Magic wavelengths for the State-insensitive Optical Trapping of Cs Atoms: case studies using the linearly and circularly polarized lights

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We demonstrate magic wavelengths for the $6s\ ^2S_{1/2} - 6p\ ^2P_{3/2,1/2}$ transitions in the Cs atom due to the circularly polarized light. In addition, we also obtain magic wavelengths using the linearly polarized light in order to verify the previously reported values and make a comparative study with the values obtained for the circularly polarized light. Most of these wavelengths are found to be in the optical region and could be of immense interest to the experimentalists for carrying out high precision measurements. To obtain these wavelengths, we have calculated dynamic dipole polarizabilities of the ground, $6p\ ^2P_{1/2}$ and $6p\ ^2P_{3/2}$ states. We use the available precise values of the electric dipole (E1) matrix elements for the most dominantly contributing transitions from the lifetime measurements of the above excited states and obtain other relevant important E1 matrix elements by employing a relativistic coupled-cluster method approximating at the singles and doubles level. Accuracies in these quantities are substantiated by comparing our evaluated scalar polarizability values with the corresponding experimental results.

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I. INTRODUCTION

Techniques involving laser cooling and trapping of neutral atoms are of immense interest for many scientific applications that are capable of probing new physics of elementary particles [1] and searching for exotic quantum phase transitions using ultracold atoms [2]. In particular, trapping atoms using optical lasers have many advantages since trapped atoms can be easily accessed [3] and can also offer long storage time [2, 4, 5]. It is commendable to optically trap neutral atoms without getting affected by the change in the internal energy levels due to the laser fields that are applied to trap the atoms. One can achieve this by trapping the atoms using the lasers at the wavelengths for which the differential Stark shifts of a transition effectively get nullified though each state associated with the transition may have finite Stark shift. These wavelengths are popularly known as magic wavelengths ($\lambda_{\rm magic}$s) [6]. These wavelengths play crucial roles in the state-insensitive quantum engineering and atomic-optical lock experiments [5, 7, 8], at which the measurements can attain relative uncertainties as as small $\sim 10^{-18}$, enabling many sophisticated applications such as investigation on fundamental physics [9], precise determination of fundamental constants [10], advancements in global positioning systems [11] etc.

Alkali atoms are mostly prefered for performing experiments related to laser cooling and trapping. The reason being that their closed optical transitions lie in the convenient spectral range. Owing to a large number of applications of trapped Cs atoms such as in the atomic fountain clock, quantum computation and communication schemes etc., it is imperative to further investigate more possible magic wavelengths for the Cs atom so that it can be conveniently trapped by a preferable laser when needed. Mckeever *et al.* had experimentally demonstrated $\lambda_{\rm magic}$ at 935.6 nm for the $6S\ ^2S_{1/2}$-$6p\ ^2P_{3/2}$ transition in Cs [12] using linearly polarized light. Following this, magic wavelengths for many alkali atoms including Cs atom were determined using linearly polarized light by Arora *et al.* [13] by evaluating the dynamic polarizabilities of these atoms using the relativistic coupled cluster (RCC) method. The estimation of ac Stark shifts using the circularly polarized light can be expedient due to the predominant role played by the vector polarizability, which does not contribute in the case of linearly polarized light, and can be manipulated shrewdly to offer a large number of magic wavelengths. Recently, we had investigated magic wavelengths in the lighter alkali atoms for the circularly polarized light and realized more possible magic wavelengths, especially for the ns $^2S_{1/2}$-np $^2P_{3/2}$ transitions with the ground state principal quantum number n [5, 14, 15], and also compare our results with the available results for the linearly polarized light. However, magic wavelengths for the circularly polarized light in the Cs atom have not been explored much. The fact that among all the previously investigated alkali metal atoms for the magic wavelengths, Cs atom has large nuclear spin and is heavier, it shows wider hyperfine splittings making it suitable for the microwave clock and quantum computation. Thus, searching for $\lambda_{\rm magic}$s in this atom will be quite useful.

In this paper, we intend to determine the magic wavelengths for the $6s\ ^2S_{1/2}$-$6p\ ^2P_{3/2,1/2}$ transitions in the Cs atom using the circularly polarized light. For this purpose we calculate the dipole polarizabilities of the ground, $6p\ ^2P_{1/2}$ and $6p\ ^2P_{3/2}$ states very precisely.

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TABLE I: Static polarizabilities of the $6S$, $6P_{1/2}$ and $6P_{3/2}$ states of Cs atom. Uncertainties are given in the parentheses.

