

Modeling X-ray Emission Line Profiles from Massive Star Winds - A Review

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Abstract

The *Chandra* and *XMM-Newton* X-ray telescopes have led to numerous advances in the study and understanding of astrophysical X-ray sources. Particularly important has been the much increased spectral resolution of modern X-ray instrumentation. Wind-broadened emission lines have been spectroscopically resolved for many massive stars. This contribution reviews approaches to the modeling of X-ray emission line profile shapes from single stars, including smooth winds, winds with clumping, optically thin versus thick lines, and the effect of a radius-dependent photoabsorption coefficient.

Keywords: X-rays; Massive Stars; Stellar Winds; Line Profile Modeling; Stellar Mass Loss

1. Introduction

Massive stars have long been known to be X-ray sources (Cassinelli & Olson, 1979; Harnden *et al.*, 1979; Long & White, 1980; Cassinelli *et al.*, 1981). Early X-ray studies of massive stars (i.e., non-degenerate OB stars) were limited to pass-band fluxes or low-resolution spectra (e.g., Berghöfer *et al.*, 1997). Recent instrumentation with *Chandra* and *XMM-Newton* have since permitted observations of resolved broad-emission lines from several massive stars, which have represented a major forward step in studies of the X-ray properties of massive stars (e.g., Kahn *et al.*, 2001; Waldron & Cassinelli, 2001; Cassinelli *et al.*, 2001; Skinner *et al.*, 2001; Oskinova, Feldmeier, & Hamann, 2006; Waldron & Cassinelli, 2007; Güdel & Nazé, 1981; Oskinova *et al.*, 2012; Leutenegger *et al.*, 2013; Cohen *et al.*, 2014a,b).

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Winds of massive stars typically have wind terminal speeds of order 10^3 km s^{-1} . Shocks involving speeds at this level easily produce peak temperatures at several MK. At such temperatures a thermal plasma will cool primarily via emission lines (Cox & Tucker, 1969; Raymond & Smith, 1977). There are many scenarios that can lead to strong shocks in massive star winds. Some massive stars are in binary systems, and the winds of the two stars can collide to produce relatively hard and luminous X-ray emission. Another scenario involves stellar magnetism. In some massive stars, the stellar magnetic field is strong enough to deflect or even channel a portion of the wind flow. The channeling can lead to head-on collisions of counter-moving streams of plasma, leading to strong shocks and a significant X-ray luminosity. The calculation of line profile shapes for colliding wind binaries and magnetically channeled winds is not reviewed in this contribution. A review of X-ray emission from colliding winds appears in Rauw *et al.* (2015); and the influence of stellar magnetism for X-ray emission from massive stars is reviewed in ud-Doula & Nazé (2015).

This review focuses on approaches for modeling X-ray emission line profile shapes for single massive stars. Modeling of the line shape is important for extracting information about the source, such as the mass-loss rate of the wind. This paper emphasizes line profile calculations; results derived from model fitting to observed X-ray spectra of massive stars are reviewed by Oskinova (2015).

For single massive stars, the leading culprit for the production of multi-million degree gas is found in the same mechanism that propels their fast winds, namely the line-driving force (Lucy & Solomon, 1970; Castor *et al.*, 1975; Pauldrach *et al.*, 1986; Müller & Vink, 2008). This force is subject to the line deshadowing instability (LDI) that results in the formation of wind shocks (Milen, 1926; Lucy & White, 1980; Owocki *et al.*, 1988). As a result, a highly structured, supersonic wind flow develops (e.g., Dessart & Owocki, 2003), with a distribution of wind shocks capable of emitting X-rays at observed temperatures (Feldmeier *et al.*, 1997a). Modeling of the line shapes has grown more complex to match the observations. This is exciting because the data have pushed the line modeling to include greater physical realism.

Section 2 provides an overview of the evolution of X-ray emission line profile calculations. Section 2.1 begins with a description of the emissive process for the production of X-rays from single massive-star winds, followed by a description of the properties of spherical stellar winds in section 2.2. Then 2.3 details expressions to calculate line profile shapes for smooth winds. A review of the exospheric approximation is given in section 2.4 to illustrate

basic scalings, followed by 2.5 that compares effects for thin versus thick lines. The special case of a constant expansion wind is handled in 2.6. The topic of clumping is covered in 2.7. Using a linear (or, homologous) velocity law, a selection of illustrative profile calculations are provided in section 2.8. A summary and conclusions are given in section 3. Appendix A details the derivation of profile shapes for constant spherical expansion with a power-law volume filling factor. Appendix B presents a derivation for the photoabsorbing optical depth in the case that the absorbing coefficient is a power law in the wind velocity.

2. Modeling of Stellar Wind X-ray Emission Line Profiles

2.1. The Line Emissivity

X-ray line profile shape modeling begins by specifying the source geometry and the emissivity process. For geometry the winds are assumed to be spherically symmetric in time average. This assumption can accommodate the inclusion of stochastic structure in the wind, normally referred to as “clumping”.

The X-ray emission from single and non-magnetic massive star winds is normally attributed to embedded wind shocks. This “hot plasma” component at millions of Kelvin is a thermal plasma that emits a spectrum dominated by lines of highly ionized metals (e.g., [Cox & Tucker, 1969](#); [Raymond & Smith, 1977](#)). The bulk of the line photons arises from collisional excitation followed by radiative decay. Consequently, the line emissivity is a density-squared process. In addition, wind shocks are expected to display a range of temperatures as the post-shock gas undergoes cooling (e.g., [Feldmeier *et al.*, 1997b](#); [Cassinelli *et al.*, 2008](#); [Krtićka, *et al.*, 2009](#); [Gayley, 2014](#)).

The volume emissivity for a line is denoted as j_l [$\text{erg s}^{-1} \text{cm}^{-3}$] and is given by

$$j_l(T, E_l) = \Lambda_l(T, E_l) (n_i n_e)_X, \quad (1)$$

where the “ l ” subscript identifies a particular line, $\Lambda_l(T, E_l)$ [$\text{erg s}^{-1} \text{cm}^3$] is the cooling function for a line at energy E_l , and n_i and n_e are number densities for the ions and electrons in the X-ray emitting gas (hence the “X” subscript). Here Λ_l is frequency (or energy, or wavelength) integrated over the line profile; its value depends on the temperature, T , of the plasma. Note that another form of the emissivity is the volume emissivity per unit solid angle, η_l . For isotropic emission one has that $j_l = 4\pi \eta_l$.

For an optically thin plasma in which there is no line transfer and no photoabsorption of the X-rays, the line luminosity generated in a differential volume element is

$$dL_l(T, E_l) = j_l(T, E_l) dV \quad (2)$$

$$= \Lambda_l(T, E_l) (n_i n_e)_X dV \quad (3)$$

$$= \Lambda_l(T, E_l) dEM_X, \quad (4)$$

where EM_X is the emission measure of the X-ray emitting gas.

The total luminosity generated from a multi-temperature plasma in a particular line becomes

$$L_l = \int \Lambda_l(T, E_l) \frac{dEM_X}{dT} dT, \quad (5)$$

where dEM_X/dT signifies the differential emission measure and represents the relative amounts of plasma at different temperatures.

Although equation (5) is correct, the integration over differential emission measure is not normally how line profile shapes are modelled. Instead, most approaches for the line modeling tend to start with equation (3). The properties of the stellar wind density and temperature distribution are specified. Taking account of the wind velocity distribution, the contribution by a differential volume element to the line profile depends on the volume's Doppler shift with respect to the observer.

2.2. The Stellar Wind Model

For a spherically symmetric wind, the density of the gas, ρ , is determined by the continuity equation

$$\rho(r) = \frac{\dot{M}}{4\pi r^2 v(r)}, \quad (6)$$

where \dot{M} is the mass-loss rate, r is the radius in the wind, and $v(r)$ is the wind speed. The wind speed starts with a low initial value of v_0 at the wind base, that is frequently taken to be the stellar radius R_* . The flow achieves an asymptotic terminal speed, v_∞ , for $r \gg R_*$.

The wind velocity profile is often parametrized in terms of a ‘‘beta law’’ (Pauldrach *et al.*, 1986), with

$$v(r) = v_\infty \left(1 - \frac{bR_*}{r}\right)^\beta, \quad (7)$$

where $0 < b < 1$ is a parameter that sets the initial wind speed, with $v_0 = v_\infty(1 - b)^\beta$.

Frequently, the presumption is that the hot plasma is a minority component (in terms of relative mass) of the wind flow; however, this may not always be the case. Some have considered “coronal” wind models (Lucy, 2012) to explain the so-called “weak-wind stars”. Huenemoerder *et al.* (2012) have reported evidence for a case in which the X-ray emitting plasma is the dominant component of the wind. However, in the majority of stars with highly resolved X-ray lines, the X-ray emitting plasma is the minority component; this review focuses on these cases.

Recall from the previous section that the total emission measure for X-ray production is

$$EM_X = \int_{\text{wind}} (n_i n_e)_X dV. \quad (8)$$

It is common to express the X-ray emissivity in terms of the wind density. This is accomplished through the introduction of a volume filling factor, f_V (e.g., Owocki & Cohen, 1999; Ignace *et al.*, 2000). The total emission measure available in the wind is defined to be

$$EM_w = \int_{\text{wind}} (n_i n_e)_w dV. \quad (9)$$

A wind-averaged volume filling factor, in terms of emission measure, is $\langle f_V \rangle = EM_X/EM_w$. Thus, $f_V \leq 1$. (Note that slightly different definitions of f_V appear in the literature.)

