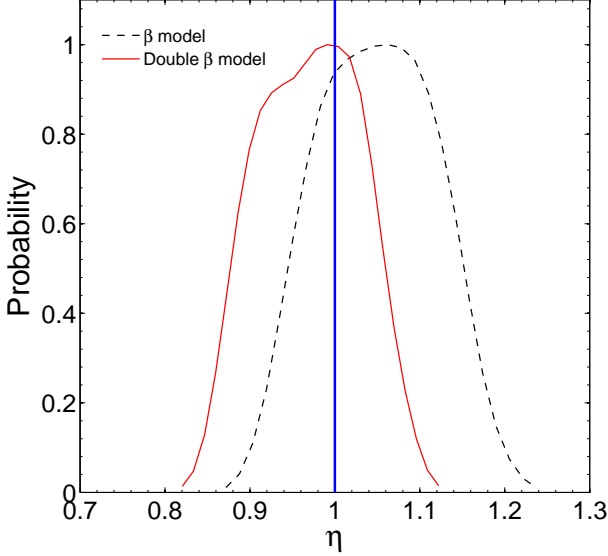


**Figure 3.** Confidence contours and marginalized likelihood distribution functions for the parameters in  $\eta(z) = \eta_0 + \eta_{P1}z$  and  $\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$  relation. Black dashed lines and red lines correspond to the fits obtained on the full  $n = 29$  sub-sample under the assumption of isothermal  $\beta$  model and non-isothermal double- $\beta$  model, respectively. The blue cross represents the expected case when the DDR holds exactly ( $\eta_0 = 1$ ,  $\eta_{P1,2} = 0$ ).

(LaRoque et al. 2006), the results of  $f_{\text{gas}}$  derived from models including the single- $\beta$  model and double- $\beta$  model fit to the X-ray data agree well enough to claim that the cluster core can be accounted for by either excluding it from the fit or modeling the gas with the double  $\beta$ -model.

## 4 CONCLUSIONS

Clusters of galaxies are the largest virialized objects in the Universe. Therefore they can serve as excellent probes of cosmology: their number density can be predicted and tested

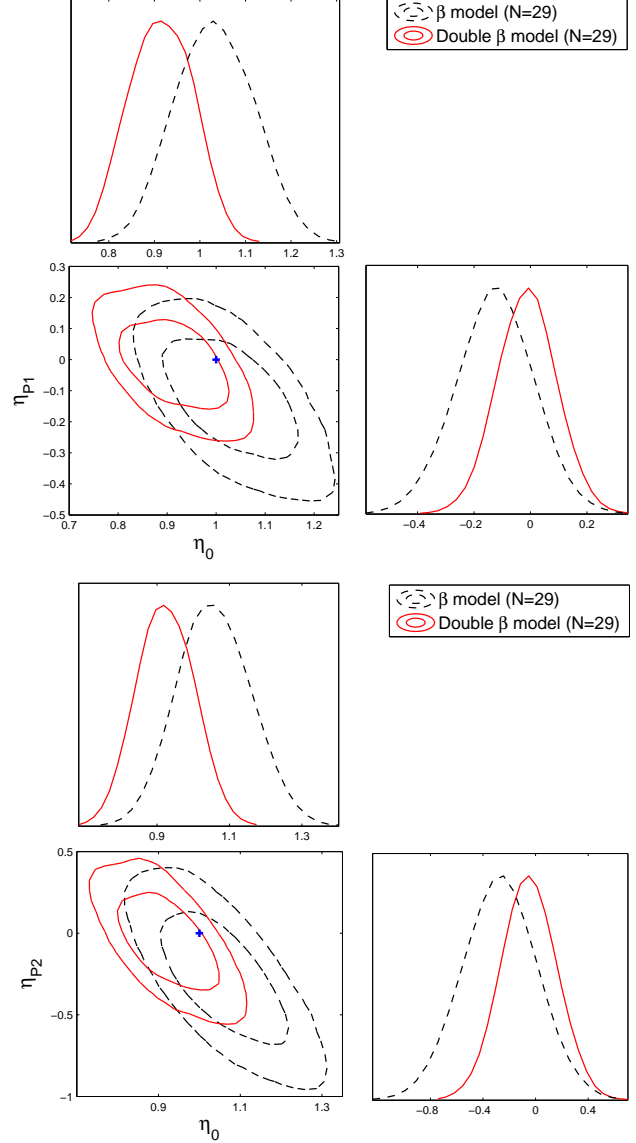


**Figure 4.** Likelihood functions for  $\eta$  parameter corresponding to the isothermal  $\beta$  model (black dashed line) and non-isothermal double- $\beta$  model (red continuous line). Results are obtained with the  $n = 29$  sub-sample of clusters. WMAP9 best fitted  $\Lambda$ CDM parameters were taken to represent a “true” cosmology. The blue vertical line represents the expected case when the DDR holds exactly.

against observations. More than that, combined X-ray and Sunyaev-Zeldovich observations can in principle be used to measure absolute distances to the clusters and to test cosmology (the Hubble constant, dark energy etc.). However, for cosmological applications we need to have at least a reliable “proxy” for the gas mass distribution in clusters and this is otherwise known to be complicated (e.g. from strong and weak lensing studies). So we need to compromise by making assumptions like isothermal  $\beta$  model or its “off-spring” – the non-isothermal double- $\beta$  model. In this paper we addressed the question of which of these two proxies is more supported by the data.

