SPECIAL MULTSERIAL ALGEBRAS ARE QUOTIENTS OF SYMMETRIC SPECIAL MULTISERIAL ALGEBRAS

EDWARD L. GREEN AND SIBYLLE SCHROLL

Abstract. In this paper we give a new definition of symmetric special multiserial algebras in terms of *defining cycles*. As a consequence, we show that every special multiserial algebra is a quotient of a symmetric special multiserial algebra.

1. Introduction

A major breakthrough in representation theory of finite dimensional algebras is the classification of algebras in terms of their representation type. This is either finite, tame or wild [D]. Algebras of finite representation type have only finitely many isomorphism classes of indecomposable modules, the infinitely many indecomposable modules of a tame algebra can be parametrized by one-parameter families whereas the representation theory of a wild algebra contains that of the free algebra in two generators and so in some sense contains that of any finite dimensional algebra. Thus no hope of a parametrization of the isomorphism classes of the indecomposable modules can exist.

For this reason, algebras of finite and tame representation type have been the focus of much of the representation theory of finite dimensional algebras. An important family of tame algebras are special biserial algebras defined in [SW]. This class contains many of the tame group algebras of finite groups and tame subalgebras of group algebras of finite groups [E], gentle algebras, string algebras and symmetric special biserial algebras [WW], also known as Brauer graph algebras [R, S], algebras of quasi-qurternion type [La] and the intensely studied Jacobian algebras of surface triangulations with marked points in the boundary arising in cluster theory [ABCP].

The strength of the well-studied representation theory of special biserial algebras, derives from the underlying string combinatorics. Namely, by [WW] every indecomposable non-projective module over a special biserial algebra is a string or band module. Not only does this give rise to a formidable tool for calculations and proofs but it also shows that special biserial algebras are of tame representation type.

Special multiserial algebras, defined in [VHW], are in general of wild representation type and as a consequence their indecomposable modules cannot be classified in a similar way. It is therefore remarkable that many of the results that are known to hold for special biserial algebras still hold for special multiserial algebras. For example, a very surprising fact about the indecomposable modules of these wild algebras was shown

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in [GS2]. Namely, the indecomposable modules over a special multiserial algebra are multiserial, that is, their radical is either 0 or a sum of uniserial submodules. Thus generalizing the analogous result for special biserial algebras [SW]. However, given the absence of string combinatorics in the multiserial case, the proof is built on an entirely different strategy. The same holds true for the result and proofs in this paper.

We start by giving the definition of an algebra *defined by cycles*. This definition is built on the notion of a *defining pair*. We show that such an algebra is symmetric special multiserial and that conversely, every symmetric special multiserial algebra is an algebra defined by cycles. Note that in this context a symmetric algebra is an algebra over a field endowed with a symmetric linear form with no non-zero left ideal in its kernel. Symmetric algebras play an important role in representation theory and many examples of well-known algebras are symmetric such as group algebras of finite groups or Hecke algebras.

Given the new definitions of defining pairs and algebras defined by cycles we show that we can construct a defining pair for every special multiserial algebra A and that A is a quotient of the corresponding algebra defined by cycles. Thus we prove that every special multiserial algebra is a quotient of a symmetric special multiserial algebra. This result is an analogue of the corresponding result for special biserial algebras [WW]. Moreover, the special biserial case follows from our our result omitting thus the need for the string combinatorics on which the proof in [WW] is based.

2. Preliminaries

We let K denote a field and Q a quiver. An ideal I in the path algebra KQ is *admissible* if $J^N \subseteq I \subseteq J^2$ for some $N \geq 2$, where J is the ideal in KQ generated by the arrows of Q. We begin by recalling the definition of a special multiserial algebra. A K-algebra is a *special multiserial algebra* if it is Morita equivalent to a quotient of a path algebra, KQ/I, by an admissible ideal I which satisfies the following condition:

(M) For every arrow $a \in Q$ there is at most one arrow $b \in Q$ such that $ab \notin I$ and

$$\text{there is at most one arrow } c \in Q \text{ such that } ca \notin I$$

Throughout this paper we assume that all algebras are indecomposable and that if they are finite dimensional, the Jacobson radical squared is nonzero. Note that the only non-semisimple special multiserial algebra of the form KQ/I with radical squared zero is $K[x]/(x^2)$.