The volume filling factor can be treated as a radius-dependent parameter (e.g., Hillier *et al.*, 1993; Ignace, 2001; Owocki & Cohen, 2001; Runacres & Owocki, 2002). Now the emission measure available for X-rays is

$$EM_X = \int_{\text{wind}} f_V(r) (n_i n_e)_w dV. \quad (10)$$

Allowing for a radius dependence of f_V introduces a free parameter to alter emission profile shapes when fitting observed lines.

2.3. Formalism for X-ray Line Profile Modeling

The X-ray emission lines from massive star winds are expected to be optically thin. However, it is possible that in some rare cases, a line could be optically thick (Ignace & Gayley, 2002; Leutenegger *et al.*, 2007). To handle stellar wind lines of general optical depth, the Sobolev approximation is a useful technique for calculating the transfer of line radiation (Sobolev,

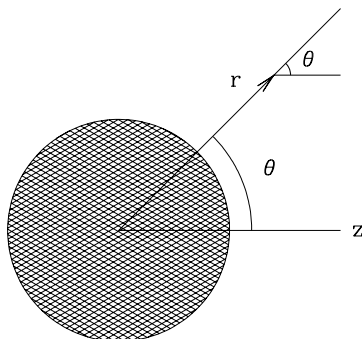


Figure 1: Schematic for the adopted coordinate system. The observer is located right along the z -axis. The vector indicates a point in the wind at (r, θ) , with θ the polar angle from the observer axis. The impact parameter of this point is $p = r \sin \theta$ (not shown).

1960; Rybicki & Hummer, 1978). The Sobolev approach simplifies the line transfer when the flow speeds are large¹ compared to the thermal speed.

In dealing with high-speed stellar wind outflows, an understanding of the line profile shape is facilitated through the use of “isovelocity zones”. Isovelocity zones represent spatial “sectors” through the emitting volume, with each zone contributing its emission to a different velocity shift in the line profile. Spatially, the vector velocity of the flow throughout one of these zones is not uniform; however, all points within a zone share the same Doppler-shifted velocity with respect to the observer. The shape of the resolved spectral line is related to the velocity distribution of the wind, since $v(r)$ sets the spatial configuration of the isovelocity zones.

To develop a prescription for line profile calculation, consider a spherically symmetric wind with a velocity beta law as given in equation (7). The observer view of this source is axially symmetric. Figure 1 shows a ray

¹The Sobolev approximation relates to velocity gradients. However, expansion and/or rotation of flow about a star involves both physical gradients of the flow as well as geometrical line-of-sight gradients. Consequently, large bulk flow speeds as compared to the thermal speed is often a sufficient condition for the Sobolev approach to be valid.

from the observer to a point in the wind at (r, θ) or equivalently (p, z) . The normalized line-of-sight (“los”) Doppler velocity shift of the flow at that point is given by

$$w_z = -\mu w(r), \quad (11)$$

where $w(r) = v(r)/v_\infty$ and $\mu = \cos \theta$. An isovelocity zone is a surface of revolution about the observer’s axis to the star center, for which w_z is a constant, as given by the condition that $\mu = -w_z/w(r)$.

To calculate the line profile, the thin line case is considered first, with modification for the effects of line optical depth following. Guided by [Owocki & Cohen \(2001\)](#), the emission line profile is given by

$$\frac{dL_l}{dw_z} = \int 4\pi \eta(r) \delta(w_{\text{obs}} - w_z) dV, \quad (12)$$

where the δ -function signifies application of the Sobolev approximation, and w_{obs} is an observed normalized Doppler shift in the line profile. The integration is carried out over the zone for which $w_{\text{obs}} = w_z = -\mu w(r)$.

The delta function can be expressed as

$$\delta(w_{\text{obs}} - w_z) = \delta[\mu - \mu(p, z)] \left| \frac{dw_z}{d\mu} \right|^{-1}, \quad (13)$$

with

$$\frac{dw_z}{d\mu} = -w(r). \quad (14)$$

The emissivity function is

$$\eta(r) = \eta_0 \frac{\Lambda_l(T, E_l)}{\Lambda_0} \left[\frac{\rho(r)}{\rho_0} \right]^2, \quad (15)$$

with Λ_0 a scaling factor for the cooling function, and

$$\rho_0 = \frac{\dot{M}}{4\pi R_*^2 v_\infty}. \quad (16)$$

The emissivity scaling factor is then

$$\eta_0 = \Lambda_0 \rho_0^2. \quad (17)$$

The expression for the line profile becomes:

$$\frac{dL_l}{dw_z} = 4\pi \eta_0 R_*^3 \int_{w_z} f_V(r) g[T(r)] \left[\frac{R_*^4}{r^4 w^2(r)} \right] \left[\frac{1}{w(r)} \right] \frac{2\pi r^2 dr}{R_*^3}, \quad (18)$$

where again the integral is taken over a particular isovelocity zone, with account of stellar occultation implied. Note that dL_l/dw_z can easily be converted to a specific luminosity, in appropriate units for any given dataset, through the use of the Doppler formula. The combination of factors leading the integral has units of luminosity. The function $g[T(r)]$ allows for temperature variations with radius. In principle, the factor g can be subsumed into the volume filling factor, that would then be interpreted as a volume filling factor specific to a given ionic species and line transition (c.f., [Owocki & Cohen, 2001](#)). It can be useful to introduce such a line-specific volume filling factor, with

$$f_l(r) = f_V(r) g[T(r)], \quad (19)$$

which will be employed later.

In evaluating the integration, a change of variable is introduced with $u = R_*/r$. The line profile shape is now given by

$$\frac{dL_l}{dw_z} = L_0 \int_0^{u(w_z)} \frac{f_V(u) g(u)}{w^3(u)} du, \quad (20)$$

where

$$L_0(E_l) = 8\pi^2 \eta_0(E_l) R_*^3. \quad (21)$$

The lower limit of zero to the integral of equation (20) corresponds to $r \rightarrow \infty$. The upper limit depends on the Doppler shift. The upper limit is the stellar radius, which is $u = 1$, for the observer-facing side of the star where $w_z \leq 0$. However, on the far side of the star where $w_z < 0$, a portion of the emission from an isovelocity zone is occulted by the star. The upper limit to the integral must take this into account. Emission only reaches the observer for $\mu \geq \mu_{\text{occ}} = \sqrt{1 - u_{\text{occ}}^2(w_z)}$.

It is also possible to allow for an ‘‘onset radius’’ below which there is no X-ray emitting gas. Let this radius be r_X , and let $u_X = R_*/r_X$. This provides additional flexibility in the line profile modeling to account for where wind shocks become strong enough to produce sufficiently high-temperature gas to emit X-rays. So, the upper limit to equation (20) can be generalized as u_{max} , where for blueshifts, u_{max} is the minimum of u_X and 1, and for redshifts, u_{max} is the minimum of u_X and $u_{\text{occ}}(w_z)$.

Equation (20) does not include the effect of line optical depth. One can think of the integrand of that equation as a photon generation rate per unit interval in u . When the line is thin, photon escape is locally isotropic. When the line is thick to resonance scattering, the escape of X-ray photons can be non-isotropic. Following [Leutenegger *et al.* \(2007\)](#), the direction-dependent optical depth that governs photon escape is

$$\tau_{S,\mu} = \frac{\tau_{S,0}}{1 + \sigma \mu^2}, \quad (22)$$

where the ‘‘S’’ subscript is used to signify the Sobolev line optical depth, $\tau_{S,0}$ is a characteristic line optical depth for the line of interest, and

$$\sigma = \frac{d \ln v}{d \ln r} - 1 \quad (23)$$

is the ‘‘anisotropy’’ factor. The escape of line photons is isotropic if $\sigma = 0$, which occurs for a linear velocity law with $v(r) \propto r$ (e.g., [Ignace & Hendry, 2000](#)). Equation (22) shows that the line optical depth has a directional dependence on μ , and the strength of that dependence is set by the velocity gradient term in σ .

The Sobolev escape probability \mathcal{P}_S represents the probability of a line photon escaping into a certain direction. This parameter depends on the Sobolev optical depth, with

$$\mathcal{P}_S(r, \mu) = \frac{1 - e^{-\tau_{S,\mu}}}{\tau_{S,\mu}}. \quad (24)$$

If the line is optically thin, $\mathcal{P}_S \rightarrow 1$, regardless of the value of σ . [Leutenegger *et al.* \(2007\)](#) introduced an angle-averaged quantity $\bar{\mathcal{P}}_S$ as

$$\bar{\mathcal{P}}_S(r) = \frac{1}{2} \int_{-1}^{+1} \bar{\mathcal{P}}(r, \mu) d\mu. \quad (25)$$

The ratio $\mathcal{P}_S/\bar{\mathcal{P}}_S$ becomes a correction factor to the case of pure optically thin emission.

The new integral for the line profile shape that accounts for the possibility of anisotropic photon escape is:

$$\frac{dL_l}{dw_z} = L_0 \int_0^{u(w_z)} \frac{f_V(u) g(u)}{w^3(u)} \left[\frac{\mathcal{P}_S(u, \mu)}{\bar{\mathcal{P}}_S(u)} \right] du. \quad (26)$$

Note that $\mu = \mu(u)$ by virtue of the shape of the isovelocity surface. Equation (26) reduces to equation (20) when $\tau_{S,0} \lesssim 1$.

Equation (26) still lacks the possibility of photoabsorption of X-ray photons by the wind. Photoabsorption by interstellar gas reduces the observed number of source line photons received at the Earth, but it does not alter the line profile shape. Photoabsorption by the wind can alter the line profile shape because the amount of absorption depends on the column density of the wind between the point of emission and the observer. The column density increases monotonically from the near side toward the back side along a given ray; consequently, more severe photoabsorption is to be expected for rearward isovelocity zones (i.e., with $w_z > 0$) than for forward ones (i.e., with $w_z < 0$).

