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Moduli spaces of AdS_5 vacua in $\mathcal{N} = 2$ supergravity

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ABSTRACT

We determine the conditions for maximally supersymmetric AdS_5 vacua of five-dimensional gauged $\mathcal{N} = 2$ supergravity coupled to vector-, tensor- and hypermultiplets charged under an arbitrary gauge group. In particular, we show that the unbroken gauge group of the AdS_5 vacua has to contain an $U(1)_R$-factor. Moreover we prove that the scalar deformations which preserve all supercharges form a Kähler submanifold of the ambient quaternionic Kähler manifold spanned by the scalars in the hypermultiplets.

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Contents

1 Introduction

Anti-de Sitter (AdS) backgrounds of supergravity are an essential part of the AdS/CFT correspondence [1] and have been studied in recent years from varying perspectives. On the one hand they can be constructed as compactifications of higher-dimensional supergravities as is the natural set up in the AdS/CFT correspondence.[1] Alternatively, one can investigate and, if possible, classify their appearance directly in a given supergravity without relating it to any compactification.

For a given AdS background it is also of interest to study its properties and in particular its moduli space $\mathcal{M}$, i.e. the subspace of the scalar field space that is spanned by flat directions of the AdS background. This moduli space has been heavily investigated in Minkowskian backgrounds of string theory as it prominently appears in its low energy effective theory. For AdS backgrounds much less is known about $\mathcal{M}$, partly because the defining equations are more involved and furthermore quantum corrections contribute unprotected.

In [5, 6] supersymmetric AdS$_4$ vacua and their classical supersymmetric moduli spaces were studied in four-dimensional ($d = 4$) supergravities with $\mathcal{N} = 1, 2, 4$ supersymmetry without considering their relation to higher-dimensional theories.[2] For $\mathcal{N} = 1$ it was found that the supersymmetric moduli space is at best a real submanifold of the original Kähler field space. Similarly, for $\mathcal{N} = 2$ the supersymmetric moduli space is at best a product of a real manifold times a Kähler manifold while $\mathcal{N} = 4$ AdS backgrounds have no supersymmetric moduli space. This analysis was repeated for AdS$_5$ vacua in $d = 5$ gauged supergravity with 16 supercharges ($\mathcal{N} = 4$) in [7] and for AdS$_7$ vacua in $d = 7$ gauged supergravity with 16 supercharges in [8]. For the $d = 5$, $\mathcal{N} = 4$ theories it was shown that the supersymmetric moduli space is the coset $\mathcal{M} = SU(1, m)/(U(1) \times SU(m))$ while in $d = 7$ it was proven that

[1]See [2, 3] for earlier work and e.g. [4] and references therein for a more recent review.

[2]Throughout this paper we only consider AdS backgrounds that preserve all supercharges of a given supergravity and furthermore only consider the subspace of the moduli space that preserves all these supercharges. This is what we mean by supersymmetric AdS backgrounds and supersymmetric moduli spaces.

again no supersymmetric moduli space exists.

In this paper we focus on supersymmetric AdS_5 vacua in $d = 5$ gauged supergravities with eight supercharges ($\mathcal{N} = 2$) coupled to an arbitrary number of vector-, tensor- and hypermultiplets. A related analysis was carried out in [9] for the coupling of Abelian vector multiplets and hypermultiplets. We confirm the results of [9] and generalize the analysis by including tensor multiplets and non-Abelian vector multiplets. In particular, we show that also in this more general case the unbroken gauge group has to be of the form $H \times U(1)_R$ where the $U(1)_R$-factor is gauged by the graviphoton. This specifically forbids unbroken semisimple gauge groups in AdS backgrounds.

In a second step we study the supersymmetric moduli space $\mathcal{M}$ of the previously obtained AdS_5 backgrounds and show that it necessarily is a Kähler submanifold of the quaternionic scalar field space $\mathcal{T}_H$ spanned by all scalars in the hypermultiplets.[3] This is indeed consistent with the AdS/CFT correspondence where the moduli space $\mathcal{M}$ is mapped to the conformal manifold of the dual superconformal field theory (SCFT). For the gauged supergravities considered here the dual theories are $d = 4$, $\mathcal{N} = 1$ SCFTs. In [10] it was indeed shown that the conformal manifold of these SCFTs is a Kähler manifold.

The organization of this paper is as follows. In section 2 we briefly review gauged $\mathcal{N} = 2$ supergravities in five dimensions. This will then be used to study the conditions for the existence of supersymmetric AdS_5 vacua and determine some of their properties in section 3. Finally, in section 4 we compute the conditions on the moduli space of these vacua and show that it is a Kähler manifold.

2 Gauged $\mathcal{N} = 2$ supergravity in five dimensions

To begin with let us review five-dimensional gauged $\mathcal{N} = 2$ supergravity following [11–13].[4] The theory consists of the gravity multiplet with field content

$$\{g_{\mu\nu}, \Psi_\mu^{\mathcal{A}}, A_\mu^0\} \ , \quad \mu, \nu = 0, ..., 4 \ , \quad \mathcal{A} = 1, 2 \ , \tag{2.1}$$

where $g_{\mu\nu}$ is the metric of space-time, $\Psi_\mu^{\mathcal{A}}$ is an $SU(2)_R$-doublet of symplectic Majorana gravitini and A_μ^0 is the graviphoton. In this paper we consider theories that additionally contain n_V vector multiplets, n_H hypermultiplets and n_T tensor multiplets. A vector multiplet $\{A_\mu, \lambda^{\mathcal{A}}, \phi\}$ transforms in the adjoint representation of the gauge group G and contains a vector A_μ, a doublet of gauginos $\lambda^{\mathcal{A}}$ and a real scalar ϕ. In $d = 5$ a vector is Poincaré dual to an antisymmetric tensor field $B_{\mu\nu}$ which carry an arbitrary representation of G.

[3]This result was also obtained in [9]. Our results is more general as we include tensor multiplets and non-Abelian vector multiplet in the analysis.

[4]Ref. [13] constructed the most general version of five-dimensional gauged $\mathcal{N} = 2$ supergravity.

This gives rise to tensor multiplets which have the same field content as vector multiplets, but with a two-form instead of a vector. Since vector- and tensor multiplets mix in the Lagrangian, we label their scalars ϕ^i by the same index $i, j = 1, ..., n_V + n_T$. Moreover, we label the vector fields (including the graviphoton) by $I, J = 0, 1, ..., n_V$, the tensor fields by $M, N = n_V + 1, ..., n_V + n_T$ and also introduce a combined index $\tilde{I} = (I, M)$. Finally, the n_H hypermultiplets

$$\{q^u, \zeta^\alpha\}, \quad u = 1, 2, ..., 4n_H \ , \quad \alpha = 1, 2, ..., 2n_H \ , \tag{2.2}$$

contain $4n_H$ real scalars q^u and $2n_H$ hyperini ζ^α.