We introduce notation and definitions needed for what follows. We say that a nonzero element $x \in KQ$ is *uniform* if there exists vertices v and w in Q, corresponding to idempotents e_v and e_w in KQ such that $x = e_v x e_w$. If a and b are arrows in Q, we let ab denote the path consisting of the arrow a followed by the arrow b. We say a cycle in Q is *simple* if it has no repeated arrows. If C is a cycle in Q and p is a path, we say *p lies in C* if p is a subpath of C^s, for some $s \geq 1$. If p is a path in Q, the *length of* p, denoted $\ell(p)$, is the number of arrows in p.

If p is a path in Q, then the start vertex of p is denoted $\mathfrak{s}(p)$ and the end vertex of p is denoted $\mathfrak{t}(p)$.

3. Defining pairs

In this section we give a method for constructing symmetric special multiserial algebras. Suppose that Q is a quiver. We say the pair $(\mathcal{S},\mu)$ is a *defining pair in* Q if $\mathcal{S}$ is a set of simple cycles in Q and $\mu\colon \mathcal{S}\to\mathbb{Z}_{>0}$ which satisfy the following conditions:

(D0) If C is a loop at a vertex v and $C\in\mathcal{S}$, then $\mu(C)>1$.

(D1) If a simple cycle is in $\mathcal{S}$, every cyclic permutation of the cycle is in $\mathcal{S}$.

(D2) If $C\in\mathcal{S}$ and C' is a cyclic permutation of C then $\mu(C)=\mu(C')$.

(D3) Every arrow occurs in some simple cycle in $\mathcal{S}$.

(D4) If an arrow occurs in two cycles in $\mathcal{S}$, the cycles are cyclic permutations of each other.

If $(\mathcal{S},\mu)$ is a defining pair in Q then the K-algebra they define has quiver Q and ideal of relations generated by all relations of the following three types:

Type 1 $C^{\mu(C)}-C'^{\mu(C')}$, if C and C' are cycles in $\mathcal{S}$ at some vertex $v\in Q_0$.

Type 2 $C^{\mu(C)}a$, if $C\in\mathcal{S}$ and a is the first arrow in C.

Type 3 ab, if $a,b\in Q_1$ and ab is not a subpath of any $C\in\mathcal{S}$.

The algebra $A=KQ/I$, where I is generated by all relations of Types 1, 2, and 3, is called *the algebra defined by* $(\mathcal{S},\mu)$ and we call A an *algebra defined by cycles*. We note that some of the generators of Types 1,2, and 3 are in general redundant.

Theorem 3.1. *Let Q be a quiver, K a field, and $(\mathcal{S},\mu)$ a defining pair for Q. Let $A=KQ/I$ be the algebra defined by $(\mathcal{S},\mu)$. Then A is a symmetric special multiserial algebra.*

Proof. We begin by showing that I is an admissible ideal. Clearly, I is contained in the ideal generated by paths of length 2. Let $N=\max\{\mu(C)\ell(C)\mid C\in\mathcal{S}\}$. We claim that all paths of length $N+1$ are in I. Let p be such a path. If there are arrows a and a' such that aa' is a subpath of p and aa' is a Type 3 relation, then $p\in I$. Suppose that p contains no Type 3 relations. Then (D3) and the definition of N imply that there is a simple cycle $C\in\mathcal{S}$ and an arrow b so that either $bC^{\mu(C)}$ or $C^{\mu(C)}b$ is a subpath of p. Since $C\in\mathcal{S}$, we suppose first that $C^{\mu(C)}b$ is a subpath of p since the length of $p>N$. Then $C^{\mu(C)}b$ either is a Type 2 relation or Cb contains a Type 3 relation. Finally, suppose that $bC^{\mu(C)}$ is a subpath of p. If b is the last arrow in C, then $bC^{\mu(C)}$ is a Type 2 relation using (D1) and (D2). If b is not the last arrow in C, then ba is a Type 3 relation where a is the first arrow of C.