Let $\kappa(r)$ be the photoabsorptive coefficient, then the optical depth τ for wind attenuation of X-rays will be given by

$$\tau = \int \kappa(r) \rho(r) dz, \quad (27)$$

where τ is the optical depth along a ray to some point in the wind (see Fig. 1). Then the effect of photoabsorption for calculation of the line profile can be included in equation (26) as follows:

$$\frac{dL_l}{dw_z} = L_0 \int_0^{u(w_z)} \frac{f_V(u) g(u)}{w^3(u)} \left[\frac{\mathcal{P}_S(u, \mu)}{\overline{\mathcal{P}_S(u)}} \right] e^{-\tau} du, \quad (28)$$

For completeness the total line luminosity is

$$L_l = \int_{-1}^{+1} \frac{dL_l}{dw_z} dw_z. \quad (29)$$

At this point equation (28) is a fairly sophisticated expression for the wind emission line profile shape under the following assumptions: spherical symmetry, smooth wind, monotonically increasing wind velocity (not necessarily a beta law), and the Sobolev approximation. Clumping of the emitting gas is implied by virtue of using a volume filling factor; however, it is assumed that the integral representation remains valid. Clumping by the absorbing component generally requires modification of equation (28) (see Sect. 2.7); however, if the clumps are individually optically thin, then the expression remains valid.

2.4. The Exospheric Approximation

Before exploring applications of equation (28), it is useful first to review the “exospheric approximation” (e.g., Owocki & Cohen, 1999; Ignace & Oskinova, 1999), which represents a quick and easy approach to understanding

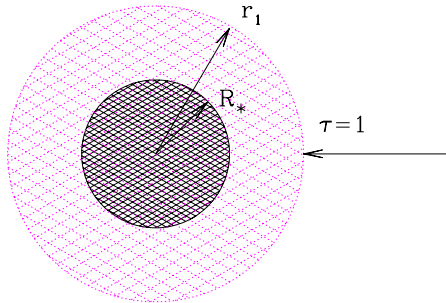


Figure 2: Geometry for the exospheric approximation. The star is indicated as the central sphere with radius R_* . The location of optical unity ($\tau = 1$) along the line-of-sight to the star center for a distant observer to the right is indicated by radius r_1 . That radius is then taken to demarcate a sphere, shaded in magenta, for which X-rays can be seen only exterior to its boundary.

some of the principle factors that influence the X-ray line emission from stellar winds. This approximation is based on a severe simplification, and yields inaccurate results in detail (e.g., [Leutenegger *et al.*, 2010](#)); still, it is intuitive in nature and can yield overall useful scalings, and so it is worth review.

The exospheric approximation is a core-halo approach in which the radius r_1 , at which $\tau = 1$ in photoabsorption along the los from the star to the observer, demarcates a division between inner radii where no X-ray photons escape the wind versus outer radii where all X-ray photons escape the wind (see Fig. 2). Using equation (27), this location is determined by the condition

$$\tau = 1 = \int_{r_1}^{\infty} \kappa(r) \rho(r) dr. \quad (30)$$

This expression is an implicit relation for r_1 , and its solution requires that $\kappa(r)$ be provided.

Frequently a constant absorption coefficient with $\kappa(r) = \kappa_0$ has been adopted (e.g., see [Leutenegger *et al.*, 2010](#)). Using this case for illustration, the relation for the radius of optical unity becomes

$$1 = \tau_0 \int_0^{u_1} \frac{du}{w(u)}, \quad (31)$$

where the integral is reexpressed in terms of the inverse radius, u , and $u_1 = R_*/r_1$. An optical depth scaling coefficient is also introduced as $\tau_0 = \kappa_0 \rho_0 R_*$. Note that it has been common in the literature to use τ_* as the optical depth through the wind to the stellar radius (e.g., [Owocki & Cohen, 2001](#)). For $\kappa(r)$ a constant, the relation between τ_* and τ_0 is

$$\tau_* = \tau_0 \int_0^1 \frac{du}{w(u)}. \quad (32)$$

With constant expansion with $w(u) = 1$, the solution for the radius of optical depth unity is $r_1 = \tau_0 R_*$. Bear in mind that $\tau_0 = \tau(\lambda)$, so the extent of $r_1(\lambda)$ may vary from one line to another. The case of $r_1(\lambda) < R_*$ means that the wind is largely transparent to X-rays at that wavelength. The photoabsorptive absorption coefficient is comprised in large part by bound-free opacity from H and He and from K-shell ionization of metals. The overall energy trend can be crudely approximated as a power law with $\kappa(E) \sim E^{-2.6}$ ([Cassinelli & Olson, 1979](#)).

The radius of optical unity can be derived analytically for a beta velocity law. With $\beta \neq 1$, the solution for r_1 is

$$r_1 = \frac{b R_*}{1 - \left[1 + \frac{b(\beta-1)}{\tau_0}\right]^{-1/(\beta-1)}}. \quad (33)$$

For $\beta = 1$, the solution becomes

$$r_1 = \frac{b R_*}{1 - e^{-b/\tau_0}}. \quad (34)$$

Note that when $\tau_0 \gg 1$, both of these expressions reduce to $r_1 \propto \tau_0$, which is the solution for a constant expansion wind. This arises because the optical depth integral is an inward evaluation, not an outward one; so $\tau = 1$ occurs far from the acceleration zone of the flow where, in fact, the wind is in constant expansion.

In the exospheric approximation, the total line emission is

$$L_l = \int_{r_1}^{\infty} 4\pi \eta(r) dV \quad (35)$$

where for simplicity an optically thin line is assumed. The lower limit of the integral takes account of the wind absorption of X-ray photons. For a

constant expansion wind in which $f_V = f_0$ and $g = g_0$ are constants, the solution to the integral becomes

$$L_l = L_0 u_1 = \frac{L_0(E_l)}{\tau_0(E_l)}, \quad (36)$$

where L_0 is given in equation (21). More broadly, this result represents an overall scaling for the luminosity of an optically thin line that forms in a wind that is optically thick to X-rays. The implication is that a distribution of normalized line luminosities L_l/L_0 should scale inversely with τ_0 . Allowance for $v(r)$, $f_V(r)$, and/or $g(r)$ will alter the result in detail, but overall one still expects a trend of normalized line luminosities with E_l . The variation of the optical depth with energy therefore provides a diagnostic of the wind mass-loss rate, if the absorbing opacity is known (c.f., [Cohen *et al.*, 2010](#)).

Now consider the case of a beta velocity law. The total line luminosity is given by

$$L_l = L_0 f_0 g_0 \int_0^{u_1} \frac{du}{w^2(u)} \quad (37)$$

$$= \frac{L_0 f_0 g_0}{b(2\beta - 1)} \left[\frac{1}{(1 - bu_1)^{2\beta-1}} - 1 \right]. \quad (38)$$

with a reminder that $u_1 = u_1(E_l)$. Again, the ratio L_l/L_0 depends on the energy of the line in question.

But what are the implications for the line profile shape? The radius r_1 is treated like the stellar photosphere in this core-halo approximation in terms of (a) being a lower boundary for the integration that determines the line emission and (b) acting in the form of stellar occultation. The isovelocity zones are taken to terminate at the r_1 -sphere. In the core-halo approach, the r_1 -sphere leads to occultation for some of the emission with redshifted velocities, which leads to line asymmetry.

To be explicit, consider the case of a constant expansion wind, still with f_V and g as constants in the wind. An optically thin line would produce a flat-top profile if occultation could be ignored. In the exospheric approximation, the flat-top morphology remains for blueshifted velocities, whereas the redshifted portion takes on the shape $\sqrt{1 - w_z^2}$. However, when the wind is optically thick to X-rays, a proper treatment of the photoabsorption reveals that generally no portion of the line is flat-topped in shape.

2.5. Thin and Thick Lines for Smooth Winds

Optically thin lines were explored in [Owocki & Cohen \(2001\)](#), who considered different velocity laws and filling factors. That paper also considered different onset radii, r_X , for the hot plasma. This parameter allows for an offset of the X-ray emitting gas from the wind base, although recent work indicates that wind shocks can develop quite close to the photosphere (for more details, see [Sundqvist & Owocki, 2013](#)).

As a general rule, even without photoabsorption, thin X-ray emission lines are asymmetric owing to stellar occultation. The profiles tend to become more symmetric in appearance as the onset radius for the production of the X-ray emitting gas is made larger. This corresponds to smaller values of $u(w_z)$ for the upper limit to the integral in equation (28), and therefore reduces the influence of stellar occultation. Increasing the onset radius also tends to produce flat-top portions near line center.

The effect of photoabsorption is to enhance the asymmetry of the line. In addition to reducing the overall line luminosity, photoabsorption by the wind shifts location of peak emission away from line center toward blueshifted velocities, although other parameters also influence where the peak occurs. In changing the line shape, the line width (i.e., FWHM) is also altered.

When a line becomes optically thick to resonance scattering, the effect of anisotropic escape from a Sobolev zone can make the line more symmetric, an effect that is in opposition to the influence of photoabsorption. For optically thick resonance line scattering (i.e., $\tau_{S,0} \gg 1$), [Leutenegger et al. \(2007\)](#) noted that the direction-dependent correction for photon escape is given by

$$\mathcal{P}_S/\bar{\mathcal{P}}_S = \frac{1 + \sigma(u)\mu^2}{1 + \sigma(u)/3}, \quad (39)$$

where σ was given in equation (23). In this equation $\mu = \cos \theta$, with θ as in Figure 1. The σ parameter is a somewhat complicated function of radius. However, photons appearing at line center correspond to $\mu = 0$. Photon escape toward the observer is enhanced where $\sigma < 0$, which occurs at radii close to the star. In this way line optical depth can act to reduce the line asymmetry caused by the photoabsorption.