$6S_{1/2}$ state			$6P_{1/2}$ state			$6P_{3/2}$ state			
Transition	E1 amplitude	$\alpha^{(0)}$	Transition	E1 amplitude	$\alpha^{(0)}$	Transition	E1 amplitude	$\alpha^{(0)}$	$\alpha^{(2)}$
$6S_{1/2}-6P_{1/2}$	4.489(7)	131.88(2)	$6P_{1/2}-6S_{1/2}$	4.489(7)	-131.88(2)	$6P_{3/2}-6S_{1/2}$	6.324(7)	-124.69(2)	124.69(2)
$6S_{1/2}-7P_{1/2}$	0.30(3)	0.30	$6P_{1/2}-7S_{1/2}$	4.236(21)	178.43(7)	$6P_{3/2}-7S_{1/2}$	6.47(3)	225.3(1)	-225.3(1)
$6S_{1/2}-8P_{1/2}$	0.09(1)	0.02	$6P_{1/2}-8S_{1/2}$	1.0(1)	5.86(1)	$6P_{3/2}-8S_{1/2}$	1.5(1)	6.21(2)	-6.21(2)
$6S_{1/2}-9P_{1/2}$	0.04	$\sim$0	$6P_{1/2}-9S_{1/2}$	0.55(6)	1.41	$6P_{3/2}-9S_{1/2}$	0.77(8)	1.44	-1.44
$6S_{1/2}-10P_{1/2}$	0.02	$\sim$0	$6P_{1/2}-10S_{1/2}$	0.36(4)	0.57	$6P_{3/2}-10S_{1/2}$	0.51(5)	0.57	-0.57
$6S_{1/2}-11P_{1/2}$	0.02	$\sim$0	$6P_{1/2}-11S_{1/2}$	0.27(3)	0.29	$6P_{3/2}-11S_{1/2}$	0.37(4)	0.29	-0.29
$6S_{1/2}-12P_{1/2}$	0.01	$\sim$0	$6P_{1/2}-12S_{1/2}$	0.20(2)	0.17	$6P_{3/2}-12S_{1/2}$	0.29(3)	0.17	-0.17
$6S_{1/2}-6P_{3/2}$	6.324(7)	249.38(3)	$6P_{1/2}-5D_{3/2}$	7.016(24)	1084.3(5)	$6P_{3/2}-5D_{3/2}$	3.166(16)	132.51(4)	106.01(3)
$6S_{1/2}-7P_{3/2}$	0.60(6)	1.20	$6P_{1/2}-6D_{3/2}$	4.3(4)	120.98(9)	$6P_{3/2}-6D_{3/2}$	2.1(2)	15.54(7)	12.43(5)
$6S_{1/2}-8P_{3/2}$	0.23(2)	0.15	$6P_{1/2}-7D_{3/2}$	2.1(2)	21.03(8)	$6P_{3/2}-7D_{3/2}$	1.0(1)	2.47	1.97
$6S_{1/2}-9P_{3/2}$	0.13(1)	0.05	$6P_{1/2}-8D_{3/2}$	1.3(1)	7.43(1)	$6P_{3/2}-8D_{3/2}$	0.61(6)	0.84	0.67
$6S_{1/2}-10P_{3/2}$	0.09(1)	0.02	$6P_{1/2}-9D_{3/2}$	0.93(9)	3.55	$6P_{3/2}-9D_{3/2}$	0.43(4)	0.40	0.32
$6S_{1/2}-11P_{3/2}$	0.06(1)	0.01	$6P_{1/2}-10D_{3/2}$	0.71(7)	2.01	$6P_{3/2}-10D_{3/2}$	0.33(3)	0.22	0.18
$6S_{1/2}-12P_{3/2}$	0.05	0.01				$6P_{3/2}-5D_{5/2}$	9.59(8)	1174(2)	-234.9(4)
						$6P_{3/2}-6D_{5/2}$	6.3(6)	132(2)	-26.4(3)
						$6P_{3/2}-7D_{5/2}$	2.9(3)	21.6(1)	-4.32(3)
						$6P_{3/2}-8D_{5/2}$	1.8(2)	7.46(3)	-1.49
						$6P_{3/2}-9D_{5/2}$	1.3(1)	3.53	-0.71
						$6P_{3/2}-10D_{5/2}$	1.0(1)	1.98	-0.40
Main(α_v^v)		383.03(4)	Main(α_v^v)		1294.2(5)	Main(α_v^v)		1602(3)	-255.9(5)
Tail(α_v^v)		0.15(8)	Tail(α_v^v)		24(12)	Tail(α_v^v)		25(13)	-5(2)
α_v^{cv}		-0.47(0)	α_v^{cv}		$\sim$0	α_v^{cv}		$\sim$0	$\sim$ 0
α_0^c		16.8(8)	α_0^c		16.8(8)	α_0^c		16.8(8)	
Total		399.5(8)	Total		1335(12)	Total		1644(13)	-261(2)
Others		399.9[16] 399[17]	Others		1338[13] 1290[18]	Others		1650[13] 1600[18]	-261[13] -233 [18]
Experiment		401.0(6)[19]	Experiment		1328.4(6)[20]	Experiment		1641(2)[21]	-262(2)[21]

These values are also compared these values with the other available precise results. Moreover, we also determine magic wavelengths due to the linearly polarized light for the $6s\ ^2S_{1/2}$-$6p\ ^2P_{3/2,1/2}$ transitions in order to compare with the previously reported values and with the magic wavelengths obtained using the circularly polarized light. All these quantities are given in atomic units (a.u.) unless stated otherwise.