Our judgement was based on the assumed validity of the DDR — the distance duality relation (for which there are good reasons to believe that it’s true). To be specific, we have studied two samples of cluster gas mass fraction data obtained from 38 X-ray luminous galaxy clusters observed by *Chandra* in the redshift range  $0.14 \sim 0.89$  (LaRoque et al. 2006) the full sample and its  $n = 29$  sub-sample produced by excluding some “suspect” clusters.

Bearing in mind, that in practice some systematic effects might disturb the DDR relation, we parameterized it in two ways:  $\eta(z) = \eta_0 + \eta_{P1}z$  and  $\eta(z) = \eta_0 + \eta_{P2}z/(1+z)$ . Then we checked which of the two “proxy” models for gas mass distribution (isothermal  $\beta$ -model and non-isothermal double  $\beta$ -model) performs better. Our result is that within standard concordance cosmology ( $\Lambda$ CDM1) double- $\beta$  model is marginally better respecting the DDR (at  $1 \sigma$  level). If one takes instead  $\Lambda$ CDM parameters best fitted to Planck data, both models are compatible. However, within the quintessential XCDM cosmology double- $\beta$  model is preferred at  $2 \sigma$  level. The preference of the double- $\beta$  model over isothermal  $\beta$  model can be best seen on marginalized dis-



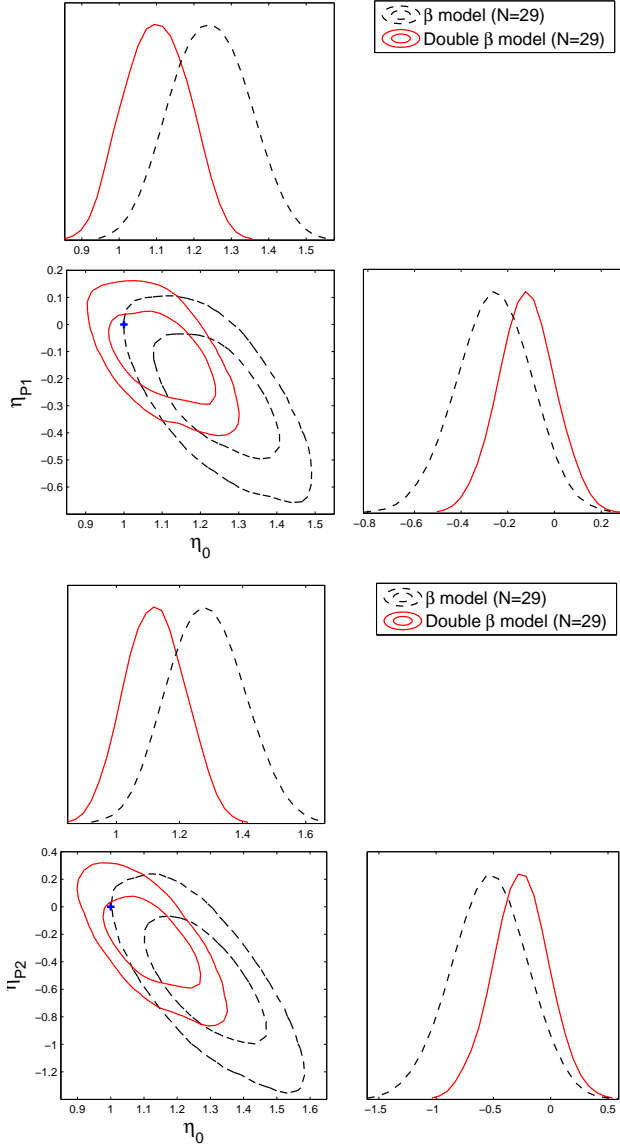
**Figure 5.** Confidence contours and marginalized likelihood distribution functions for the  $\eta$  parameters for the case when Planck best fitted  $\Lambda$ CDM parameters were taken to represent a “true” cosmology. Black dashed lines and red lines correspond to the fits obtained on the  $n = 29$  sub-sample under the assumption of isothermal  $\beta$  model and non-isothermal double- $\beta$  model. The blue cross represents the expected case when the DDR holds exactly.

tributions of  $\eta_0$ ,  $\eta_P$  parameters where it shows up irrespectively of the cosmology assumed.

We conclude by saying that as the cluster sample size increases with upcoming X-ray cluster surveys, we hope the method proposed in this paper may prove useful to improve the constraints on cluster gas mass density profiles.

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**Figure 6.** The same as Fig 5, but for the case when XCDM model was taken to represent a “true” cosmology.

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