2.6. The Special Case of a Constant Expansion Wind

It can be fruitful to consider limiting behavior to gain an understanding of the influences of the different model parameters. Here the asymptotic behavior of thin and thick lines are described for a wind with a constant speed

of spherical expansion. Aside from insights gained from asymptotic behavior, applications may be found for winds that have a high optical depth in photoabsorption. The Wolf-Rayet (WR) stars are the evolved counterparts of the most massive stars (e.g., [Langer, 2012](#)). These stars are known to have high mass-loss rates and dense winds. Their X-rays are thought to be strongly absorbed (e.g., [Pollock, 1987](#); [Skinner et al., 2002](#); [Oskinova et al., 2003, 2012](#)). At some wavelengths the X-rays will emerge predominantly from the terminal speed flow of the wind.

2.6.1. The Limiting Behavior for Thin Lines

Following [MacFarlane et al. \(1991\)](#), [Ignace \(2001\)](#) showed that for a wind that expands from a star at constant speed, the solution for the X-ray emission line profile is analytic with

$$\frac{dL_l}{dw_z} = L_0 \times \begin{cases} \frac{1 - \exp[-\tau_0 s_1(w_z)]}{\tau_0 s_1(w_z)} & \text{for } w_z \leq 0 \\ \frac{1 - \exp[-\tau_0 s_2(w_z)]}{\tau_0 s_1(w_z)} & \text{for } w_z > 0, \end{cases} \quad (40)$$

where the difference between the redshifted and blueshifted velocities arises from considerations of stellar occultation. For $w_z \leq 0$, the fraction is an escape probability that depends on τ_0 and velocity shift. For $w_z > 0$, the fraction is similar to an escape probability, but at low optical depth, the fraction recovers the effect of occultation. The two functions s_1 and s_2 are given by:

$$s_1(w_z) = \frac{\theta}{\sin \theta} = \frac{\cos^{-1}(-w_z)}{\sqrt{1 - w_z^2}} \quad (41)$$

$$s_2(w_z) = \theta = \cos^{-1}(-w_z) \quad (42)$$

In the limit of large photoabsorptive optical depth, the solution reduces to

$$\frac{dL_l}{dw_z} = \frac{L_0(E_l)/\tau_0(E_l)}{s_1(w_z)}, \quad (43)$$

where the energy dependence is made explicit. This profile shape has peak emission at $w_z = -1$ and declines smoothly to zero at $w_z = +1$.

[Ignace \(2001\)](#) showed that a volume filling factor with a power-law dependence on radius gives a semi-analytic result. Recalling $f_l(u) = f_V(u)g(u)$ from equation (19), a power-law dependence is introduced for the line-specific filling factor, with $f_l = f_{l,0} u^q$, for $f_{l,0}$ a constant and $q > -1$. The line profile shape becomes

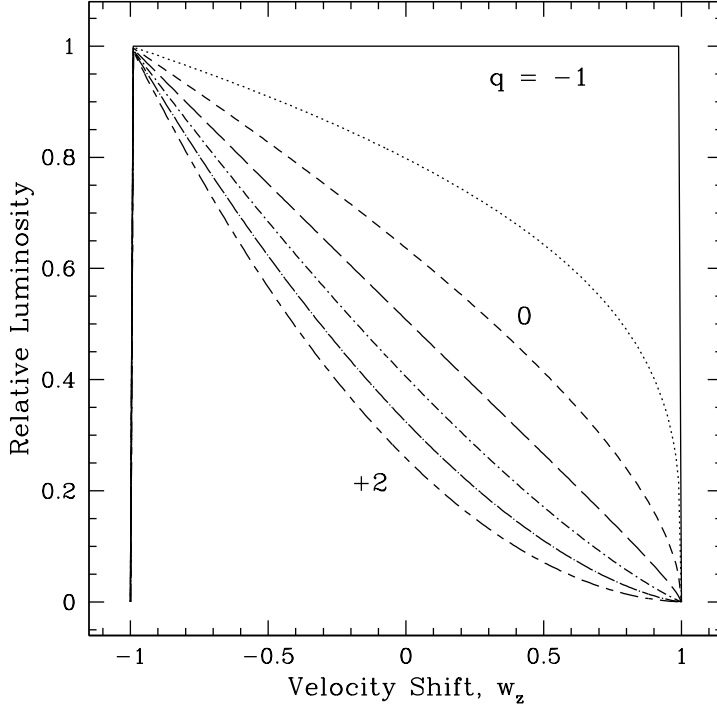


Figure 3: Explicitly thin emission lines in the regime of large photoabsorbing optical depth for a smooth wind. The different curves are for different line-specific volume filling-factor power laws (see eq. [19]), with $f_l \propto u^q$. The value of q ranges from -1 (solid curve) to $+2$ (long dash dotted curve) in intervals of 0.5 . The case of constant filling factor ($q = 0$) is the short dashed curve. Note that all of the line profiles are normalized to have a peak value of unity.

$$\frac{dL_l}{dw_z} = L_0 \frac{\Gamma(1 + q, u_{\max} \tau_0 s_1)}{(\tau_0 s_1)^{1+q}} \quad (44)$$

where u_{\max} corresponds to the lower radius bound to the X-ray emission, and takes account of stellar occultation (which depends on w_z).

For $\tau_0 \gg 1$, the Gamma function varies only weakly with w_z (see [Appendix A.](#)), so that there is little error in taking $u_{\max} = 1$. Consequently in this limit, the line profile shape is well described by

$$\frac{dL_l}{dw_z} \propto s_1^{-(1+q)}. \quad (45)$$

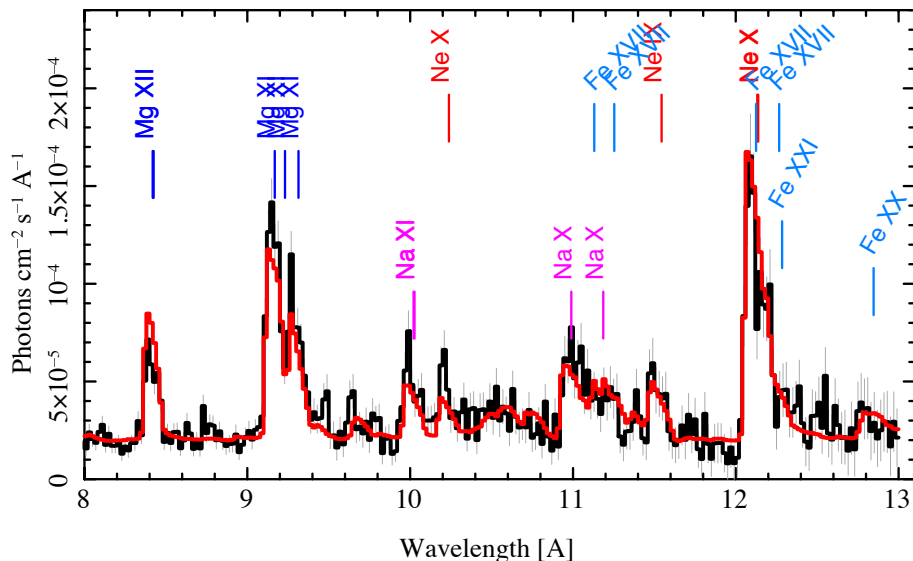


Figure 4: A portion of a *Chandra* HETG spectrum of the nitrogen-rich WR star WR 6 from 8 to 13 Å. Black is for the spectral data. Strong lines are labeled. Red is a model fit assuming a constant expansion wind, as suggested by the sharp blue wings of the emission lines. Note that some lines are blends. (Figure courtesy of D. Huenemoerder.)

Several example line profiles using equation (45) are displayed in Figure 3, ranging from $q = -1$ to $q = +2$ in intervals of 0.5. The case of $q = 0$ is for f_l a constant (shown as short dashed). The case² of $q = -1$ actually recovers a flat-top profile shape (shown as solid) across the entire line. The value of q serves mainly to alter the steepness of decline for the line profile as it moves from peak emission at extreme blueshift to no emission at extreme redshift. As a result, the FWHM of the line profile declines monotonically with increasing q for $w(u) = 1$.

An interesting special case for applications of the preceding results can be found among the WR stars. WR winds can be so thick to X-rays, that observed lines form in the terminal-speed flow. The only example of high-resolution X-ray lines from a (putatively) single WR star is WR 6 (Huenemoerder *et al.*, 2015). When the line emission emerges from large radii, line-profile fitting no longer offers constraints on \dot{M} for the wind, aside from

²Formally, $q = -1$ leads to a line luminosity that diverges, which is unphysical. The example has only heuristic value in showing that a flat-top profile is a limiting case.

requiring that $\tau_0 \gg 1$. On the other hand, ambiguities about the choice of wind velocity law (e.g., the value of β) or the onset radius, r_X , are no longer a concern.

Figure 4 shows an application of equation (45) to a *Chandra* HETG spectrum of WR 6 (based on [Huenemoerder et al., 2015](#)). The figure shows a portion of the spectrum, with data in black and model line profiles in red. Steep blue wings to the line profiles do indeed suggest that the lines form in the terminal-speed flow. However, different lines require different values of q .

One may interpret the different values of q for different lines as indicating a temperature distribution in the hot plasma at large radius. In this consideration a range in FWHMs among lines does not translate to lines being formed in different velocity regimes, since all the model profiles assume $v(r) = v_\infty$. Indeed, allowing for the line width to be a free parameter of the model, [Huenemoerder et al. \(2015\)](#) find that fits to the observed lines are consistent with all the resolved lines being formed in a constant-speed outflow.