II. THEORY AND METHOD OF EVALUATION

In the time independent second order perturbation theory, the Stark shift in the energy of vth level of an atom placed in a static electric field ($\boldsymbol{\mathcal{E}}$) is expressed as [22]

$$\Delta E_v = \sum_{k\neq v} \frac{|\langle\psi_v|V|\psi_k\rangle|^2}{E_v^0 - E_k^0}, \tag{1}$$

where $V = -\boldsymbol{D}\cdot\boldsymbol{\mathcal{E}}$ is the perturbing electric-dipole interaction Hamiltonian, E_i^0 refers to the unperturbed energy of the corresponding ith level and states with subscript k are the intermediate states to which transition from the vth state is allowed by the dipole selection rules. For convenience, Eq. (1) is simplified using the tensor analysis as

$$\Delta E_v = -\frac{1}{2}\alpha_v \mathcal{E}^2, \tag{2}$$

where, α_v is the static dipole polarizability and is given by

$$\alpha_v = -2\sum_{k\neq v} \frac{(p^*)_{vk}(p)_{kv}}{\delta E_{vk}}. \tag{3}$$

Here, $\delta E_{vk} = E_v^0 - E_k^0$ and $(p)_{kv} = \langle\psi_k|D|\psi_v\rangle$ is the electric dipole (E1) matrix element between the states $|\psi_v\rangle$ and $|\psi_k\rangle$. Since in a number of applications oscillating electric fields are being used, the above expression is slightly modified in this case with polarizability as a function of frequency of the electric field by

$$\alpha_v(\omega) = -\sum_{k\neq v} (p^*)_{vk}(p)_{kv}\left[\frac{1}{\delta E_{vk}+\omega} + \frac{1}{\delta E_{vk}-\omega}\right]. \tag{4}$$

In the absence of magnetic field, the above expression is further parameterized in terms of rank 0, rank 1 and rank 2 components of the tensor products that are popularly known as scalar ($\alpha_v^{(0)}$), vector ($\alpha_v^{(1)}$) and tensor ($\alpha_v^{(2)}$)

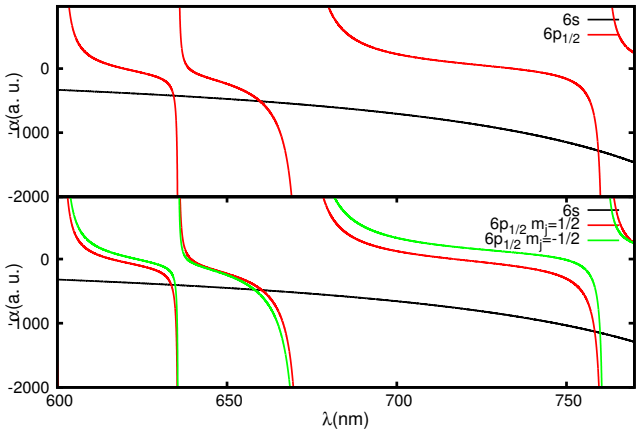

FIG. 1: (Color online) Dynamic polarizabilities (in a.u.) for the $6S_{1/2}$ and $6P_{1/2}$ states of Cs with different m_j values in the wavelength range 600-770 nm for the linearly polarized light (upper half) and the circularly polarized light (lower half).

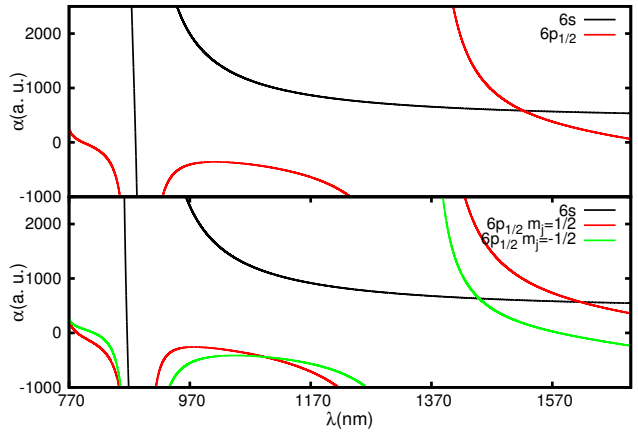

FIG. 2: (Color online) Dynamic polarizabilities (in a.u.) for the $6S_{1/2}$ and $6P_{1/2}$ states in Cs with different m_j values in the wavelength range 770-1600 nm for the linearly polarized light (upper half) and the circularly polarized light (lower half).