Next we show that A is a special multiserial algebra. Let a be an arrow. Suppose that ab is a path of length 2 in Q. By D(3) and (D4), a is in a cycle C in $\mathcal{S}$ that is unique up to cyclic permutation. Either b is the unique arrow such that ab lies in C or ab is a Type 3 relation and hence in I. Thus, there is at most one arrow b such that $ab\notin I$. Similarly, there is at most one arrow c such that $ca\notin I$ and we see that A is a special multiserial algebra.

Finally we show that A is a symmetric algebra. We let $\pi\colon KQ \to A$ denote the canonical surjection. Define $f\colon KQ \to K$ as follows. If p is a path in Q, define

$$f(p) = \begin{cases} 1, & \text{if } p = C^{\mu(C)} \text{ for some } C \in \mathcal{S} \\ 0, & \text{otherwise.} \end{cases}$$

Linearly extend f to KQ. The reader may check that f induces a linear map $\bar{f}\colon A \to K$. Let $\mathcal{B}$ be the set of paths in Q. All sums will have only a finite number of nonzero terms. Note that if $x = \sum_{p\in\mathcal{B}} \alpha_p p \in KQ$ with $\alpha_p \in K$, then $f(x) = \sum_{C\in\mathcal{S}} \alpha_{C^{\mu(C)}}$.

First we show that if $\lambda, \lambda' \in A$, then $\bar{f}(\lambda\lambda') = \bar{f}(\lambda'\lambda)$. Let $x = \sum_{p\in\mathcal{B}} \alpha_p p \in KQ$ and $y = \sum_{q\in\mathcal{B}} \beta_q q \in KQ$ such that $\pi(x) = \lambda$ and $\pi(y) = \lambda'$. Then

$$f(xy) = f((\sum_{p\in\mathcal{B}} \alpha_p p)(\sum_{q\in\mathcal{B}} \beta_q q)) = \sum_{p\in\mathcal{B}}\sum_{q\in\mathcal{B}} \alpha_p\beta_q f(pq) = \sum_{pq=C^{\mu(C)} \text{ for } C\in\mathcal{S}} \alpha_p\beta_q$$

and

$$f(yx) = f((\sum_{q\in\mathcal{B}} \beta_q q)(\sum_{p\in\mathcal{B}} \alpha_p p)) = \sum_{q\in\mathcal{B}}\sum_{p\in\mathcal{B}} \beta_q\alpha_p f(qp) = \sum_{qp=C^{\mu(C)} \text{ for } C\in\mathcal{S}} \beta_q\alpha_p$$

It follows that $f(xy) = f(yx)$ and hence $\bar{f}(\lambda\lambda') = \bar{f}(\lambda'\lambda)$.

Finally we claim that $\ker(\bar{f})$ contains no nonzero left or right ideals. Supoose that $\mathfrak{I}$ is a right ideal of A contained in the kernel of $\bar{f}$. Assume $\mathfrak{I} \neq (0)$ and let $\lambda \in \mathfrak{I}$ with $\lambda \neq 0$. Then $\lambda = \pi(\sum_{p\in\mathcal{B}} \alpha_p p)$ where $\alpha_p \in K$ and all but a finite number of $\alpha_p \neq 0$. Without loss of generality, we may assume that if $\alpha_p \neq 0$, then $p \notin I$. First suppose that there is a path $p^* \notin I$ such that $\alpha_p^* \neq 0$ and p^* is not of the form $C^{\mu(C)}$ for any $C \in \mathcal{S}$. Then there is a unique $C \in \mathcal{S}$ and path q such that $p^*q = C^{\mu(C)}$. By (D1)-(D4), if $p' \neq p^*$, then $p'q$ is not of the form $C'^{\mu(C')}$ for any $C' \in \mathcal{S}$. Hence

$$\bar{f}(\lambda\pi(q)) = \bar{f}(\pi((\sum_{p\in\mathcal{B}} \alpha_p p)q)) = f(\sum_{p\in\mathcal{B}} \alpha_p pq) = \alpha_{p^*} \neq 0.$$

But this contradicts $\mathfrak{I} \subseteq \ker(\bar{f})$ since $\lambda\pi(q) \in \mathfrak{I}$. The proof that there a no left ideals in $\ker(\bar{f})$ is similar to the proof for right ideals.