Figure 5 illustrates the contribution function for the total line luminosity, dL_l/dr . This refers to how shells of width dr contribute to the total emergent line luminosity, L_l . The calculation accounts for wind absorption. The curves are for the constant expansion case, with each curve for a different value of q as labeled. The plot is a log-log plot against inverse radius, u . In this example a wind optical depth of $\tau_0 = 10$ is chosen. Note that optical depth unity along the line-of-sight occurs at $u_1 = 1/\tau_0 = 0.1$, as indicated in the figure. With the exception of $q = -0.5$, the most luminous shell for each curve actually occurs below the optical depth unity location.

Note that the results described above assume that $\kappa(r) = \kappa_0$. This is understood not always to be the case. ([Leutenegger et al., 2010](#); [Hervé et al., 2012, 2013](#)). However, it is expected that κ approaches a constant value at large radii, and so the asymptotic results should hold if photoabsorption is sufficiently strong.

2.6.2. The Limiting Behavior for Thick Lines

[Ignace & Gayley \(2002\)](#) derived the influence of resonance line scattering on X-ray emission lines in the limit of a constant expansion wind. For strong wind absorption, the profile shape for a thick line can be obtained from the result for a thin line as multiplied by an additional factor that depends on w_z . Taking the solution equation (43) for large τ_0 , and multiplying by $\tau_{S,0}(1 - w_z^2)$ gives the profile shape for a thick line ([Ignace & Gayley, 2002](#)):

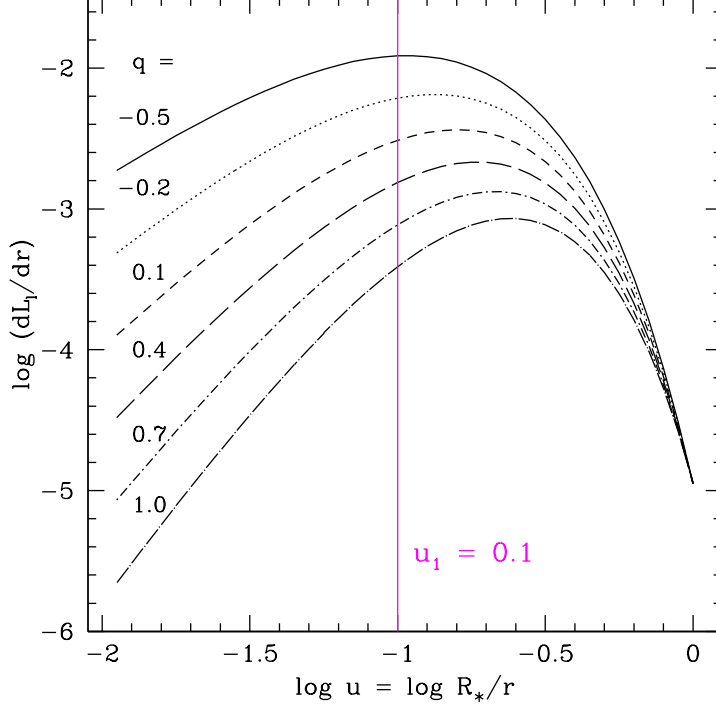


Figure 5: The luminosity contribution of individual shells to the total line luminosity is dL_l/dr . For the case of constant expansion, the function is shown for different q values against $u = R_*/r$ in a logarithmic plot. These curves are for $\tau_0 = 10$; the location of optical depth unity u_1 is indicated by the vertical magenta line.

$$\frac{dL_l}{dw_z} \propto \frac{\tau_{S,0}}{\tau_0} \times \frac{(1 - w_z^2)}{s_1(w_z)}. \quad (46)$$

This result can be extended to the case of a power law in f_l , all other assumptions being the same. Using equation (45) and multiplying by the factor as above, the thick line result for a constant expansion wind with large τ_0 is

$$\frac{dL_l}{dw_z} \propto \frac{(1 - w_z^2)}{s_1^{1+q}} \quad (47)$$

Example line profiles for different values of q are shown in Figure 5 in the same way as displayed in Figure 3. For thin lines, $q = -1$ recovers a flat-top

profile, as if no wind absorption was present. For thick lines the case $q = -1$ recovers a downward-opening parabolic profile of the form $(1 - w_z^2)$, which is the result for a thick line with $\tau_0 = 0$.

2.7. Line Profiles from Winds with Clumping

There is considerable evidence for departures from a smooth wind, indicating that massive star winds are structured. This evidence derives from discrete absorption components (or DACs) in the absorption troughs of UV P Cygni resonance lines (e.g., [Prinja & Howarth, 1986](#); [Massa *et al.*, 1995](#)), optical emission line variability ([Lépine *et al.*, 1996](#); [Lépine & Moffat, 2008](#)), polarimetric variability ([Robert *et al.*, 1989](#); [Brown *et al.*, 1995](#); [Rodrigues & Magalhães, 1995](#)), and a relative absence of X-ray variability ([Cassinelli & Swank, 1983](#); [Nazé *et al.*, 2013](#)). The wind structure consists of a stochastic component and possibly a globally-ordered component. For the stochastic structure, a natural explanation is the intrinsic instability associated with the line-driving force ([Castor *et al.*, 1975](#)). A popular candidate for globally ordered structure is found in co-rotating interaction regions, or CIRs (e.g., [Mullan, 1986](#); [Cranmer & Owocki, 1996](#); [Dessart, 2004](#); [St-Louis *et al.*, 2009](#); [Ignace *et al.*, 2015](#)).

“Clumping” is the term that is associated with the ubiquitous stochastic component of structured massive star winds. There have been several simulations of radial clumping effects in 1-dimensional (1D) flow. (e.g., [Owocki *et al.*, 1988](#); [Feldmeier *et al.*, 1997a](#)). Shocks form to produce a hot plasma component of the wind. Being 1D, the structures take the form of spherical shells. Many researchers use the 1D results as motivation for 3D clumping scenarios. The structured flow seen in 1D are taken to occur independently in a large number of sectors about the star. Although fully 3D simulations have not been reported, 2D simulations have been explored ([Dessart & Owocki, 2003](#)). In 2D, clumped structures take the form of rings, and the results support a picture of highly fractured and evolving wind structure with stellar latitude.

How does clumping influence the calculation of X-ray line profiles? Certainly, it adds complexity to the evaluation, depending on the nature of the adopted assumptions. Inspired by [Feldmeier *et al.* \(1997a,b\)](#), [Feldmeier *et al.* \(2003\)](#) developed a model for a “fragmented” wind in which the cool-wind absorbing component took the form of an ensemble of circular “pancake-shaped” structures that propagate radially from the star. These compressed fragments lead to geometric avenues for increasing the escape of X-ray photons from the far hemisphere of the star relative to the smooth wind case. At high photoabsorptive optical depths, model emission line

profiles are more symmetric as compared to the smooth wind case. Within the context of these fragmented wind models, [Oskinova *et al.* \(2004\)](#) used a monte carlo approach to confirm the semi-analytic results, and considered a variety of clump structures in a parameter study of line profile shapes.

One important consideration is making clear what is clumped. The winds are normally considered to have 2 components: the X-ray emitting gas and the cooler gas that can give rise to photoabsorption of X-rays. The volume filling factor, f_V , that has been used up to this point refers to the hot plasma component. In a smooth wind, this hot component is considered to be uniformly “intermingled” with the cooler component; details about the spatial relationship between the two components is often not specified beyond characterization in terms of f_V . However, in [Feldmeier *et al.* \(2003\)](#) and subsequent work, much attention is given to the clumping of the cool component. Consequently, a new volume filling factor must be introduced, f_{abs} , for the cool, absorbing component as distinct from the X-ray emitting gas.

Similar in spirit to [Feldmeier *et al.* \(2003\)](#), [Owocki & Cohen \(2006\)](#) introduced a formalism based on “porosity”. Their approach is a time-averaging of the stochastic flow, and is described in terms of a characteristic porosity length-scale. The two methods of [Feldmeier *et al.* \(2003\)](#) and [Owocki & Cohen \(2006\)](#) are generally commensurate. Whereas [Feldmeier *et al.* \(2003\)](#) used “fragmentation frequency” to parametrize the degree of clumping in the wind, [Owocki & Cohen \(2006\)](#) used the porosity length.

At this point it is useful to establish terminology. Macroclumping and porosity have been used somewhat interchangeably. Here I suggest that macroclumping explicitly refers to the approach that treats clumps as discretized structures. Such is the case of [Oskinova *et al.* \(2004\)](#), who modeled line profile shapes using monte carlo simulations. Porosity is then macroclumping when the radiative transfer through the clumped medium can be described with integral relations. In the macroclumping approach, a clumpy wind can produce time-variable line shapes that are not necessarily smooth, owing to discrete structures that evolve through the flow. The porosity approach yields smooth line shapes that do not vary in time. Finally, microclumping is the limit in which all wind clumps are optically thin so that radiative transfer effects through clumps can be ignored.