polarizabilities respectively i.e. [23]

$$\alpha_v(\omega) = \alpha_v^{(0)} + \frac{Am_j}{2j_v}\alpha_v^{(1)} + \frac{3m_j^2 - j_v(j_v+1)}{j_v(2j_v-1)}\alpha_v^{(2)}, \quad (5)$$

where

$$\begin{aligned}\alpha_v^{(0)} &= \frac{1}{3(2j_v+1)}\sum_{j_k}|\langle\psi_v||D||\psi_k\rangle|^2 \\ &\times\left[\frac{1}{\delta E_{kv}+\omega}+\frac{1}{\delta E_{kv}-\omega}\right], \quad (6)\\ \alpha_v^{(1)} &= -\sqrt{\frac{6j_v}{(j_v+1)(2j_v+1)}}\sum_{j_k}(-1)^{j_k+j_v+1} \\ &\times\left\{\begin{array}{ccc}1 & 1 & 1\\ j_v & j_v & j_k\end{array}\right\}|\langle\psi_v||D||\psi_k\rangle|^2 \\ &\times\left[\frac{1}{\delta E_{kv}+\omega}-\frac{1}{\delta E_{kv}-\omega}\right] \quad (7)\end{aligned}$$

and

$$\begin{aligned}\alpha_v^{(2)} &= -2\sqrt{\frac{5j_v(2j_v-1)}{6(j_v+1)(2j_v+3)(2j_v+1)}} \\ &\times\sum_{j_k}(-1)^{j_k+j_v+1}\left\{\begin{array}{ccc}1 & 1 & 2\\ j_v & j_v & j_k\end{array}\right\}|\langle\psi_v||D||\psi_k\rangle|^2 \\ &\times\left[\frac{1}{\delta E_{kv}+\omega}+\frac{1}{\delta E_{kv}-\omega}\right], \quad (8)\end{aligned}$$

for the angular momentum j_v and its azimuthal quantum number m_j, and A representing the degree of polarization which can have values 1 and -1 for the right handed and left handed circularly polarized light, respectively, whereas $A = 0$ for the linearly polarized light.

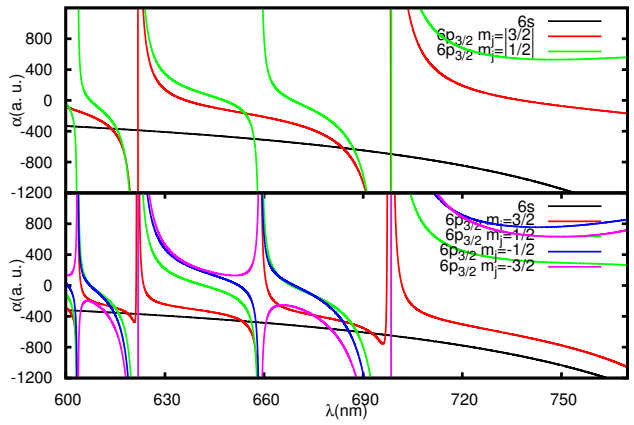

FIG. 3: (Color online) Dynamic polarizabilities (in a.u.) of the Cs for $6S_{1/2}$ and $6P_{3/2}$ states with different m_j values in the wavelength range 6000-770 nm for the linearly polarized light (upper half) and circularly polarized light (lower half).

The differential ac Stark shift of a transition containing the ground state and an excited state is the difference between the ac Stark shifts of the two states and is given by

$$\begin{aligned}\delta(\Delta E)_{ge}(\omega) &= \Delta E_g(\omega) - \Delta E_e(\omega) \\ &= -\frac{1}{2}\left[\alpha_g(\omega) - \alpha_e(\omega)\right]\mathcal{E}^2. \quad (9)\end{aligned}$$

Here, subscripts 'g' and 'e' represent the ground and excited states, respectively. Our aim is to find out the ω values at which $\delta(\Delta E)_{ge}(\omega)$ will be zero.

For a state of an atomic system having a closed core and a valence electron, dipole polarizabilitiy can be conveniently evaluated by calculating contributions sepa-

TABLE II: Magic wavelengths ($\lambda_{\rm magic}$s) (in nm) with corresponding polarizabilities ($\alpha_v(\omega)$s) (in a. u.) for the $6S-6P_{1/2}$ transition in the Cs atom with the linearly and the circularly polarized lights along with the resonant wavelengths ($\lambda_{\rm res}$s) (in nm).

		Linearly Polarization $m_j=\|1/2\|$				Circularly Polarization $m_j=1/2$		$m_j=-1/2$	
		Present		Ref.[13]					
Resonance	$\lambda_{\rm res}$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$
$6P_{1/2}-8D_{3/2}$	601.22								
		634.3(2)	-423	634.3(2)	-424(2)	633.5(3)	-401	634.7(1)	-405
$6P_{1/2}-9S_{1/2}$	635.63								
		659.8(8)	-510	660.1(6)	-513(3)	660.3(8)	-485	658.1(7)	-477
$6P_{1/2}-7D_{3/2}$	672.51								
		759.38(5)	-1281	759.40(3)	-1282(3)	758.31(7)	-1130	759.91(6)	-1150
$6P_{1/2}-8S_{1/2}$	761.10								
$6P_{1/2}-6D_{3/2}$	876.38								
$6P_{1/2}-6S_{1/2}$	894.59								
$6P_{1/2}-7S_{1/2}$	1359.20								
		1522(3)	583	1520(3)	583(2)	1617(5)	568	1449(1)	632
$6P_{1/2}-5D_{3/2}$	3011.15								

TABLE III: The $\lambda_{\rm magic}$ values (in nm) with the corresponding $\alpha_v(\omega)$s (in a.u.) with $m_j=3/2$ and $1/2$ for the $6S-6P_{3/2}$ transition in the Cs atom with the linearly polarized light along with the $\lambda_{\rm res}$s (in nm).