The bosonic Lagrangian of $\mathcal{N} = 2$ gauged supergravity in five dimensions reads[5] [13]

$$\begin{aligned} e^{-1}\mathcal{L} = & \tfrac{1}{2}R - \tfrac{1}{4}a_{\tilde{I}\tilde{J}}H^{\tilde{I}}_{\mu\nu}H^{\tilde{J}\mu\nu} - \tfrac{1}{2}g_{ij}\mathcal{D}_\mu\phi^i\mathcal{D}^\mu\phi^j - \tfrac{1}{2}G_{uv}\mathcal{D}_\mu q^u\mathcal{D}^\mu q^v - g^2V(\phi, q) \\ & + \tfrac{1}{16g}e^{-1}\epsilon^{\mu\nu\rho\sigma\tau}\Omega_{MN}B^M_{\mu\nu}\left(\partial_\rho B^N_{\sigma\tau} + 2gt^N_{IJ}A^I_\rho F^J_{\sigma\tau} + gt^N_{IP}A^I_\rho B^P_{\sigma\tau}\right) \\ & + \tfrac{1}{12}\sqrt{\tfrac{2}{3}}e^{-1}\epsilon^{\mu\nu\rho\sigma\tau}C_{IJK}A^I_\mu\left[F^J_{\nu\rho}F_{\sigma\tau} + f^J_{FG}A^F_\nu A^G_\rho\left(-\tfrac{1}{2}F^K_{\sigma\tau} + \tfrac{g^2}{10}f^K_{HL}A^H_\sigma A^L_\tau\right)\right] \\ & - \tfrac{1}{8}e^{-1}\epsilon^{\mu\nu\rho\sigma\tau}\Omega_{MN}t^M_{IK}t^N_{FG}A^I_\mu A^F_\nu A^G_\rho\left(-\tfrac{g}{2}F^K_{\sigma\tau} + \tfrac{g^2}{10}f^K_{HL}A^H_\sigma A^L_\tau\right) \ . \end{aligned} \tag{2.3}$$

In the rest of this section we recall the various ingredients which enter this Lagrangian. First of all $H^{\tilde{I}}_{\mu\nu} = (F^I_{\mu\nu}, B^M_{\mu\nu})$ where $F^I_{\mu\nu} = 2\partial_{[\mu}A^I_{\nu]} + gf^I_{JK}A^J_\mu A^K_\nu$ are the field strengths with g being the gauge coupling constant. The scalar fields in $\mathcal{L}$ can be interpreted as coordinate charts from spacetime M_5 to a target space $\mathcal{T}$,

$$\phi^i \otimes q^u : M_5 \longrightarrow \mathcal{T}. \tag{2.4}$$

Locally $\mathcal{T}$ is a product $\mathcal{T}_{VT} \times \mathcal{T}_H$ where the first factor is a projective special real manifold $(\mathcal{T}_{VT}, g)$ of dimension $n_V + n_T$. It is constructed as a hypersurface in an $(n_V + n_T + 1)$-dimensional real manifold $\mathcal{H}$ with local coordinates $h^{\tilde{I}}$. This hypersurface is defined by

$$P(h^{\tilde{I}}(\phi)) = C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{I}}h^{\tilde{J}}h^{\tilde{K}} = 1, \tag{2.5}$$

where $P(h^{\tilde{I}}(\phi))$ is a cubic homogeneous polynomial with $C_{\tilde{I}\tilde{J}\tilde{K}}$ constant and completely symmetric. Thus $\mathcal{T}_{VT} = \{P = 1\} \subset \mathcal{H}$.

The generalized gauge couplings in (2.3) correspond to a positive metric on the ambient space $\mathcal{H}$, given by

$$a_{\tilde{I}\tilde{J}} := -2C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{K}} + 3h_{\tilde{I}}h_{\tilde{J}} \ , \tag{2.6}$$

where

$$h_{\tilde{I}} = C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{J}}h^{\tilde{K}} \ . \tag{2.7}$$

[5] Note that we set the gravitational constant $\kappa = 1$ in this paper.

The pullback metric g_{ij} is the (positive) metric on the hypersurface $\mathcal{T}_{VT}$ and is given by

$$g_{ij} := h_i^{\tilde{I}} h_j^{\tilde{J}} a_{\tilde{I}\tilde{J}} , \tag{2.8}$$

where

$$h_i^{\tilde{I}} := -\sqrt{\tfrac{3}{2}}\, \partial_i h^{\tilde{I}}(\phi) . \tag{2.9}$$

These quantities satisfy (see Appendix C in [13] for more details)

$$h^{\tilde{I}} h_{\tilde{I}} = 1 , \qquad h_{\tilde{I}} h_i^{\tilde{I}} = 0 , \qquad h_{\tilde{I}} h_{\tilde{J}} + h_{\tilde{I}}^i h_{\tilde{J}i} = a_{\tilde{I}\tilde{J}} , \tag{2.10}$$

where we raise and lower indices with the appropriate metrics $a_{\tilde{I}\tilde{J}}$ or g_{ij} respectively. The metric g_{ij} induces a covariant derivative which acts on the $h_i^{\tilde{I}}$ via

$$\nabla_i h_j^{\tilde{I}} = -\sqrt{\tfrac{2}{3}}\, (h^{\tilde{I}} g_{ij} + T_{ijk} h^{\tilde{I}k}) , \tag{2.11}$$

where $T_{ijk} := C_{\tilde{I}\tilde{J}\tilde{K}} h_i^{\tilde{I}} h_j^{\tilde{J}} h_k^{\tilde{K}}$ is a completely symmetric tensor.

The second factor of $\mathcal{T}$ in (2.4) is a quaternionic Kähler manifold $(\mathcal{T}_H, G, Q)$ of real dimension $4n_H$ (see [14] for a more extensive introduction). Here G_{uv} is a Riemannian metric and Q denotes a ∇^G invariant rank three subbundle $Q \subset \mathrm{End}(T\mathcal{T}_H)$ that is locally spanned by a triplet J^n, $n = 1, 2, 3$ of almost complex structures which satisfy $J^1 J^2 = J^3$ and $(J^n)^2 = -\mathrm{Id}$. Moreover the metric G_{uv} is hermitian with respect to all three J^n and one defines the associated triplet of two-forms $\omega_{uv}^n := G_{uw}(J^n)_v^w$. In contrast to the Kählerian case, the almost complex structures are not parallel but the Levi-Civita connection ∇^G of G rotates the endomorphisms inside Q, i.e.

$$\nabla J^n := \nabla^G J^n - \epsilon^{npq} \theta^p J^q = 0 . \tag{2.12}$$

Note that ∇ differs from ∇^G by an $SU(2)$-connection with connection one-forms θ^p. For later use let us note that the metric G_{uv} can be expressed in terms of vielbeins $\mathcal{U}_u^{\alpha\mathcal{A}}$ as

$$G_{uv} = C_{\alpha\beta} \epsilon_{\mathcal{A}\mathcal{B}} \mathcal{U}_u^{\alpha\mathcal{A}} \mathcal{U}_v^{\beta\mathcal{B}} , \tag{2.13}$$

where $C_{\alpha\beta}$ denotes the flat metric on $Sp(2n_H, \mathbb{R})$ and the $SU(2)$-indices $\mathcal{A}, \mathcal{B}$ are raised and lowered with $\epsilon_{\mathcal{A}\mathcal{B}}$.