Thus we may assume that if $\alpha_p \neq 0$ then $p = C^{\mu(C)}$ for some $C \in \mathcal{S}$. Since $\lambda \neq 0$, we see there is some vertex v such that $e_v\lambda e_v \neq 0$. Let $S_v = \{C \in \mathcal{S} \mid C \text{ is a cycle at v}\}$. Then

$$0 = \bar{f}(e_v\lambda e_v) = \bar{f}(\pi(\sum_{C\in\mathcal{S}_v} \alpha_{C^{\mu(C)}} C^{\mu(C)}) = f(\sum_{C\in\mathcal{S}_v} \alpha_{C^{\mu(C)}} C^{\mu(C)}) = \sum_{C\in\mathcal{S}_v} \alpha_{C^{\mu(C)}}.$$

Choose some $C^* \in \mathcal{S}_v$. Then, using that, for $C \in \mathcal{S}_v$, $(C^{\mu(C)} - C^{*\mu(C^*)})$ is a Type 1 relation and $\sum_{C\in\mathcal{S}_v} \alpha_{C^{\mu(C)}} = 0$, we see that

$$e_v\lambda e_v = \pi(\sum_{C\in\mathcal{S}_v} \alpha_{C^{\mu(C)}} C^{\mu(C)}) - (\sum_{C\in\mathcal{S}_v} \alpha_{C^{\mu(C)}})\pi(C^{*\mu(C^*)}) =$$
$$\sum_{C\in\mathcal{S}_v} \alpha_{C^{\mu(C)}} \pi(C^{\mu(C)} - C^{*\mu(C^*)}) = 0,$$

contradicting $e_v\lambda e_v \neq 0$. This completes the proof. $\square$

The converse of the above Theorem is also true. The Theorem below is not used in the remainder of the paper and we only sketch the proof. The sketch below assumes knowledge of Brauer configuration algebras found in [GS1].

Theorem 3.2. *Let $A = KQ/I$ be an indecomposable symmetric special multiserial algebra with Jacobson radical squared nonzero, where I is an admissible ideal in KQ. Then there is a defining pair $(\mathcal{S}, \mu)$ in Q such that the algebra defined by $(\mathcal{S}, \mu)$ is isomorphic to A.*

Proof. By [GS2], we may assume that A is the Brauer configuration algebra associated to a Brauer configuration $\Gamma = (\Gamma_0, \Gamma_1, \mu, \mathfrak{o})$. Let $\mathcal{S}$ be the set of special cycles in the quiver of A. If C is a special α-cycle for some $\alpha \in \Gamma_0$, define $\mu^*(C) = \mu(\alpha)$. Using the properties of special cycles of a Brauer configuration algebra, the reader my check that $(\mathcal{S}, \mu^*)$ is a defining pair in Q. It is straightforward to see that the Brauer configuration algebra A is isomorphic to the algebra defined by $(\mathcal{S}, \mu^*)$. □

4. QUOTIENTS

For the remainder of this section, we let $A = KQ/I$ be a special multiserial algebra satisfying condition (M). We introduce two functions associated to A which play a central role. Let $\diamond$ be some element not in Q_1 and set $\mathcal{A} = Q_1 \cup \{\diamond\}$. Define $\sigma\colon Q_1 \to \mathcal{A}$ and $\tau\colon Q_1 \to \mathcal{A}$ by

$$\sigma(a) = \begin{cases} b & \text{if } ab \notin I \\ \diamond & \text{if } ab \in I \text{ for all } b \in Q_1 \end{cases}$$

and

$$\tau(a) = \begin{cases} c & \text{if } ca \notin I \\ \diamond & \text{if } ca \notin I \text{ for all } c \in Q_1 \end{cases}.$$

where $a, b, c \in Q_1$. From the definition of a special multiserial algebra, we see that these functions are well-defined. Since A is finite dimensional, one of two things occur for σ when repeatedly applied to an arrow $a \in Q_1$. Either there is a smallest positive integer m_a such that $\sigma^{m_a}(a) = \diamond$ or there is a smallest positive integer $\hat{m}_a$ such that $\sigma^{\hat{m}_a}(a) = a$. Similarly, either there is a smallest positive integer n_a such that $\tau^{n_a}(a) = \diamond$ or there is a positive integer $\hat{n}_a$ such that $\tau^{\hat{n}_a}(a) = a$.