		$m_j=\|1/2\|$				$m_j=\|3/2\|$			
		Present		Ref. [13]		Present		Ref. [13]	
Resonance	$\lambda_{\rm res}$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$
$6P_{3/2}-9D_{5/2}$	584.68								
		602.6(3)	-338	602.6(4)	-339(1)				
$6P_{3/2}-10S_{1/2}$	603.58								
		615.4(10)	-370	615.5(8)	-371(3)	614(1)	-365	614(3)	-367(8)
$6P_{3/2}-8D_{5/2}$	621.48								
		621.924(4)	-387	621.924(2)	-388(1)	621.85(4)	-387	621.844(3)	-388(1)
$6P_{3/2}-8D_{3/2}$	621.93								
		657.0(2)	-500	657.05(9)	-500(1)				
$6P_{3/2}-9S_{1/2}$	658.83								
		687(1)	-633	687.3(3)	-635(3)	684(1)	-617	684.1(5)	-618(4)
$6P_{3/2}-7D_{5/2}$	697.52								
		698.5(5)	-697	698.524(2)	-697(2)	698.3(7)	-695	698.346(4)	-696(2)
$6P_{3/2}-7D_{3/2}$	698.54								
		793.1(2)	-2072	793.07(2)	-2074(5)				
$6P_{3/2}-8S_{1/2}$	794.61								
$6S_{1/2}-6P_{3/2}$	852.35								
		888(2)	-5690	887.95(10)	-5600(100)	884(3)	-1618	883.4(2)	-1550(90)
$6S_{1/2}-6P_{1/2}$	894.59								
$6P_{3/2}-6D_{5/2}$	917.48								
		921.0(9)	4088	921.01(3)	4088(10)	920(2)	4180	920.18(6)	4180(14)
$6P_{3/2}-6D_{3/2}$	921.11								
		933(8)	3153	932.4(8)	3197(50)	941.7(3)	2752	940.2(1.7)	2810(70)

rately due to the core, core-valence and valence correlations [24, 25]. In other words, we can write

$$\alpha_v(\omega) = \alpha_0^c(\omega) + \alpha_v^{cv}(\omega) + \alpha_v^v(\omega), \tag{10}$$

where $\alpha_0^c(\omega)$, $\alpha_v^{cv}(\omega)$ and $\alpha_v^v(\omega)$ are the contributions from the core, core-valence and valence correlation effects, respectively. The subscript '0' in $\alpha_0^c(\omega)$ means it is independent of the valence orbital in a state. For estimating the dominant $\alpha_v^v(\omega)$ contributions, we calculate wave functions of many low-lying excited states ($|\psi_k\rangle s$) using a linearized version of the RCC method in the singles and doubles excitations approximation (SD method) [26–28]. In this method, the wave functions of the ground, $6p\ ^2P_{1/2}$ and $6p\ ^2P_{3/2}$ states in Cs that have a common

TABLE IV: The $\lambda_{\rm magic}$ values (in nm) with the corresponding $\alpha_v(\omega)$s (in a.u.) with $m_j = 3/2, 1/2, -1/2$ and $-3/2$ for the $6S - 6P_{3/2}$ transition in the Cs atom for the circularly polarized light along with resonant $\lambda_{\rm res}$s (in nm).

		$m_j = 3/2$		$m_j = 1/2$		$m_j = -1/2$		$m_j = -3/2$	
Resonance	$\lambda_{\rm res}$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$	$\lambda_{\rm magic}$	$\alpha_v(\omega)$
$6P_{3/2} - 9D_{5/2}$	584.68								
		601(1)	-321	602.1(6)	-324	602.9(2)	-326	604.5(5)	-329
$6P_{3/2} - 10S_{1/2}$	603.58								
		619(4)	-363	616(1)	-356	614.6(9)	-353	612(1)	-346
		621.15(8)	-368						
$6P_{3/2} - 8D_{5/2}$	621.48								
		621.6(1)	-369	621.9(3)	-370			621.9(4)	-370
$6P_{3/2} - 8D_{3/2}$	621.93								
		653(3)	-461	656.1(3)	-471	657.5(1)	-475	660.6(2)	-486
$6P_{3/2} - 9S_{1/2}$	658.83								
		693(6)	-620	688(1)	-597	686(1)	-586	681(1)	-562
		696.7(7)	-640						
$6P_{3/2} - 7D_{5/2}$	697.52								
		697.9(4)	-646	698.5(1)	-649			698.5(4)	-649
$6P_{3/2} - 7D_{3/2}$	698.54								
		786(3)	-1581	792.1(2)	-1731	793.5(2)	-1766	796.0(2)	-1837
$6P_{3/2} - 8S_{1/2}$	794.61								
$6S_{1/2} - 6P_{3/2}$	852.35								
		838(3)	-5393	878(2)	-4741	882(2)	-7213	880(2)	-5647
$6S_{1/2} - 6P_{1/2}$	894.59								
$6P_{3/2} - 6D_{5/2}$	917.48								
		917(4)	6441	920.80(4)	5587			920.8(1)	5595
$6P_{3/2} - 6D_{3/2}$	921.11								
		924(25)	4945	927(23)	4622	933(17)	4072	948(2)	3083
								1249(6)	791
$6P_{3/2} - 7S_{1/2}$	1469.89								