[Oskinova, Feldmeier, & Hamann \(2006\)](#) and [Sundqvist *et al.* \(2012\)](#) provide a summary of the porosity formalism with application to X-ray line profiles. Again, the porosity is in reference to the absorbing medium through which the source X-rays must escape to be observed. An effective absorption coefficient is introduced ([Feldmeier *et al.*, 2003](#); [Oskinova *et al.*, 2004](#)):

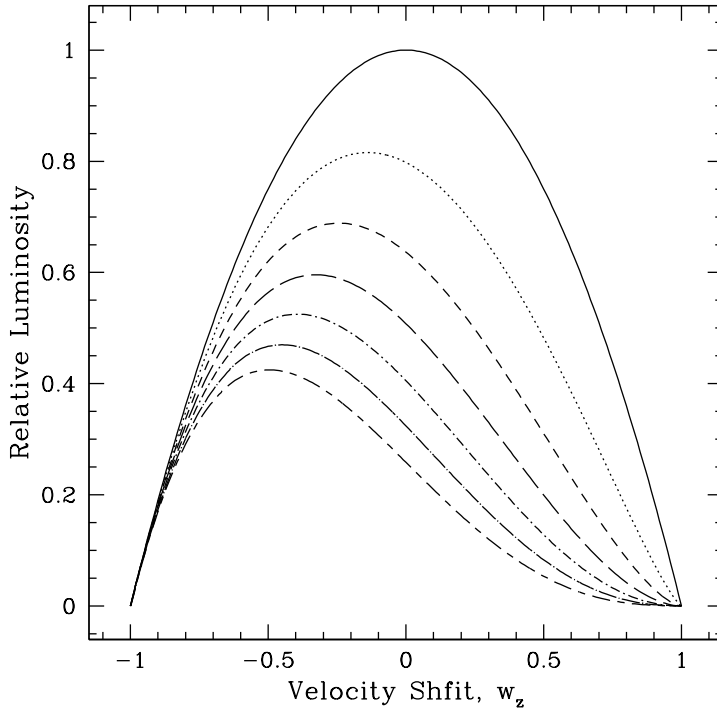


Figure 6: Model line profiles like those of Fig. 5, but now for optically thick lines instead of thin ones. The profiles are again for different q values from -1 (highest) to $+2$ (lowest). The profiles are relative to the case of $q = -1$.

$$\kappa_{\text{eff}}\rho = \chi_{\text{eff}} = n_{\text{cl}} A_{\text{cl}} \mathcal{P}_{\text{cl}}. \quad (48)$$

The opacity χ is noted here since it is often used in the literature. The number density of clumps is n_{cl} , the cross-section of a clump is A_{cl} , and the probability that a photon will be absorbed by a clump is \mathcal{P}_{cl} . This probability can be expressed as

$$\mathcal{P}_{\text{cl}} = 1 - e^{-\tau_{\text{cl}}}, \quad (49)$$

for τ_{cl} the optical depth of a photoabsorbing clump. The clump optical depth is given by

$$\tau_{\text{cl}} = \kappa \bar{\rho} h, \quad (50)$$

where $\bar{\rho}$ is the average density, and h is the porosity length with

$$h = \frac{1}{n_{\text{cl}} A_{\text{cl}}}. \quad (51)$$

Then the relation between the effective and average absorption coefficients is

$$(\kappa\rho)_{\text{eff}} = \left(\frac{1 - e^{-\tau_{\text{cl}}}}{\tau_{\text{cl}}} \right) \kappa\bar{\rho}. \quad (52)$$

Here the factor in parentheses involving the clump optical depth is an escape probability from the clump (however, see [Sundqvist *et al.*, 2012](#), for further discussion on “bridging laws” for the effective opacity). The factor reduces to unity for thin clumps – the limit of microclumping – and becomes $1/\tau_{\text{cl}}$ for thick clumps. When the absorbing clumps are optically thin, the smooth wind result for the line profile calculation is recovered. It is only when clumps become optically thick that porosity significantly influences the line shape, an influence that in detail depends on the clump geometry.

As an example, [Figure 7](#) shows model X-ray line profiles in the limiting case of constant spherical expansion using the porosity formalism. Generally, the porosity length can vary with location in the wind. With $v(r) = v_{\infty}$, the porosity length is a constant as a function of r , with a value given by h_{∞} (e.g., [Sundqvist *et al.*, 2012](#)). Here the subscript “ ∞ ” is used in analogy with the wind velocity; v_{∞} is the asymptotic wind speed, and h_{∞} is the asymptotic porosity length. The profiles of [Figure 7](#) were calculated for $\tau_0 = 1$ and optically thin lines (i.e., $\tau_{S,0} \ll 1$) assuming “pancake”-like clump fragments. The fragment geometry and the constant wind speed give a porosity length $h(r, \mu) = h_{\infty}/|\mu|$, which can also be expressed as $h = h_{\infty}/|w_z|$.

In [Figure 7](#), the red curve is the analytic solution for a smooth wind from [Ignace \(2001\)](#) at $\tau_0 = 1$. The blue curve corresponds to the smooth wind case with zero photoabsorption (i.e., $\tau_0 = 0$). Stellar occultation is included. The four other curves in black are for $h_{\infty}/R_* = 0.01, 0.1, 1.0$, and 10, with the smallest value corresponding to the solid curve (lying nearly atop of the red profile), and the largest value to the long-dashed curve (nearest to the blue profile). The figure illustrates how at fixed τ_0 , large porosity lengths make the wind more transparent to X-rays, as the absorbing opacity becomes more spatially concentrated.

2.8. The Special Case of a Linear Velocity Law

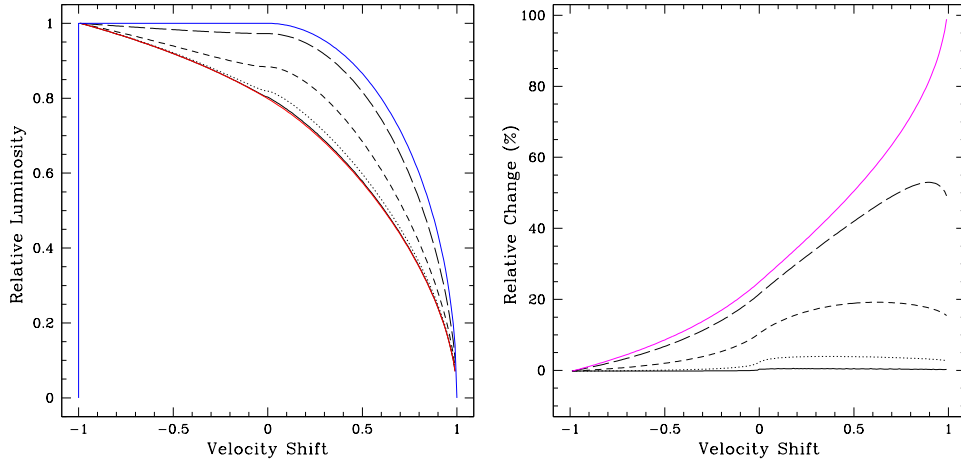


Figure 7: *Left*: An illustration of porosity effects on line profile shapes. The profiles are normalized to a peak value of unity. Red is for a smooth wind with $\tau_0 = 1$; blue is $\tau_0 = 0$. In black from solid to long dash are profiles with $h_\infty = 0.01, 0.1, 1.0,$ and $10,$ respectively, all with $\tau_0 = 1$. *Right*: Shown is the percent difference between normalized line profiles appearing in the left panel, with the red curve as the reference profile. Magenta (top) is the percent difference between the blue curve and the red one. The black curves are for winds with porosity corresponding to the lines in the left panel. Moving from small to large porosity lengths leads to a wind that is increasingly optically thin to photoabsorption for fixed τ_0 .

Owocki & Cohen (2001) presented a parameter study for line profile shapes for smooth winds. As previously noted, it is standard to adopt a beta velocity law for the wind velocity. The rise in speed, from an inner value of v_0 to an asymptotic value of v_∞ , is approximately linear with radius for a portion of the inner wind. As a way of illustrating the influences of different model parameters, example line profiles are presented here using a linear velocity, with $v(r) = kr$,

One motivation for a linear velocity law is that the photoabsorbing optical depth has an analytic solution for $\kappa(r)$ a constant. A second motivation is the interesting property that the escape of photons is always isotropic for a linear velocity, even when the line is optically thick. This means that $\mathcal{P}_S/\bar{\mathcal{P}}_S = 1$ for all values of $\tau_{S,0}$.

Conceptually, the emission profile contribution from a geometrically thin spherical shell is flat-topped in shape, regardless of optical depth for a linear law. Ignoring stellar occultation, the FWHM of the shell's contribution is

$2v(r)$. A radius-dependent volume filling factor serves to modify the emission amplitude of this flat-top contribution. By contrast photoabsorption serves to make the emission contribution deviate from a flat-top shape. Stellar occultation blocks a portion of the blueshifted side of the shell, in a way that depends on the shell's radius.

It is useful now to introduce a characteristic speed for normalizing the Doppler velocity shifts. Let r_c be the radius where a characteristic velocity v_c is achieved. Then $v_c = k r_c$. Stellar winds have terminal speeds, whereas the linear law formally has no maximum speed value. So, $v_c = v_\infty$ is chosen for convenience, even though the wind speed never achieves a terminal value. However, the r^{-3} decline in density ensures that the large radius wind makes relatively little contribution to the emission profile in the examples that will be shown. It is also useful to introduce a minimum speed, $v_0 = k R_*$. So, the normalized wind speed is $w = v(r)/v_c$, for which $w = 1$ occurs where $v = v_c$, and $w_0 = v_0/v_c$.

The normalized Doppler velocity shift is $w_z = -v(r)\mu/v_c = -z/r_c = -\mu r/r_c$. Thus, w_z is a constant for z a constant, and isovelocity zones are therefore parallel planes that are oriented orthogonal to the observer los.