We list some basic properties of σ and τ.

- B1 If $\sigma(a) \in Q_1$, then $\tau\sigma(a) = a$.
- B2 If $\tau(a) \in Q_1$, then $\sigma\tau(a) = a$.
- B3 For $a \in Q_1$, m_a exists if and only if n_a exists.
- B4 For $a \in Q_1$, $\hat{m}_a$ exists if and only if $\hat{n}_a$ exists.

Along with σ and τ, we need one more concept. We say a path $M = a_1 a_2 \cdots a_r$, with $a_i \in Q_1$ is (σ, τ)-*maximal* if $\sigma(a_r) = \diamond = \tau(a_1)$. Let $\mathcal{M}$ denote the set of (σ, τ)-maximal paths.

Suppose $a \in Q_1$ is such that $\hat{m}_a$ exists. Then we have a simple oriented cycle in Q, denoted C_a such that $C_a = a\sigma(a)\sigma^2(a) \cdots \sigma^{\hat{m}_a - 1}(a)$ since $\sigma^{\hat{m}_a}(a) = a$ implies that $\mathfrak{s}(C_a) = \mathfrak{t}(C_a)$. It is easy to check that C_a is a simple cycle. We let the set of such simple cycles be denoted by $\mathcal{C}$. Note that if $C \in \mathcal{C}$, then every cyclic permutation of C is in $\mathcal{C}$.

Now suppose that $a \in Q_1$ is such that m_a exists. Then

$$M_a = \tau^{n_a - 1}(a)\tau^{n_a - 2}(a) \cdots \tau(a) a \sigma(a) \cdots \sigma^{m_a - 1}(a)$$

is a (σ,τ)-maximal path in which a occurs.

The next lemma lists some basic properties of the above constructions. The proof is straightforward and left to the reader.

Lemma 4.1. *Let $A = KQ/I$ be a special multiserial algebra and let $a \in Q_1$. Then*

(1) *Either a occurs in the (σ,τ)-maximal path M_a or the simple cycle C_a but not both.*

(2) *The arrow a occurs in at most one (σ,τ)-maximal path.*

(3) *If a is an arrow in a simple cycle $C_b \in \mathcal{C}$, for some arrow b, then C_b is a cyclic permutation of C_a.*

(4) *The length of C_a, if it exists, is $\hat{m}_a = \hat{n}_a$.*

(5) *If $M \in \mathcal{M}$ is a maximal path, M has no repeated arrows.*

(6) *If $M \in \mathcal{M}$ and a is an arrow in p, then $M = M_a$.*

(7) *If a occurs in the (σ,τ)-maximal path M_a then the length of M_a is $m_a + n_a - 1$.*

We now construct a new quiver, Q^*, from Q. Set $Q_0^* = Q_0$. For each $M \in \mathcal{M}$, let a_M be an arrow from $\mathfrak{t}(M)$ to $\mathfrak{s}(M)$. Set $Q_1^* = Q_1 \cup \{a_M \mid M \in \mathcal{M}\}$. Note that Ma_M is a simple cycle in Q^* at $\mathfrak{s}(M)$. Let $\mathcal{M}^*$ denote the set of cycles in Q^* consisting of all cyclic permutations of the Ma_M, for $M \in \mathcal{M}$.

Since Q is a subquiver of Q^*, we will freely view paths and cycles in Q as paths or cycles in Q^*. Let

$$\mathcal{S} = \{C \in \mathcal{C}\} \cup \mathcal{M}^*,$$

viewed as a set of simple cycles in Q^*. Next, since I is admissble, there is a smallest positive integer N, $N \geq 2$, such that all paths of length N or larger, are in I. Define $\mu\colon \mathcal{S} \to \mathbb{Z}_{>0}$ by $\mu(C) = N$ for all $C \in \mathcal{S}$

Proposition 4.2. *Keeping the above notation, $(\mathcal{S},\mu)$ is a defining set in Q^*.*