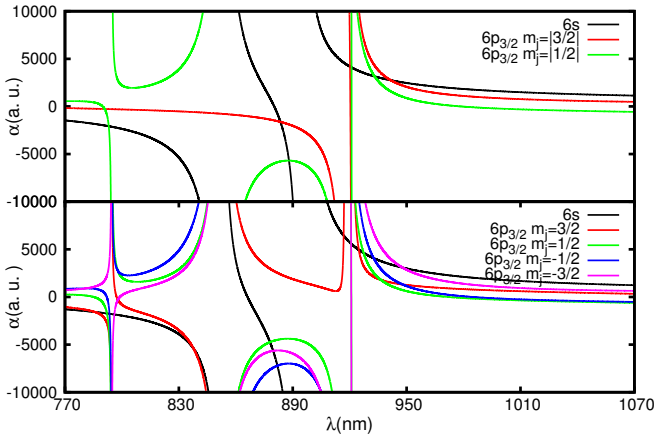

FIG. 4: (Color online) Dynamic polarizabilities (in a.u.) of the $6S_{1/2}$ and $6P_{3/2}$ states with different m_j values in the wavelength range 770-1070 nm for the linearly polarized light (upper half) and the circularly polarized light (lower half).

core $[5p^6]$ are expressed as

$$\begin{aligned}|\Psi_v\rangle &= [1 + T_1 + T_2 + S_{1v} + S_{2v}]|\Phi_v\rangle \\ &= [1 + \sum_{ma} \rho_{ma} a_m^\dagger a_a^\dagger + \frac{1}{2} \sum_{nmab} \rho_{mnab} a_m^\dagger a_n^\dagger a_b a_a \\ &\quad + \sum_{m\neq v} \rho_{mv} a_m^\dagger a_v + \sum_{mna} \rho_{mnva} a_m^\dagger a_n^\dagger a_a a_v]|\Phi_v\rangle, \quad (11)\end{aligned}$$

where, $a_i^\dagger$s and a_js are the creation and annihilation operators. Indices m, n and r refer to the virtual orbitals, indices a and b represent the core orbitals and v corresponds to the valence orbital. The coefficients ρ_{ma} and ρ_{mv} are the singles and doubles excitation amplitudes involving the core electrons alone while ρ_{mnab} and ρ_{mnva} refer to the singles and doubles excitation amplitudes involving the valence orbital v from $|\Phi_v\rangle$. We obtain $|\Phi_v\rangle$ by expressing

$$|\Phi_v\rangle = a_v^\dagger |\Phi_0\rangle, \quad (12)$$

where $|\Phi_0\rangle$ is the Dirac-Hartree-Fock (DHF) wave function of a closed core $[5p^6]$.

The E1 matrix element for a transition between the

states $|\Psi_v\rangle$ and $|\Psi_k\rangle$ are calculated using the expression

$$D_{vk} = \frac{\langle\Psi_v|D|\Psi_k\rangle}{\sqrt{\langle\Psi_v|\Psi_v\rangle\langle\Psi_k|\Psi_k\rangle}} = \frac{\langle\Phi_v|\tilde{D}|\Phi_k\rangle}{\sqrt{\langle\Phi_v|\{1+\tilde{N}_v\}|\Phi_v\rangle\langle\Phi_k|\{1+\tilde{N}_k\}|\Phi_k\rangle}}, \quad (13)$$

where $\tilde{D} = \{1+S_{1v}^\dagger+S_{2v}^\dagger+T_1^\dagger+T_2^\dagger\}D\{1+S_{1k}+S_{2k}+T_1+T_2\}$ and $\tilde{N}_v = \{S_{1v}^\dagger+S_{2v}^\dagger+T_1^\dagger+T_2^\dagger\}\{S_{1v}+S_{2v}+T_1+T_2\}$. For the practical purpose, we calculate E1 matrix elements of the low-lying transitions which contribute dominantly to α_v^v and refer to the result as "Main(α_v^v)" contribution. Contributions from the other high-lying excited states including the continuum are estimated using the DHF method and given as "Tail(α_v^v)". We, again, estimate α_v^{cv} and α_0^c contributions using the DHF method.