The gauge group G is specified by the generators t_I of its Lie algebra $\mathfrak{g}$ and the structure constants f_{IJ}^K,

$$[t_I, t_J] = -f_{IJ}^K t_K . \tag{2.14}$$

The vector fields transform in the adjoint representation of the gauge group, i.e. $t_{IJ}^K = f_{IJ}^K$ while the tensor fields can carry an arbitrary representation. The most general representation for n_V vector multiplets and n_T tensor multiplets has been found in [12] and is given by

$$t_{I\tilde{J}}^{\tilde{K}} = \begin{pmatrix} f_{IJ}^K & t_{IJ}^N \\ 0 & t_{IM}^N \end{pmatrix} . \tag{2.15}$$

We see that the block matrix t_{IJ}^N mixes vector- and tensor fields. However the t_{IJ}^N are only nonzero if the chosen representation of the gauge group is not completely reducible. This never occurs for compact gauge groups but there exist non-compact gauge groups containing an Abelian ideal that admit representations of this type, see [12]. There it is also shown that the construction of a generalized Chern-Simons term in the action for vector- and tensor multiplets requires the existence of an invertible and antisymmetric matrix Ω_{MN}. In particular, the $t_{I\tilde{J}}^N$ are of the form

$$t_{I\tilde{J}}^N = C_{I\tilde{J}P}\Omega^{PN} \ . \tag{2.16}$$

The gauge group is realized on the scalar fields via the action of Killing vectors ξ_I for the vector- and tensor multiplets and k_I for the hypermultiplets that satisfy the Lie algebra $\mathfrak{g}$ of G,

$$\begin{aligned} [\xi_I, \xi_J]^i &:= \xi_I^j \partial_j \xi_J^i - \xi_J^j \partial_j \xi_I^i = -f_{IJ}^K \, \xi_K^i \ , \\ [k_I, k_J]^u &:= k_I^v \partial_v k_J^u - k_J^v \partial_v k_I^u = -f_{IJ}^K \, k_K^u \ . \end{aligned} \tag{2.17}$$

In the case of the projective special real manifold, one can obtain an explicit expression for the Killing vectors ξ_I^i given by [13]

$$\xi_I^i := -\sqrt{\tfrac{3}{2}}\, t_{I\tilde{J}}^{\tilde{K}} h^{\tilde{J}} h_{\tilde{K}}^i = -\sqrt{\tfrac{3}{2}}\, t_{I\tilde{J}}^{\tilde{K}} h^{\tilde{J}i} h_{\tilde{K}} \ . \tag{2.18}$$

The second equality is due to the fact that [15]

$$t_{I\tilde{J}}^{\tilde{K}}\, h^{\tilde{J}} h_{\tilde{K}} = 0 \ , \tag{2.19}$$

and thus

$$0 = \partial_i (t_{I\tilde{J}}^{\tilde{K}} h^{\tilde{J}} h_{\tilde{K}}) = t_{I\tilde{J}}^{\tilde{K}} h^{\tilde{J}} \partial_i h_{\tilde{K}} + t_{I\tilde{J}}^{\tilde{K}} (\partial_i h^{\tilde{J}}) h_{\tilde{K}} \ , \tag{2.20}$$

which implies[6]

$$t_{I\tilde{J}}^{\tilde{K}} h^{\tilde{J}} h_{\tilde{K}}^i = t_{I\tilde{J}}^{\tilde{K}} h^{\tilde{J}i} h_{\tilde{K}} \ . \tag{2.21}$$

The Killing vectors k_I^u on the quaternionic Kähler manifold $\mathcal{T}_H$ [12, 14, 16] have to be triholomorphic which implies

$$\nabla_u k_w^I (J^n)_v^w - (J^n)_u^w \nabla_w k_v^I = 2\epsilon^{npq} \omega_{uv}^p \mu^{Iq} \ . \tag{2.22}$$

Here μ_I^n is a triplet of moment maps which also satisfy

$$\tfrac{1}{2}\omega_{uv}^n k_I^v = -\nabla_u \mu_I^n \ , \tag{2.23}$$

[6]Note that the derivative $h_{\tilde{I}i} = \sqrt{\frac{3}{2}}\,\partial_i h_{\tilde{I}}$ has an additional minus sign compared to (2.9) which can be shown by lowering the index with $a_{\tilde{I}\tilde{J}}$ given in (2.6).

and the equivariance condition

$$f_{IJ}^{K}\mu_K^n = \tfrac{1}{2}\omega_{uv}^n k_I^u k_J^v - 2\epsilon^{npq}\mu_{Ip}\mu_{Jq} \ . \tag{2.24}$$

Furthermore the covariant derivative of the Killing vectors obeys [16, 17]

$$\nabla_u k_{Iv} + \nabla_v k_{Iu} = 0 \ , \qquad \nabla_u k_{Iv} - \nabla_v k_{Iu} = \omega_{uv}^n \mu_{nI} + L_{Iuv} \ , \tag{2.25}$$

where the L_{Iuv} are related to the gaugino mass matrix and commute with J^n. For later use we define

$$S_{Iuv}^n := L_{Iuw}(J^n)_v^{\ w} \ , \qquad L_{uv} := h^I L_{Iuv} \ , \qquad S_{uv}^n := h^I S_{Iuv}^n \ , \tag{2.26}$$

where the S_{Iuv}^n are symmetric in u, v [16].

Before we proceed let us note that for $n_H = 0$, i.e. when there are no hypermultiplets, constant Fayet-Iliopoulos (FI) terms can exist which have to satisfy the equivariance condition (2.24). In this case the first term on the right hand side of (2.24) vanishes which implies that there are only two possible solutions [13]. If the gauge group contains an $SU(2)$-factor, the FI-terms have to be of the form

$$\mu_I^n = c e_I^n \ , \quad c \in \mathbb{R} \ , \tag{2.27}$$

where the e_I^n are nonzero constant vectors for $I = 1, 2, 3$ of the $SU(2)$-factor that satisfy

$$\epsilon^{mnp} e_I^m e_J^n = f_{IJ}^K e_K^p \ . \tag{2.28}$$

The second solution has $U(1)$-factors in the gauge group and the constant moment maps are given by

$$\mu_I^n = c_I e^n \ , \quad c_I \in \mathbb{R} \ , \tag{2.29}$$

where e^n is a constant $SU(2)$-vector and I labels the $U(1)$-factors.