Assuming $\kappa(r) = \kappa_0$ throughout the wind, and using equation (27), the photoabsorbing optical depth to any point in an isovelocity zone has an analytic solution. Again, $v \propto r$ implies $\rho \propto r^{-3}$, and the integral for the optical depth along a ray of impact p is

$$\tau(r, w_z) = \tau_0 \int_{z(w_z)}^{\infty} \frac{R_*^2 dz}{r^3}.$$

As before, the optical depth scaling $\tau_0 = \kappa_0 \rho_0 R_*$, and $r^2 = p^2 + z^2$. Using a change of variable with $z = p/\tan \theta$, the integral becomes

$$\tau(r, w_z) = \tau_0 \frac{R_*^2}{p^2} \int_0^\theta \sin \theta' d\theta'.$$

Changing to the inverse radius u , and noting that $p = r \sin \theta$, the solution for optical depth becomes:

$$\begin{aligned} \tau(r, w_z) &= \tau_0 u^2 \left(\frac{1 - \cos \theta}{\sin^2 \theta} \right), \\ &= \tau_0 \left(\frac{u^2}{1 - w_z u/u_c} \right), \end{aligned} \tag{53}$$

where $u_c = R_*/r_c$, and

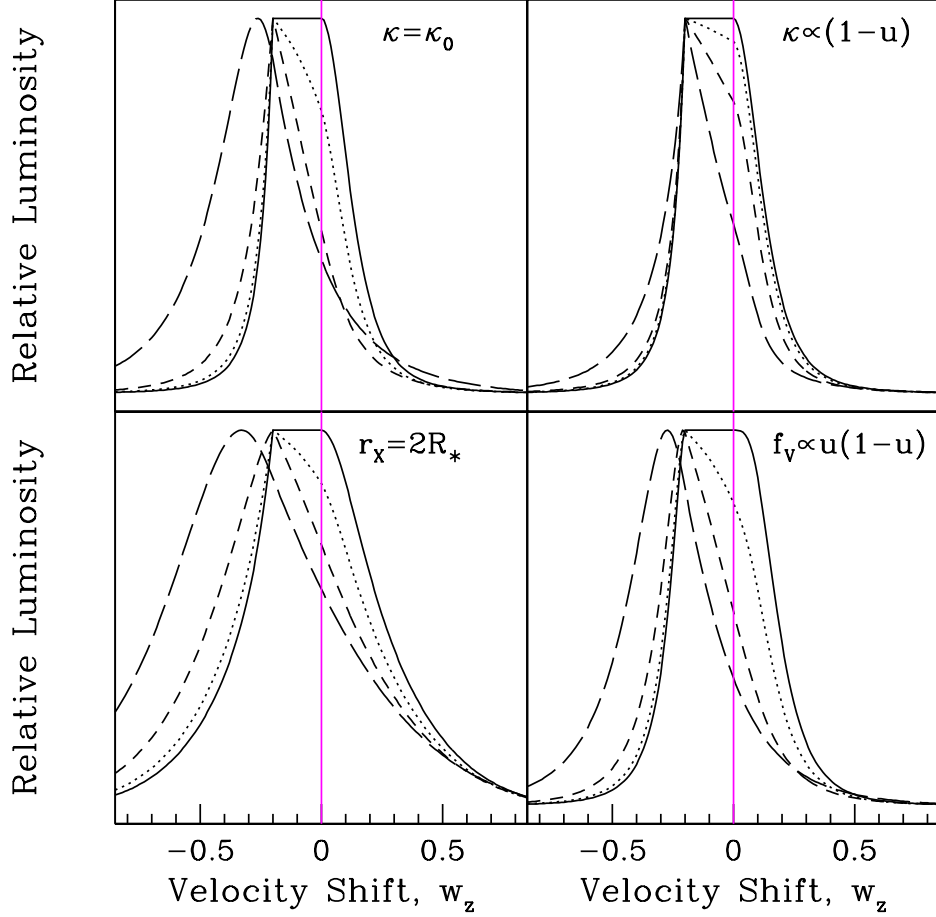


Figure 8: Example emission line profile shapes using $v(r) \propto r$, with each profile normalized to peak emission. Each panel shows four model profiles for $\tau_0 = 0$ (solid), 1 (dotted), 4 (short dash) and 14 (long dash). Upper left is for a constant photoabsorbing opacity; upper right is for one that varies with radius (see text). Lower panels are for constant absorption coefficient. Lower left is for $r_X = 1.4R_*$, and lower right is for a radius-dependent filling factor (see text).

$$\cos \theta = -w_z u / u_c. \quad (54)$$

However, the wind opacity can be influenced by the radial dependence of the ionization in the wind, such as the recombining of ionized He ([Hervé](#)

et al., 2012). To illustrate such effects for the parameter study, $\kappa(r) = \kappa_0(1 - u)$ is chosen³. In this case the resultant optical depth is more complicated, but still analytic. Following the steps that led to equation (53), the optical depth with the above $\kappa(r)$ is

$$\tau(r, w_z) = \tau_0 \left[u^2 \left(\frac{1 - \cos \theta}{\sin^2 \theta} \right) - \frac{u^3}{2} \left(\frac{\theta - \sin \theta \cos \theta}{\sin^3 \theta} \right) \right]. \quad (55)$$

The volume filling factor is another parameter that is commonly allowed to vary. The examples used so far in this article have been power laws for f_V . Here a new form is chosen, with $f_V \propto u(1 - u)$. This version achieves a maximum at $r = 2R_*$. Based on wind simulations, it is reasonable that measures for the wind structure can achieve peak values at intermediate radii (e.g., Runacres & Owocki, 2002). For this example the functional form and location of the maximum are arbitrary and used for illustrative purposes only. Finally, line profiles are calculated with an onset radius, $r_X \geq R_*$. Figure 8 shows model line profiles for different combinations of $\kappa(r)$, $f_V(r)$, and r_X . Line optical depth is not a consideration since the emissivity is always isotropic for a linear velocity law. In all of these examples, $w_0 = 0.2$ is used.

The upper left panel of the figure shows line profiles with a constant absorption coefficient, $\kappa = \kappa_0$, using equation (53). The optical depth coefficient values are $\tau_0 = 0$ (solid), 1 (dotted), 4 (short dashed), and 14 (long dashed). Upper right has the same values of τ_0 , but for $\kappa(r) \propto (1 - u)$ and using equation (55). For a given value of τ_0 , lowering the inner absorption coefficient increases the relative prominence of the inner wind as making the dominant contribution to the line emission. Note that in both of the upper panels, $r_X = R_*$ and f_V is a constant.

At lower left in Figure 8, the profiles are for $r_X = 2R_*$, constant filling factor, constant absorption coefficient, and the same values of τ_0 as in the upper panels. Lower right shows the variable filling factor described above, now with $r_X = R_*$ restored, also a constant absorption coefficient, and again the same four values of τ_0 .

As expected, increasing the optical depth alters the line shape, by shifting peak emission blueward, and changing the line width. Other parameters have an influence on the shape as well. For profiles in the upper right

³Note that if $\kappa \propto w(u)$, the wind photoabsorption optical depth actually reduces to the case of a constant expansion flow, as in Ignace (2001), although the emissivity function and the isovelocity surfaces still depend on $w(u)$. A consideration of $\kappa \propto w^m$ for beta laws is given in Appendix B.

panel, where the opacity starts low and increases toward a constant at large radius, the line shape is still significantly altered, although interestingly the blueward shift of the line emission peak is much less pronounced.

It should be noted that the chosen value of w_0 is rather large, at 20% of v_c . For a beta law, the value of w_0 is typically much smaller (more like 0.03). In Figure 8, the relatively high value of w_0 leads to a central blueshifted portion of the line profile that is flat-topped for low τ_0 . The relatively large value of w_0 serves to emphasize the influence that stellar occultation can have.

3. Summary and Conclusions

Modeling X-ray emission lines from single, spherically symmetric, smooth, massive star winds requires specification of the velocity structure for the hot plasma, the photoabsorbing opacity with radius, and the temperature structure. The velocity structure determines the density structure, and since the emissivity scales as ρ^2 , the density sets the overall amplitude for the line emission throughout the wind. A line volume filling factor can be included as a free parameter to match observed line shapes. Physically, the filling factor is motivated by a picture of embedded wind shocks.

Inclusion of a structured wind, as opposed to a smooth one, is certainly justified given the vast amount of multi-wavelength evidence for clumping. The influence of wind clumping for modeling of X-ray lines has been addressed in several papers (Feldmeier *et al.*, 2003; Oskinova *et al.*, 2004; Oskinova, Feldmeier, & Hamann, 2006; Owocki & Cohen, 2006; Sundqvist *et al.*, 2012).

The papers that deal with macroclumping or porosity effects usually treat the wind as having two components: the cool wind and the hot plasma. The cool wind is treated as clumped, and the hot plasma is effectively smoothly distributed in most applications. If the hot plasma is not smoothly distributed, then specification of its spatial distribution in relation to the cool, absorbing clumps is required for line profile calculations to proceed (e.g., as in Feldmeier *et al.*, 2003; Oskinova *et al.*, 2004).

Cassinelli *et al.* (2008) adopted a different approach to modeling X-ray lines from a structured wind flow. They used the idea of a wind consisting of two cool components: dense clumps and an interclump medium (c.f., Zsargó *et al.*, 2008; Sundqvist *et al.*, 2010; Šurlan *et al.*, 2012). If the two components have a velocity difference, then a bow shock develops around the cool clumps to produce yet a third component, called the “clump bow shock”. For a sufficiently large velocity difference, the post-shock gas of this

third component will be hot enough to emit X-rays. The properties of an isolated clump bow shock were investigated in 2D hydrodynamic simulations. Assuming adiabatic cooling, the simulations led to a prediction for the differential emission measure arising from a single bow shock.

[Ignace *et al.* \(2012\)](#) developed a phenomenological model that “peppered” a wind flow with clump bow-shocks to calculate X-ray emission line profiles from the ensemble. Those results represent an extension for a considered in [Feldmeier *et al.* \(2003\)](#), who modeled flattened shock fragments with spatially correlated emitting and absorbing components (their Sect. 4.4 on “natal fragment absorption”). The line profile shapes showed features that are inconsistent with observations. By contrast the case of clump bow-shocks indicate that the spatial correlation of cool and hot components can in principle produce reasonable emission line profiles. However, the simulations of [Cassinelli *et al.* \(2008\)](#) did not evolve a clump through the flow. Instead, [Ignace *et al.* \(2012\)](#) made use of beta velocity laws to impose a velocity difference between the clumps and interclump gas. Future modeling should address the viability of significant velocity differences for producing hot gas, and should include radiative cooling of the post-shock gas to better match conditions in some massive star winds.