III. RESULTS AND DISCUSSION

To find precise values of $\lambda_{\rm magic}$s for the $6S-6P_{3/2,1/2}$ transitions in the Cs atom, accurate values of the dynamic dipole polarizabilities of the involved states are prerequisites. To evince accuracies of these results, we first evaluate the static polarizabilities ($\alpha_v(0)$) of these states and compare them with their respective experimental values and previously reported precise calculations. We give both scalar and tensor polarizabilities of the considered ground, $6p\ ^2P_{1/2}$ and $6p\ ^2P_{3/2}$ states of Cs in Table I using our calculations along with other results. Contributions from "Main" and "Tail" to α_v^v, core-valence and core contributions to our calculations are given explicitly in this table. We also tabulate the E1 matrix elements used for determining the "Main" contributions to α_v^v.

To reduce the uncertainties in the evaluation of the polarizabilities, we use the E1 matrix elements for the $6S-6P$ transitions extracting from the very precisely measured lifetimes of the $6p\ ^2P_{3/2}$ and $6p\ ^2P_{1/2}$ states of Cs by Rafac *et al.* [29]. We also use the E1 matrix elements for the $6P-7S$ transitions compiled in Ref. [30], which are derived from the measured lifetime of the $7S$ state. Similarly, the E1 matrix element of the $6P_{1/2}-5D_{3/2}$ transition has been derived by combining the measured differential Stark shift of the D1 line with the experimental value of the ground state dipole polarizability of Amini *et al.* [19]. The values of these matrix elements along with their experimental uncertainties are listed in Table I. Otherwise, the required E1 matrix elements for the transitions up to the $12S$, $12P$ and $10D$ states are obtained by employing the SD method as described in the previous section. The uncertainties in these matrix elements are calculated by comparing matrix elements for the $6S-6P$ and $6P-7S$ transitions calculated using our method and available experimental values. The maximum difference between our and experimental results for these matrix elements is 6%. Therefore, we assign a 10% uncertainties to the other matrix elements given in Table I. We have used 70 B-spline functions confined within a cavity of radius $R = 220$ a.u. to construct the single-particle orbitals. We use experimental values of the excitation energies of these transitions from the National Institute of Science and Technology (NIST) database [31] to reduce further the uncertainties in the evaluation of the polarizabilities.

Our calculated value of $\alpha_v(0)$ for the ground state is 399.5 a.u., which matches very well with other theoretical values 399 a.u. and 399.9 a.u. estimated by Borschevsky *et al.* [17] and Derevianko *et al.* [16] using the other variants of RCC method, respectively. These results are also in very good agreement with the experimental result 401.0(6) a.u. measured by Amini *et al.* [19] using the time-of-flight technique. Similarly our calculation gives $\alpha_v(0)$ of the $6P_{1/2}$ state to be 1335 a.u., which is slightly larger than the other calculated value 1290 a.u. of Wijngaarden *et al.* [18] but agrees quite well with another calculated value 1338 a.u. reported by Arora *et al.* [13] and the measured value 1328.4(6) a.u. reported in Ref. [20]. In the work of Wijngaarden *et al.*, polarizabilities were evaluated using the oscillator strengths from the method of Bates and Damgaard [32]. The scalar and tensor polarizabilities of the $6P_{3/2}$ state using our method are obtained as 1644 a.u. and -261 a.u., respectively. They are also in very good agreement with the experimental values reported in Ref. [21] and are in reasonable agreement with the theoretical values reported by Arora *et al.* [13] and Wijngaarden *et al.* [18]. The above analysis shows we have obtained very accurate values of the dipole polarizabilities using our method of evaluation. This justifies that determining dynamic polarizabilities using the same procedure can also provide competent results. Hence, $\lambda_{\rm magic}$ values of the $6S-6P_{1/2,3/2}$ transitions in Cs can be determined without any ambiguity using these accurate values of the dipole polarizabilities.

We are now proceeding towards determining $\lambda_{\rm magic}$s for the $6S-6P_{1/2,3/2}$ transitions in the Cs atom. For this purpose, we plot the dynamic dipole polarizabilities of the $6S$ and $6P_{1/2,3/2}$ states in Figs. 1, 2, 3 and 4. They are shown in the upper and lower halves for the linearly and circularly polarized lights respectively. We use the left circularly polarized light ($A=-1$) while determining the magic wavelengths. It can be noted that the results for the left circular polarized light can be interchangeably used for the right circularly polarized light if m_j is replaced by $-m_j$ due to linear dependency of the degree of polarizability A in Eq. (5); hence it is not required to determine the magic wavelengths for both the polarizations or m_j sublevels of $6S$ states independently. So, we keep fixed the sublevel of the $6S$ state as $m_j = 1/2$, but consider all possible m_j values of the $6P_{1/2,3/2}$ states. It is evident from the above plots that for the considered wavelength range the dynamic polarizability curve for the $6S$ state is generally small except in the close vicinity of the resonant $6S_{1/2}-6P_{1/2}$ and $6S_{1/2}-6P_{3/2}$ transitions. On the other hand, polariz-