Finally, the covariant derivatives of the scalars in (2.3) are given by

$$\mathcal{D}_\mu \phi^i = \partial_\mu \phi^i + g A_\mu^I \xi_I^i(\phi) \ , \qquad \mathcal{D}_\mu q^u = \partial_\mu q^u + g A_\mu^I k_I^u(q) \ . \tag{2.30}$$

The scalar potential

$$V = 2 g_{ij} W^{i\mathcal{AB}} W_{\mathcal{AB}}^j + 2 g_{ij} \mathcal{K}^i \mathcal{K}^j + 2 N_{\mathcal{A}}^\alpha N_\alpha^{\mathcal{A}} - 4 S_{\mathcal{AB}} S^{\mathcal{AB}}, \tag{2.31}$$

is defined in terms of the couplings[7]

$$\begin{aligned} S^{\mathcal{AB}} &:= h^I \mu_I^n \sigma_n^{\mathcal{AB}} \ , \qquad & W_i^{\mathcal{AB}} &:= h_i^I \mu_I^n \sigma_n^{\mathcal{AB}} \ , \\ \mathcal{K}^i &:= \tfrac{\sqrt{6}}{4} h^I \xi_I^i \ , \qquad & N^{\alpha\mathcal{A}} &:= \tfrac{\sqrt{6}}{4} h^I k_I^u \mathcal{U}_u^{\alpha\mathcal{A}} \ . \end{aligned} \tag{2.32}$$

[7] Note that the h^M in the direction of the tensor multiplets do not appear explicitly. Nevertheless, the couplings can implicitly depend on the scalars in the tensor multiplet as they might appear in h^I after solving (2.5).

Here $\sigma^n_{\mathcal{AB}}$ are the Pauli matrices with an index lowered by $\epsilon_{\mathcal{AB}}$, i.e.

$$\sigma^1_{\mathcal{AB}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ , \quad \sigma^2_{\mathcal{AB}} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} \ , \quad \sigma^3_{\mathcal{AB}} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \ . \tag{2.33}$$

As usual the couplings (2.32) are related to the scalar parts of the supersymmetry variations of the fermions via

$$\begin{aligned} \delta_\epsilon \psi^{\mathcal{A}}_\mu &= D_\mu \epsilon^{\mathcal{A}} - \tfrac{ig}{\sqrt{6}} S^{\mathcal{AB}} \gamma_\mu \epsilon_{\mathcal{B}} + \dots \ , \\ \delta_\epsilon \lambda^{i\mathcal{A}} &= g\mathcal{K}^i \epsilon^{\mathcal{A}} - gW^{i\mathcal{AB}} \epsilon_{\mathcal{B}} + \dots \ , \\ \delta_\epsilon \zeta^\alpha &= gN^\alpha_{\mathcal{A}} \epsilon^{\mathcal{A}} + \dots \ . \end{aligned} \tag{2.34}$$

Here $\epsilon^{\mathcal{A}}$ denote the supersymmetry parameters. This concludes our review of $d = 5$ supergravity and we now turn to its possible supersymmetric AdS backgrounds.

3 Supersymmetric AdS$_5$ vacua

In this section we determine the conditions that lead to AdS$_5$ vacua which preserve all eight supercharges. This requires the vanishing of all fermionic supersymmetry transformations, i.e.

$$\langle \delta_\epsilon \psi^{\mathcal{A}}_\mu \rangle = \langle \delta_\epsilon \lambda^{i\mathcal{A}} \rangle = \langle \delta_\epsilon \zeta^\alpha \rangle = 0 \ , \tag{3.1}$$

where $\langle \ \rangle$ denotes the value of a quantity evaluated in the background. Using the fact that $W^{i\mathcal{AB}}$ and $\mathcal{K}^i$ are linearly independent [11] and (2.34), this implies the following four conditions,

$$\langle W^{\mathcal{AB}}_i \rangle = 0 \ , \quad \langle S_{\mathcal{AB}} \rangle \, \epsilon^{\mathcal{B}} = \Lambda U_{\mathcal{AB}} \, \epsilon^{\mathcal{B}} \ , \quad \langle N^{\alpha\mathcal{A}} \rangle = 0 \ , \quad \langle \mathcal{K}^i \rangle = 0 \ . \tag{3.2}$$

Here $\Lambda \in \mathbb{R}$ is related to the cosmological constant and $U_{\mathcal{AB}} = v_n \sigma^n_{\mathcal{AB}}$ for $v \in S^2$ is an $SU(2)$-matrix. $U_{\mathcal{AB}}$ appears in the Killing spinor equation for AdS$_5$ which reads [18]

$$\langle D_\mu \epsilon_{\mathcal{A}} \rangle = \tfrac{ia}{2} \, U_{\mathcal{AB}} \, \gamma_\mu \epsilon^{\mathcal{B}} \ , \quad a \in \mathbb{R} \ . \tag{3.3}$$

As required for an AdS vacuum, the conditions (3.2) give a negative background value for the scalar potential $\langle V(\phi, q) \rangle < 0$ which can be seen from (2.31). Using the definitions (2.32), we immediately see that the four conditions (3.2) can also be formulated as conditions on the moment maps and Killing vectors,

$$\langle h^I_i \mu^n_I \rangle = 0 \ , \qquad \langle h^I \mu^n_I \rangle = \Lambda v^n \ , \qquad \langle h^I k^u_I \rangle = 0 \ , \qquad \langle h^I \xi^i_I \rangle = 0 \ . \tag{3.4}$$

Note that due to (2.5), (2.8) we need to have $\langle h^I \rangle \neq 0$ for some I and $\langle h^{\tilde{I}}_i \rangle \neq 0$ for every i and some $\tilde{I}$.[8]

[8] In particular this can also hold at the origin of the scalar field space $\langle \phi^i \rangle = 0$, i.e. for unbroken gauge groups.

In order to solve (3.4) we combine the first two conditions as

$$\langle \begin{pmatrix} h^I \\ h_i^I \end{pmatrix} \mu_I^n \rangle = \begin{pmatrix} \Lambda v^n \\ 0 \end{pmatrix} . \tag{3.5}$$

Let us enlarge these equations to the tensor multiplet indices by introducing $\mu^n_{\tilde{I}}$ where we keep in mind that $\mu_N^n \equiv 0$. Then we use the fact that the matrix $(h^{\tilde{I}}, h_i^{\tilde{I}})$ is invertible in special real geometry (see Appendix C of [13]), so we can multiply (3.5) with $(h^{\tilde{I}}, h_i^{\tilde{I}})^{-1}$ to obtain a solution for both equations given by

$$\langle \mu^n_{\tilde{I}} \rangle = \Lambda v^n \langle h_{\tilde{I}} \rangle \ . \tag{3.6}$$