It is worth commenting on the nature of the data to which line profile modeling is applied. Good-quality resolved spectra for massive star winds require typical exposures of many hours with current instrumentation. Such times are comparable to, or longer than, the characteristic wind flow timescale of R_*/v_∞ . Thus, resolved X-ray line profiles are not usually reflective of a snapshot of the wind flow; instead, the flow structure will have evolved as the spectral data are accumulated. In fact, spectra may be obtained in multiple exposures that can be widely separated in time. In order to achieve better signal-to-noise, the separate exposures are combined. The end result is a measured spectrum that is a time average of the variable wind-structure. This suggests that it is reasonable to use smooth wind models, or clumped wind models that assume sphericity in time average, when fitting observed spectral lines.

Ultimately, an observed X-ray line profile shape contains information that can be used to constrain the properties of the bulk wind and the hot plasma component. A number of parameters are used in fitting model line profiles to observed ones, such as the onset radius for the hot plasma (r_X in this review), the volume filling factor $f_V(r)$, the temperature distribution $g(r)$, the wind absorbing coefficient $\kappa(r)$, and possibly the relevance of resonance scattering effects in rare cases. A strong motivation for line profile fitting has been the possibility of measuring wind mass-loss rates (e.g., [MacFarlane *et al.*](#),

1991; Kramer, Cohen, & Owocki, 2003; Oskinova, Feldmeier, & Hamann, 2006). There are many papers that address \dot{M} determinations from line fitting, with recent examples including Leutenegger *et al.* (2013); Cohen *et al.* (2014a); Rauw & Nazé (2015); Shenar *et al.* (2015). A review of these and other results from spectral modeling and line profile fitting are reviewed in a separate contribution to this special issue of the journal (Oskinova, 2015).

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Appendix A. Profile Shapes for Constant Expansion and Constant Photoabsorption Coefficient with Power-Law Filling Factors

For a power-law filling factor and $\kappa(r) = \kappa_0$, a constant, the emission profile shape for a thin line from a smooth wind in constant expansion is analytic if stellar occultation is ignored. The derivation begins with equation (28) by setting $w = 1$ and taking $f_l = f_V g = f_{l,0} u^q$. Also, the bracketed factor for resonance scattering effects will be unity for a thin line. Finally, the upper limit to the integral becomes 1.

The integral to be solved is then

$$\frac{dL}{dw_z} = L_0 \int_0^1 u^q e^{-\tau(\theta)u} du, \quad (\text{A.1})$$

for θ as in Figure 1. The solution for integer q is

$$\frac{dL}{dw_z} = L_0 \frac{q!}{\tau^{1+q}} \left\{ 1 - e^{-\tau} \sum_{k=0}^q \frac{\tau^k}{k!} \right\} \quad (\text{A.2})$$

where

$$\tau = \tau_0 \left(\frac{\theta}{\sin \theta} \right) = \tau_0 s_1(w_z), \quad (\text{A.3})$$

and s_1 is given in equation (41). With κ a constant, the optical depth coefficient $\tau_0 = \kappa_0 \rho_0 R_*$ is also the total absorbing optical depth to the

photosphere along a radial (i.e., $\tau_0 = \tau_*$). Note that if $q = 0$, the optical depth factors reduce to the escape probability $(1 - e^{-\tau})/\tau$.

There are two comments to make regarding this result. The first is that ignoring stellar occultation is reasonable if τ_0 is of order unity or larger (i.e., the inner wind is not optically thin to X-rays). The second, and more important, is that if $\tau_0 \gg 1$, it is evident that the summation term inside the curly brackets tends to zero, for all q , because of the exponential factor. In this case the solution reduces to

$$\frac{dL}{dw_z} = L_0 \frac{q!}{\tau^{1+q}}. \quad (\text{A.4})$$

To obtain corresponding results for optically thick lines (i.e., $\tau_{S,0} \gg 1$), the preceding solutions are to be multiplied by $(1 - w_z^2)$.

Appendix B. Considerations of a Radius-Dependent Opacity with $\kappa \propto w^m$

The evaluation of $\kappa(r)$ for purposes of computing photoabsorption of X-rays throughout a wind flow can depend on the ionization of He, as already noted. The trend is for κ to increase outwardly from the star, eventually to achieve an asymptotic constant value at large radius (e.g., [Hervé *et al.*, 2012](#)).

This section presents a convenient parametrization for exploring the effects of radius-dependent absorption coefficient on line profile shapes. This parametrization is not meant to reproduce, in detail, the output from numerical radiative transfer calculations of winds, such as PoWR⁴ or CMFGEN⁵. Instead, the goal is to characterize the gross trend of $\kappa(r)$ to obtain an analytic form for the wind absorbing optical depth.

Using equations 6 and (27), the optical depth to an arbitrary point in the wind is

$$\tau(u, \mu) = \tau_0 \int u^2 \frac{\gamma(r)}{w(r)} \frac{dz}{R_*}, \quad (\text{B.1})$$

where $u = R_*/r$. The integral is along a ray of fixed impact parameter p . The parameter $\gamma(r)$ allows for the radius-dependence of $\kappa(r)$, with $\kappa(r) = \kappa_\infty \gamma(r)$ for κ_∞ an asymptotic value (which may be wavelength dependent).

⁴www.astro.physik.uni-potsdam.de/wrh/PoWR/powrgrid1.html

⁵kookaburra.phyast.pitt.edu/hillier/web/CMFGEN.htm

The optical depth coefficient is then $\tau_0 = \kappa_\infty \rho_0 R_*$. The expression can be recast in terms of angle θ (see Fig. 1), with $z = p/\tan \theta$, to give

$$\tau(r, \mu) = \tau_0 \frac{R_*}{p} \int_0^\theta \frac{\gamma(u)}{w(u)} d\theta', \quad (\text{B.2})$$

where the prime indicates a variable of integration, Note that $p = r \sin \theta$, $\mu = \cos \theta$, and $u = u(\theta')$.

The wind velocity law is commonly expressed as a beta velocity law: $w = (1 - bu)^\beta$, with b a parameter that sets the inner wind speed, so that $w_0 = (1 - b)^\beta$ at $u = 1$. Choosing $\gamma = w^m$ is a way to mimic the gross trend for $\kappa(r)$. This choice allows for analytic solutions for τ under certain conditions.

The integration for optical depth becomes

$$\tau(r, \mu) = \tau_0 \frac{R_*}{p} \int_0^\theta w^{m-1} d\theta' \quad (\text{B.3})$$

$$= \tau_0 \frac{R_*}{p} \int_0^\theta (1 - bu)^{\beta(m-1)} d\theta'. \quad (\text{B.4})$$

The integral is analytic when the product $\beta(m - 1)$ is a positive integer.

For $K = \beta(m - 1)$ an integer, and using a change of variable $u = R_*/r = R_* \sin \theta'/p$, the integral becomes

$$\tau(r, \mu) = \tau_0 \frac{R_*}{p} \int_0^\theta \left(1 - b \frac{R_*}{p} \sin \theta'\right)^K d\theta'. \quad (\text{B.5})$$

With integer K , the parenthetical can be expanded to be of the form $\sum a_k \sin^k \theta'$, for k from 0 to K , and a_k are constant coefficients of the integration. Each term then gives an integral of the form

$$p^{-(1+k)} \int_0^\theta \sin^k \theta' d\theta, \quad (\text{B.6})$$

which can be individually evaluated to obtain the solution for τ .

It is useful to consider a characteristic radius r_κ , at which the absorption coefficient $\kappa \propto w^m$ achieves half of its asymptotic value. This radius can be expressed in terms of the product βm :

$$r_\kappa = \frac{2^{1/\beta m}}{2^{1/\beta m} - 1} b R_*. \quad (\text{B.7})$$

A common choice for modeling massive star winds is $\beta = 1$. With β fixed, m determines the radial extent over which κ varies significantly. For example, $r_\kappa/bR_\kappa = 2, 3.4,$ and 4.8 for $m = 1, 2,$ and 3 . Following are solutions for τ at these three values of m . In each case, after solving for the integration, the substitution $p = R_* \sin \theta/u$ is used to obtain τ in terms of the inverse radius and polar angle.

With $m = 1$ ($K = 0$), the optical depth is

$$\tau = \tau_0 u \left(\frac{\theta}{\sin \theta} \right), \quad (\text{B.8})$$

which, although the velocity follows a $\beta = 1$ law, is the same result as for a constant expansion wind. For $m = 2$ ($K = 1$), the solution is

$$\tau = \tau_0 u \left[\left(\frac{\theta}{\sin \theta} \right) - b u \left(\frac{1 - \cos \theta}{\sin^2 \theta} \right) \right]. \quad (\text{B.9})$$

Finally, the case of $m = 3$ ($K = 2$) gives

$$\tau = \tau_0 u \left[\left(\frac{\theta}{\sin \theta} \right) - 2b u \left(\frac{1 - \cos \theta}{\sin^2 \theta} \right) + \frac{b^2 u^2}{4} \left(\frac{2\theta - \sin 2\theta}{\sin^3 \theta} \right) \right]. \quad (\text{B.10})$$

Note that the results derived in this Appendix are different from expressions given in eqs. (53) and (55). Those expressions are for a linear velocity law with $v \propto r$, whereas a beta velocity law is considered in this section.

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