abilities of the $6P$ states have significant contributions from several resonant transitions. Thus, the polarizability curve of the $6P$ states cross with the polarizability curve of the $6S$ state in between these resonant transitions. The intersections of these polarizability curves are labeled as $\lambda_{\rm magic}$s in the above figures for both the linearly and circularly polarized lights. We have also tabulated these values for the $6S-6P_{1/2,3/2}$ transitions in Tables II, III and IV along with their respective uncertainties in the parentheses. These uncertainties are estimated considering maximum possible errors in the estimated differential polarizabilities between the involved states in a transition. Corresponding values of the dynamic polarizabilities are also mentioned in the above tables to provide an estimate of strengths of the electric fields required to trap the Cs atoms at those magic wavelengths. We also list the resonant wavelengths ($\lambda_{\rm res}$) in the tables to highlight the respective placements of these $\lambda_{\rm magic}$s.

As seen from Table II, we find $\lambda_{\rm magic}$s for the $6S-6P_{1/2}$ transition around 634 nm, 659 nm and 760 nm for both the linearly and circularly polarized lights. Other $\lambda_{\rm magic}$s are located at 1522 nm for the linearly polarized light and at 1617 nm and 1449 nm with m_j=1/2 and -1/2, respectively, for the circularly polarized light. The former three $\lambda_{\rm magic}$s support the blue or dark detuned trap, while the later one supports the red detuned traps. Values of $\lambda_{\rm magic}$s for the linearly polarized light are also compared with $\lambda_{\rm magic}$s of Arora *et al.* reported in Ref. [13]. Both the findings agree to each other as the method of calculation in both the works is almost similar. As evident from the table, the use of circularly polarized light does not add any advantage to the $6S-6P_{1/2}$ transition. Moreover, the m_j dependence of traps in the case of circularly polarized light could be of major concern. Thus, we do not advocate for the use of circularly polarized light in this transition, though the choice of any polarized light seem to be fine.

We list $\lambda_{\rm magic}$s for the $6S-6P_{3/2}$ transition for the linearly and circularly polarized lights separately in Tables III and IV respectively. In case of the linearly polarized light, at least ten $\lambda_{\rm magic}$s are systematically located between the resonant transitions with $m_j=|1/2|$, while only seven $\lambda_{\rm magic}$s are located for the $m_j=|3/2|$ sublevel. It, thus, implies that use of linearly polarized light does not completely support state insensitive trapping of this transition and results are dependent on the m_j sublevels of the $6P_{3/2}$ state. This is also in agreement with the results presented in Ref. [13]. The experimental magic wavelength at 935.6 nm, as demonstrated by McKeever *et. al.* [12], matches well with the average of the last two magic wavelengths obtained at 933 nm (for $m_j=|1/2|$) and 941.7 nm (for $m_j=|3/2|$). As shown in Table IV, we get a set of thirteen magic wavelengths for the circularly polarized light in between the eleven $6P_{3/2}$ resonances lying in the wavelength range 600-1600 nm. Out of these thirteen, three magic wavelengths at 621.15 nm, 696.7 nm, and 1249 nm occur only for one value of m_j. Thus, they are of limited experimental relevance. $\lambda_{\rm magic}$s at 621.6, 697.9 and 917 nms does not support state-insensitive trapping for $m_j=-1/2$ sublevel of the $6P_{3/2}$ state, here, we recommend the use of a switching trapping scheme for this transition as proposed in [15]. In this approach, the change of sign of A and m_j sublevels of the $6S$ state will lead to the same result for the positive values of m_j sublevels of the $6P_{3/2}$ state.

IV. CONCLUSION

We have investigated possible magic wavelengths within the wavelength range 600 - 1600 nm for the $6S-6P_{1/2,3/2}$ transitions in the Cs atom considering both the linearly and circularly polarized lights. Our values for the linearly polarized light were compared with the previously estimated values and they are found to be in good agreement. With the circularly polarized light, we find a large number of magic wavelengths that are in the optical region and would be of immense interest for carrying out many precision measurements at these wavelengths where the above transitions are used for the laser cooling purposes. We have used the most precise available electric dipole matrix elements extracting from the experimental observations, otherwise calculated using a relativistic coupled-cluster method to evaluate the dynamic polarizabilities of the Cs atom very precisely. These quantities are used to determine the above magic wavelengths. By comparing static values of the polarizabilities with their respective experimental results, accuracies of the polarizabilities and magic wavelengths were adjudged. In few situations, we found it would be advantageous to use the magic wavelengths of the circularly polarized light over the linearly polarized light. As an example, magic wavelengths for the circularly light are missing for some $m_j=1/2, 3/2$ sublevel but they are present for the corresponding $-m_j$ sublevel or viceversa. In this case, one can switch the polarization of the light and can successfully locate the positions of the magic wavelengths.

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