Proof. Since $N \geq 2$ and we see that (D0) holds. Since μ is constant on $\mathcal{S}$, (D2) holds. It is immediate that (D1) holds. If a is an arrow in Q, then a occurs in some $C \in \mathcal{C}$ or $M \in \mathcal{M}$. It is easy to see that for all $M \in \mathcal{M}$, the arrow a_M occurs in some cycle in $\mathcal{M}^*$. Thus, every arrow in Q^* occurs in some cycle in $\mathcal{S}$ and (D3) holds. By Lemma 4.1, and our construction, (D4) holds and the proof is complete. □

Let $A^* = KQ^*/I^*$ be the algebra defined by $(\mathcal{S},\mu)$. By Theorem 3.1, A^* is a symmetric special multiserial algebra which we call the *symmetric special multiserial algebra associated to A.*

Theorem 4.3. *Let A be a special multiserial algebra and A^* be the symmetric special multiserial algebra associated to A defined by $(\mathcal{S},\mu)$. Then A is a quotient of A^*.*

Proof. To define $F\colon KQ^* \to KQ$ we use the universal mapping property of path algebras. That is, we define F on the vertices of Q^* so that $\{F(e_v) \mid e_v \in Q_0^*\}$ is a full set of orthogonal idempotents in KQ and define F on the arrows, so that, if

$a^*\colon v^* \to w^*$ is an arrow in Q^*, then $F(a^*)$ is a uniform element of KQ such that $F(a^*) = F(e^*_v)F(a^*)F(e^*_w)$.

Let $F(e_v)$, for all $v \in Q^*_0$, be a full set of orthogonal idempotents in KQ as defined above, $F(a_M) = 0$, for all $M \in \mathcal{M}$, and $F(a) = a$ for all the remaining arrows in Q^*. The K-algebra homomorphism F is clearly surjective. The homomorphism F will induce a surjection $\hat{F}\colon A^* \to A$ if $F(I^*) \subseteq I$ since we would have an exact commutative diagram

$$\begin{tikzcd}
0 \arrow[r] & I^* \arrow[r] \arrow[d, "F_{|I^*}"] & KQ^* \arrow[r] \arrow[d, "F"] & KQ^*/I^* \arrow[r] \arrow[d, "\hat{F}"] & 0 \\
0 \arrow[r] & I \arrow[r] & KQ \arrow[r] \arrow[d] & KQ/I \arrow[r] & 0 \\
 & & 0 & &
\end{tikzcd}$$

We prove $F(I^*) \subseteq I$ by showing that F applied the generators of I^* of Types 1,2, and 3 are in I. If C and C' are in $\mathcal{S}$ then consider $C^{\mu(C)} - C'^{\mu(C')}$. If C (or C') contains an arrow of the form a_M, for some $M \in \mathcal{M}$, then F sends $C^{\mu(C)}$ (or $C'^{\mu(C')}$) to 0 which is in I. If a_M does not occur in C (or in C') then $C^{\mu(C)}$ (or $C'^{\mu(C')}$) has length greater than or equal to N since μ has constant value N. Recalling that paths of length greater or equal to N in KQ are in I, we conclude that F applied to a Type 1 relation is in I.

A similar argument works for Type 2 relations.

Finally, suppose that ab is a Type 3 relation. If either a or b is an arrow of the form a_M, for some $M \in \mathcal{M}$, then $F(ab) = 0 \in I$. Suppose that neither a nor b is of the form a_M. Then $F(ab) = ab$. Since ab does not live on any $C \in \mathcal{S}$, we see that ab does not does not live on any $C \in \mathcal{C}$, where $\mathcal{C}$ is the set of simple cycles of A as defined at the beginning of this section, nor is ab a subpath of any M in $\mathcal{M}$. It follows that $\sigma(a) \neq b$. But then $ab \in I$ and we are done. $\square$

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Edward L. Green, Department of Mathematics, Virginia Tech, Blacksburg, VA 24061, USA

E-mail address: green@math.vt.edu

Sibylle Schroll, Department of Mathematics, University of Leicester, University Road, Leicester LE1 7RH, United Kingdom

E-mail address: schroll@le.ac.uk