Note that this condition is non-trivial since it implies that the moment maps point in the same direction in $SU(2)$-space for all I. Furthermore, using the $SU(2)_R$-symmetry we can rotate the vector v^n such that $v^n = v\delta^{n3}$ and absorb the constant $v \in \mathbb{R}$ into Λ. Thus only $\langle \mu_I \rangle := \langle \mu_I^3 \rangle \neq 0$, $\forall I$ in the above equation. Since by definition $\langle \mu_N^n \rangle = 0$, this implies

$$\langle \mu_I \rangle = \Lambda \langle h_I \rangle \ , \quad \langle h_N \rangle = 0 \ . \tag{3.7}$$

In particular, this means that the first two equations in (2.10) hold in the vacuum for only the vector indices, i.e.

$$\langle h^I h_I \rangle = 1 \ , \quad \langle h_I h_i^I \rangle = 0 \ . \tag{3.8}$$

Moreover due to the explicit form of the moment maps in (3.7), the equivariance condition (2.24) reads in the background

$$f_{IJ}^K \langle \mu_K \rangle = \tfrac{1}{2} \langle \omega^3_{uv} k_I^u k_J^v \rangle . \tag{3.9}$$

Since (2.31) has to hold in the vacuum, $\langle h^I \rangle \neq 0$ for some I and thus the background necessarily has non-vanishing moment maps due to (3.7). This in turn implies that part of the R-symmetry is gauged, as can be seen from the covariant derivatives of the fermions which always contain a term of the form $A_\mu^I \langle \mu_I^3 \rangle$ [13]. More precisely, this combination gauges the $U(1)_R \subset SU(2)_R$ generated by σ^3. From (3.7) we infer $A_\mu^I \langle \mu_I^3 \rangle = \Lambda A_\mu^I \langle h_I \rangle$ which can be identified with the graviphoton [15].

We now turn to the last two equations in (3.4). Let us first prove that the third equation $\langle h^I k_I^u \rangle = 0$ implies the fourth $\langle h^I \xi_I^i \rangle = 0$. This can be shown by expressing $\langle \xi_I^i \rangle$ in terms of $\langle k_I^u \rangle$ via the equivariance condition (3.9). Note that we learn from (2.18) that the background values of the Killing vectors on the manifold $\mathcal{T}_{VT}$ are given by

$$\langle \xi_I^i \rangle = -\sqrt{\tfrac{3}{2}} \, \langle t_{I\tilde{J}}^{\tilde{K}} h^{\tilde{J}i} h_{\tilde{K}} \rangle = -\sqrt{\tfrac{3}{2}} \, \langle f_{IJ}^K h^{Ji} h_K + t_{IJ}^N h^{Ji} h_N \rangle = -\sqrt{\tfrac{3}{2}} \, \langle f_{IJ}^K h^{Ji} h_K \rangle \ , \tag{3.10}$$

where we used (2.15) and (3.7). Inserting (3.7), (3.9) into (3.10) one indeed computes

$$\langle \xi_I^i \rangle = -\sqrt{\tfrac{3}{2}} \tfrac{1}{2\Lambda} \langle h_i^J \omega_{uv}^3 k_I^u k_J^v \rangle \ . \tag{3.11}$$

But then $\langle h^I \xi_I^i \rangle = 0$ is always satisfied if $\langle h^I k_I^u \rangle = 0$. Moreover this shows that $\langle \xi_I^i \rangle \neq 0$ is only possible for $\langle k_I^u \rangle \neq 0$. Note that the reverse is not true in general as can be seen from (3.10). We are thus left with analyzing the third condition in (3.4).

Let us first note that for $n_H = 0$ there are no Killing vectors ($k_I^u \equiv 0$) and the third equation in (3.4) is automatically satisfied. However (3.7) can nevertheless hold if the constant FI-terms discussed below (2.26) are of the form given in (2.29) and thus only gauge groups with Abelian factors are allowed in this case.

Now we turn to $n_H \neq 0$. Note that then $\langle h^I k_I^u \rangle = 0$ has two possible solutions:

$$\begin{aligned} &i) \quad \langle k_I^u \rangle = 0 \ , \quad \text{for all } I \\ &ii) \quad \langle k_I^u \rangle \neq 0 \ , \quad \text{for some } I \text{ with } \langle h^I \rangle \text{ appropriately tuned.} \end{aligned} \tag{3.12}$$

By examining the covariant derivatives (2.30) of the scalars we see that in the first case there is no gauge symmetry breaking by the hypermultiplets while in the second case G is spontaneously broken. Note that not all possible gauge groups can remain unbroken in the vacuum. In fact, for case $i)$ the equivariance condition (3.9) implies

$$f_{IJ}^K \langle \mu_K \rangle = 0 \ . \tag{3.13}$$

This can only be satisfied if the adjoint representation of $\mathfrak{g}$ has a non-trivial zero eigenvector, i.e. if the center of G is non-trivial (and continuous).[9] In particular, this holds for all gauge groups with an Abelian factor but all semisimple gauge groups have to be broken in the vacuum.

In the rest of this section we discuss the spontaneous symmetry breaking for case $ii)$ and the details of the Higgs mechanism. Let us first consider the case where only a set of Abelian factors in G is spontaneously broken, i.e. $\langle k_I^u \rangle \neq 0$ for I labeling these Abelian factors. From (3.10) we then learn $\langle \xi_I^i \rangle = 0$ and thus we only have spontaneous symmetry breaking in the hypermultiplet sector and the Goldstone bosons necessarily are recruited out of these hypermultiplets. Hence the vector multiplet corresponding to a broken Abelian factor in G becomes massive by "eating" an entire hypermultiplet. It forms a "long" vector multiplet containing the massive vector, four gauginos and four scalars obeying the AdS mass relations.

Now consider spontaneously broken non-Abelian factors of G, i.e. $\langle k_I^u \rangle \neq 0$ for I labeling these non-Abelian factors. In this case we learn from (3.11) that either $\langle \xi_I^i \rangle = 0$ as before or $\langle \xi_I^i \rangle \neq 0$. However the Higgs mechanism is essentially unchanged compared to the

[9] For more details on Lie groups and their adjoint representation, see for example [20].

Abelian case in that entire hypermultiplets are eaten and all massive vectors reside in long multiplets.[10]

However there always has to exists at least one unbroken generator of G which commutes with all other unbroken generators, i.e. the unbroken gauge group in the vacuum is always of the form $H \times U(1)_R$. To see this, consider the mass matrix M_{IJ} of the gauge bosons A^I_μ. Due to (2.30) and (3.11), this is given by

$$M_{IJ} = \langle G_{uv} k^u_I k^v_J \rangle + \langle g_{ij} \xi^i_I \xi^j_J \rangle = \langle K_{uv} k^u_I k^v_J \rangle \ . \tag{3.14}$$

Here K_{uv} is an invertible matrix which can be given in terms of G_{uv} and S_{uv} defined in (2.26) as

$$K_{uv} = \langle \left(\tfrac{5}{8} G_{uv} - \tfrac{6}{8\Lambda} S_{uv} \right) \rangle \ . \tag{3.15}$$

Since $\langle h^I k^u_I \rangle = 0$ the mass matrix M_{IJ} has a zero eigenvector given by $\langle h^I \rangle$, i.e. the graviphoton $\langle h^I \rangle A^\mu_I$ always remains massless in the vacuum. In the background the commutator of the corresponding Killing vector $h^I k^u_I$ with any other isometry k_J is given by

$$\langle [h^I k_I, k_J]^u \rangle = \langle h^I (k^v_I \partial_v k^u_J - k^v_J \partial_v k^u_I) \rangle = -\langle h^I k^v_J \partial_v k^u_I \rangle \ . \tag{3.16}$$

This vanishes for $\langle k^u_J \rangle = 0$ and thus the R-symmetry commutes with every other symmetry generator of the vacuum, i.e. the unbroken gauge group is $H \times U(1)_R$. In particular, every gauge group G which is not of this form has to be broken $G \to H \times U(1)_R$.

Let us close this section with the observation that the number of broken generators is determined by the number of linearly independent $\langle k^u_I \rangle$. This coincides with the number of Goldstone bosons n_G. In fact the $\langle k^u_I \rangle$ form a basis in the space of Goldstone bosons $\mathcal{G}$ and we have $\mathcal{G} = \mathrm{span}_{\mathbb{R}}\{\langle k^u_I \rangle\}$ with $\dim(\mathcal{G}) = \mathrm{rk}\, \langle k^u_I \rangle = n_G$.

In conclusion, we have shown that the conditions for maximally supersymmetric AdS_5 vacua are given by

$$\langle \mu_I \rangle = \Lambda \, \langle h_I \rangle, \quad \langle h_M \rangle = 0, \quad \langle h^I k^u_I \rangle = \langle h^I \xi^i_I \rangle = 0 \ . \tag{3.17}$$

Note that the tensor multiplets enter in the final result only implicitly since the h^I and its derivatives are functions of all scalars ϕ^i. The first equation implies that a $U(1)_R$-symmetry is always gauged by the graviphoton while the last equation shows that the unbroken gauge group in the vacuum is of the form $H \times U(1)_R$. This reproduces the result of [9] that the $U(1)_R$ has to be unbroken and gauged in a maximally supersymmetric AdS_5 background. In the dual four-dimensional SCFT this $U(1)_R$ is defined by a-maximization. Moreover we discussed that if the gauge group is spontaneously broken the massive vector multiplets are long multiplets. Finally, we showed that space of Goldstone bosons is given by $\mathcal{G} = \mathrm{span}_{\mathbb{R}}\{\langle k^u_I \rangle\}$ which will be used in the next section to compute the moduli space $\mathcal{M}$ of these vacua.

[10] Note that short BPS vector multiplets which exist in this theory cannot appear since the breaking necessarily involves the hypermultiplets.

4 Structure of the moduli space

We now turn to the computation of the moduli space $\mathcal{M}$ of the maximally supersymmetric AdS_5 vacua determined in the previous section. Let us denote by $\mathcal{D}$ the space of all possible deformations of the scalar fields $\phi \to \langle\phi\rangle + \delta\phi$, $q \to \langle q\rangle + \delta q$ that leave the conditions (3.4) invariant. However, if the gauge group is spontaneously broken the corresponding Goldstone bosons are among these deformations but they should not be counted as moduli. Thus the moduli space is defined as the space of deformations $\mathcal{D}$ modulo the space of Goldstone bosons $\mathcal{G}$, i.e. $\mathcal{M} = \mathcal{D}/\mathcal{G}$. In order to determine $\mathcal{M}$ we vary (3.4) to linear order and characterize the space $\mathcal{D}$ spanned by $\delta\phi$ and δq that are not fixed.[11] We then show that the Goldstone bosons also satisfy the equations defining $\mathcal{D}$ and determine the quotient $\mathcal{D}/\mathcal{G}$.

Let us start by varying the second condition of (3.4). This yields

$$\langle\delta(h^I\mu_I^n)\rangle = \langle(\partial_i h^I)\,\mu_I^n\rangle\,\delta\phi^i + \langle h^I\nabla_u\mu_I^n\rangle\,\delta q^u = -\tfrac{1}{2}\langle\omega_{uv}^n h^I k_I^v\rangle\delta q^u \equiv 0\ , \tag{4.1}$$

where we used (3.4) and (2.23). Since this variation vanishes automatically, no conditions are imposed on the scalar field variation.

The variation of the first condition in (3.4) gives

$$\begin{aligned}\langle\delta(h_i^I\mu_I^n)\rangle &= \langle(\nabla_j h_i^I)\,\mu_I^n\rangle\,\delta\phi^j + \langle h_i^I\nabla_u\mu_I^n\rangle\,\delta q^u\\ &= -\sqrt{\tfrac{2}{3}}\langle\mu_I^n(h^I g_{ij} + h^{Ik}T_{ijk})\rangle\,\delta\phi^j - \tfrac{1}{2}\langle h_i^I\omega_{uv}^n k_I^v\rangle\,\delta q^u\\ &= -\sqrt{\tfrac{2}{3}}\Lambda\delta^{n3}\delta\phi_i - \tfrac{1}{2}\langle h_i^I\omega_{uv}^n k_I^v\rangle\,\delta q^u = 0\ ,\end{aligned} \tag{4.2}$$

where in the second step we used (2.11), (2.23) while in the third we used (3.4). For $n = 1, 2$ (4.2) imposes

$$\langle h_i^I\omega_{uv}^{1,2}k_I^v\rangle\,\delta q^u = 0\ , \tag{4.3}$$

while for $n = 3$ the deformations $\delta\phi_i$ can be expressed in terms of δq^u as

$$\delta\phi_i = -\sqrt{\tfrac{3}{2}}\tfrac{1}{2\Lambda}\langle h_i^I\omega_{uv}^3 k_I^v\rangle\,\delta q^u\ . \tag{4.4}$$

Thus all deformations $\delta\phi_i$ are fixed either to vanish or to be related to δq^u. In other words, the entire space of deformations can be spanned by scalars in the hypermultiplets only, i.e. $\mathcal{D} \subset \mathcal{T}_H$. Note that this is in agreement with (3.11) and also $\mathcal{G} \subset \mathcal{T}_H$.

Finally, we vary the third condition in (3.4) to obtain

$$\langle\delta(h^I k_{Iu})\rangle = \langle\partial_i h^I k_{Iu}\rangle\,\delta\phi^i + \langle h^I\nabla_v k_{Iu}\rangle\,\delta q^v = 0. \tag{4.5}$$

[11]Since we consider the variations of the vacuum equations (3.4) to first order in the scalar fields, this procedure only gives a necessary condition for the moduli space.

Inserting (4.4) and using (2.10), (3.4) we find

$$\left(\tfrac{1}{2\Lambda}\langle k^{Iu}\omega^3_{vw}k^w_I\rangle + \langle h^I\nabla_v k^u_I\rangle\right)\delta q^v = 0 \ . \tag{4.6}$$

Thus we are left with the two conditions (4.3) and (4.6) whose solutions determine $\mathcal{D}$. For a generic supergravity we will not solve them here in general. However the conditions alone suffice to prove that the moduli space is a Kähler submanifold of $\mathcal{T}_H$ as we will now show.

As a first step we prove that the Goldstone bosons satisfy (4.3) and (4.6). We know from section 3 that the Goldstone directions are of the form $\delta q^u = c^I\langle k^u_I\rangle$ where c^I are constants. Inserted into (4.3) we find

$$c^I\langle h^J_i\omega^{1,2}_{uv}k^u_I k^v_J\rangle = 2c^I\langle h^J_i f^K_{IJ}\mu^{1,2}_K\rangle = 0 \ , \tag{4.7}$$

where we used (3.9) and the fact that $\langle\mu^{1,2}_K\rangle = 0$. To show that the Goldstone bosons also satisfy (4.6) we first observe that

$$\langle h^I(\nabla_v k^u_I)k^v_J\rangle = \langle h^I(\partial_v k^u_I)k^v_J - h^I(\partial_v k^u_J)k^v_I\rangle = -\langle h^I[k_I,k_J]^u\rangle = \langle f^K_{IJ}h^I k^u_K\rangle \ , \tag{4.8}$$

where in the first step we used (3.4), added a term which vanishes in the background and then in the second step used (2.17). In addition we need to show

$$\langle f^K_{IJ}h^I k^u_K\rangle = \langle f^K_{IJ}h_K k^{Iu}\rangle \ . \tag{4.9}$$

Indeed, using (2.10) and $\langle h^I k^u_I\rangle = 0$ we find

$$\langle f^K_{IJ}h^I k^u_K\rangle = \langle f^K_{IJ}h^I k^{Lu}a_{KL}\rangle = \langle f^K_{IJ}h^I k^{Lu}h^i_K h_{Li}\rangle \ . \tag{4.10}$$

Inserting (2.21) evaluated in the vacuum, i.e. $\langle f^K_{IJ}h^J h^i_K\rangle = \langle f^K_{IJ}h^{Ji}h_K\rangle$ and using again (2.10) we obtain

$$\langle f^K_{IJ}h^I k^u_K\rangle = \langle f^K_{IJ}h^{Ii}k^{Lu}h_K h_{iL}\rangle = \langle f^K_{IJ}h_K k^{Lu}\delta^I_L\rangle = \langle f^K_{IJ}h_K k^{Iu}\rangle \ , \tag{4.11}$$

which proves (4.9) as promised.

Turning back to (4.6), we insert $\delta q^u = c^I\langle k^u_I\rangle$ and use (3.9) and (4.8) to arrive at

$$\tfrac{1}{2\Lambda}c^I\langle k^{Ju}\omega^3_{vw}k^w_J k^v_I\rangle + c^I\langle h^J\nabla_v k^u_J k^v_I\rangle = \tfrac{1}{\Lambda}c^I\langle k^{Ju}f^K_{IJ}\mu_K\rangle + c^I\langle f^K_{JI}h^J k^u_K\rangle \ . \tag{4.12}$$

Using again that $\langle\mu_I\rangle = \Lambda\langle h_I\rangle$ and applying (4.9), this yields

$$\tfrac{1}{\Lambda}c^I\langle k^{Ju}f^K_{IJ}\mu_K\rangle + c^I\langle f^K_{JI}h^J k^u_K\rangle = (f^K_{JI}+f^K_{IJ})c^I\langle h^J k^u_K\rangle = 0 \ . \tag{4.13}$$

Thus the Goldstone directions $\delta q^u = c^I\langle k^u_I\rangle$ leave the vacuum conditions (3.4) invariant and hence $\mathcal{G}\subset\mathcal{D}$.

Let us now consider the moduli space $\mathcal{M} = \mathcal{D}/\mathcal{G}$ and show that $J^3(\mathcal{M}) = \mathcal{M}$, i.e. J^3 restricts to an almost complex structure on $\mathcal{M}$. Concretely we show that the defining equations for the moduli space, (4.3) and (4.6), are invariant under J^3. For equations (4.3) this follows from the fact that J^3 interchanges the two equations. This can be seen by substituting $\delta q'^u = (J^3)^u_v \delta q^v$ and using that $J^1 J^2 = J^3$ on a quaternionic Kähler manifold.

Turning to (4.6), we note that since only $\langle \mu_I^3 \rangle \neq 0$ the covariant derivative (2.22) of the Killing vectors k_I^u commutes with J^3 in the vacuum, i.e.

$$\langle \nabla_u k^I_w (J^n)^w_v - (J^n)^w_u \nabla_w k^I_v \rangle = 2\epsilon^{npq} \langle \omega^p_{uv} \mu^{Iq} \rangle = 0 \ . \tag{4.14}$$

This implies that the second term in (4.6) is invariant under J^3 and we need to show that this also holds for the first term. In fact, we will show in the following that this term vanishes on the moduli space and is only nonzero for Goldstone directions.

Let us first note that in general $\operatorname{rk} \langle k_I^u \omega^3_{vw} k^{wI} \rangle \leq \operatorname{rk} \langle k_I^u \rangle = n_G$. However, $\langle k_I^u \omega^3_{vw} k^{wI} k_J^v \rangle \neq 0$ (as we saw in (4.12)) implies that the rank of the two matrices has to coincide. This in turn says that the first term in (4.6) can only be nonzero in the Goldstone directions and thus has to vanish for the directions spanning $\mathcal{M}$. Thus the whole equation (4.6) is J^3-invariant on $\mathcal{M}$. Therefore we have an almost complex structure $\tilde{J} := J^3|_{\mathcal{M}}$ and a compatible metric $\tilde{G} := G|_{\mathcal{M}}$ on $\mathcal{M}$. Thus $(\mathcal{M}, \tilde{G}, \tilde{J})$ is an almost hermitian submanifold of the quaternionic Kähler manifold $(\mathcal{T}_H, G, Q)$.

In the following we want to use theorem 1.12 of [19]: an almost Hermitian submanifold (M, G, J) of a quaternionic Kähler manifold $(\tilde{M}, \tilde{G}, Q)$ is Kähler if and only if it is totally complex, i.e. if there exists a section I of Q that anticommutes with J and satisfies

$$I(T_p M) \perp T_p M \quad \forall p \in M \ . \tag{4.15}$$

In particular, this condition is satisfied if the associated fundamental two-form $\omega_{uw} = G_{uw} I^w_v$ on M vanishes.

Now let us show that the moduli space $\mathcal{M}$ actually is totally complex and hence Kähler. To do so, we use (2.25) and (2.26) to note that in the vacuum (3.7) $\langle \omega^3_{uv} \rangle$ is given by

$$\langle \omega^3_{uv} \rangle = \tfrac{2}{\Lambda} \langle h^I \nabla_u k_{Iv} - L_{uv} \rangle \ . \tag{4.16}$$

We just argued that $\langle k_I^u \omega^3_{vw} k^{wI} \rangle$ vanishes on $\mathcal{M}$ and thus (4.6) projected onto $\mathcal{M}$ also implies

$$\langle h^I \nabla_u k_{vI} \rangle |_{\mathcal{M}} = 0 \ . \tag{4.17}$$

Since $\langle \omega^1_{uv} \rangle = -\langle \omega^3_{uw} (J^2)^w_v \rangle$, we can multiply (4.16) with $-(J^2)^w_v$ from the right and obtain

$$\langle \omega^1_{uv} \rangle |_{\mathcal{M}} = \tfrac{2}{\Lambda} \langle S^2_{uv} - h^I \nabla_u k_{wI} (J^2)^w_v \rangle |_{\mathcal{M}} = 0 \ , \tag{4.18}$$

where in the first step we used (2.26). This expression vanishes due to (4.17) and the fact that S^2_{uv} is symmetric while ω^1_{uv} is antisymmetric. Thus $\mathcal{M}$ is totally complex and in particular $(\mathcal{M}, \tilde{G}, \tilde{J})$ is a Kähler submanifold.

As proved in [19] a Kähler submanifold can have at most half the dimension of the ambient quaternionic Kähler manifold, i.e. $\dim(\mathcal{M}) \leq 2n_H$.[12] Note that in the case of an unbroken gauge group we have $\mathcal{G} = \{\emptyset\}$ and thus $\mathcal{D} = \mathcal{M}$. This is the case of maximal dimension of the moduli space. If the gauge group is now spontaneously broken then additional scalars are fixed by (4.3). Since $\mathcal{M}$ is J^3-invariant, every $\delta q^u \in \mathcal{M}$ can be written as $\delta q^u = (J^3)^u_v \delta q'^v$ for some $\delta q'^u \in \mathcal{M}$. Combined with the fact that $J^1 J^2 = J^3$ this implies that the two conditions in (4.3) are equivalent on $\mathcal{M}$. Furthermore we have $\mathrm{rk}\,\langle h^I_i \omega^1_{uv} k^v_I \rangle = \mathrm{rk}\,\langle k^I_u \rangle = n_G$ and thus n_G scalars are fixed by (4.3). In conclusion, we altogether have

$$\dim(\mathcal{M}) = \dim(\mathcal{D}) - \dim(\mathcal{G}) \leq (2n_H - n_G) - n_G \ , \tag{4.19}$$

so the moduli space has at most real dimension $2n_H - 2n_G$.

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[12] Applying the same method as in $d = 4$, $\mathcal{N} = 2$ this can be checked explicitly [5].

References

[1] J. M. Maldacena, "The Large N limit of superconformal field theories and supergravity," Int. J. Theor. Phys. **38** (1999) 1113 [Adv. Theor. Math. Phys. **2** (1998) 231] [hep-th/9711200].

[2] A. Kehagias, "New type IIB vacua and their F theory interpretation," Phys. Lett. B **435** (1998) 337 [hep-th/9805131].

[3] D. R. Morrison and M. R. Plesser, "Nonspherical horizons. 1.," Adv. Theor. Math. Phys. **3** (1999) 1 [hep-th/9810201].

[4] J. Polchinski, "Introduction to Gauge/Gravity Duality," arXiv:1010.6134 [hep-th]

[5] S. de Alwis, J. Louis, L. McAllister, H. Triendl and A. Westphal, "Moduli spaces in AdS_4 supergravity," JHEP **1405** (2014) 102 [arXiv:1312.5659 [hep-th]].

[6] J. Louis and H. Triendl, "Maximally supersymmetric AdS_4 vacua in $\mathrm{N} = 4$ supergravity," JHEP **1410** (2014) 007 [arXiv:1406.3363 [hep-th]].

[7] J. Louis, H. Triendl and M. Zagermann, "$\mathcal{N} = 4$ supersymmetric AdS_5 vacua and their moduli spaces," JHEP **1510** (2015) 083 [arXiv:1507.01623 [hep-th]].

[8] J. Louis and S. Lüst, "Supersymmetric AdS_7 backgrounds in half-maximal supergravity and marginal operators of (1, 0) SCFTs," JHEP **1510** (2015) 120 [arXiv:1506.08040 [hep-th]].

[9] Y. Tachikawa, "Five-dimensional supergravity dual of a-maximization," Nucl. Phys. B **733** (2006) 188 [hep-th/0507057].

[10] V. Asnin, "On metric geometry of conformal moduli spaces of four-dimensional superconformal theories," JHEP **1009** (2010) 012 [arXiv:0912.2529 [hep-th]].

[11] M. Gunaydin and M. Zagermann, "The Vacua of 5-D, N=2 gauged Yang-Mills/Einstein tensor supergravity: Abelian case," Phys. Rev. D **62** (2000) 044028 [hep-th/0002228].

[12] E. Bergshoeff, S. Cucu, T. De Wit, J. Gheerardyn, R. Halbersma, S. Vandoren and A. Van Proeyen, "Superconformal N=2, D = 5 matter with and without actions," JHEP **0210** (2002) 045 [hep-th/0205230].

[13] E. Bergshoeff, S. Cucu, T. de Wit, J. Gheerardyn, S. Vandoren and A. Van Proeyen, "N = 2 supergravity in five-dimensions revisited," Class. Quant. Grav. **21** (2004) 3015 [Class. Quant. Grav. **23** (2006) 7149] [hep-th/0403045].

[14] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara, P. Fre and T. Magri, "N=2 supergravity and N=2 superYang-Mills theory on general scalar manifolds: Symplectic covariance, gaugings and the momentum map," J. Geom. Phys. **23** (1997) 111 [hep-th/9605032].

[15] M. Gunaydin, G. Sierra and P. K. Townsend, "Gauging the d = 5 Maxwell-Einstein Supergravity Theories: More on Jordan Algebras," Nucl. Phys. B **253** (1985) 573.

[16] D. V. Alekseevsky, V. Cortes, C. Devchand and A. Van Proeyen, "Flows on quaternionic Kahler and very special real manifolds," Commun. Math. Phys. **238** (2003) 525 [hep-th/0109094].

[17] R. D'Auria and S. Ferrara, "On fermion masses, gradient flows and potential in supersymmetric theories," JHEP **0105** (2001) 034 [hep-th/0103153].

[18] E. Shuster, "Killing spinors and supersymmetry on AdS," Nucl. Phys. B **554** (1999) 198 [hep-th/9902129].

[19] D. V. Alekseevsky and S. Marchiafava, "Hermitian and Kähler submanifolds of a quaternionic Kähler manifold," Osaka J. Math. **38**, 4, (2001), 869.

[20] L. O'Raifeartaigh, "Group Structure Of Gauge Theories," Cambridge University Press, 1986.