

# Dynamic cancellation of a cosmological constant and approach to the Minkowski vacuum

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## Abstract

The  $q$ -theory approach to the cosmological constant problem is reconsidered. The new observation is that the effective classical  $q$ -theory gets modified due to the backreaction of particle production by spacetime curvature, a well-established quantum effect. Also, an arbitrary cosmological constant can be added to the energy density  $\epsilon(q)$  of the action, in order to represent the effects from zero-point energies and phase transitions. The resulting dynamical equations of a spatially-flat Friedmann–Robertson–Walker universe are then found to give a steady approach to the Minkowski vacuum, with attractor behavior for a finite domain of initial boundary conditions on the fields. The approach to the Minkowski vacuum is slow and gives rise to an inflationary behavior of the particle horizon.

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## I. INTRODUCTION

Several years ago, we have proposed a particular approach to the cosmological constant problem [1], whose motivation relies on thermodynamics and Lorentz invariance and which goes under the name of  $q$ -theory [2]. The basic idea of  $q$ -theory is to give the proper *macroscopic* description of the Lorentz-invariant quantum vacuum where the gravitational effects of a (Planck-scale) cosmological constant  $\Lambda$  have been cancelled dynamically by appropriate *microscopic* degrees of freedom. In general, there are one or more of these vacuum variables (denoted by  $q$ , with or without additional suffixes) to characterize the thermodynamics of this *static* physical system in equilibrium. Several realizations of  $q$ -theory have been given, but the most elegant is the one with  $q$  arising from a four-form field strength  $F$  (details and references are given below). The outstanding issue is the *dynamics*, namely, how the cosmological constant  $\Lambda$  is cancelled dynamically and the equilibrium state is approached.

In a follow-up paper [3], we have established the dynamic relaxation of the vacuum energy density to zero, *provided* the chemical potential  $\mu$  has the equilibrium value  $\mu_0$  corresponding to the Minkowski vacuum. But, then, the cosmological constant problem is replaced by another problem [4], namely, why does  $\mu$  have the “right” value  $\mu_0$ .

Here, we discuss how quantum effects can modify the classical  $q$ -theory and give rise to the decay of the vacuum energy density (i.e., decay of the effective chemical potential). Related work on vacuum energy decay has been presented in Refs. [5–11], but the feedback on  $q$ -theory has not been considered in detail.

Throughout, we use natural units with  $c = 1$  and  $\hbar = 1$ , unless otherwise stated.

## II. REALIZATION OF CLASSICAL $Q$ -THEORY

Start by neglecting the quantum-dissipative exchange between vacuum and matter. Then, the dynamics is described by the following classical action [3]:

$$S = - \int_{\mathbb{R}^4} d^4x \sqrt{-g} \left( \frac{R}{16\pi G(q)} + \epsilon(q) + \mathcal{L}^M(q, \psi) \right), \quad (2.1a)$$

$$q^2 \equiv -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu}, \quad F_{\kappa\lambda\mu\nu} \equiv \nabla_{[\kappa} A_{\lambda\mu\nu]}, \quad (2.1b)$$

$$F_{\kappa\lambda\mu\nu} = q\sqrt{-g} \epsilon_{\kappa\lambda\mu\nu}, \quad F^{\kappa\lambda\mu\nu} = q e^{\kappa\lambda\mu\nu} / \sqrt{-g}, \quad (2.1c)$$

where Eqs. (2.1b) and (2.1c) give a particular realization of the vacuum  $q$ -field in terms of the 4-form field strength  $F$  from a three-form gauge field  $A$  [12–19]. Furthermore, in (2.1b), the symbol  $\nabla_\mu$  denotes the covariant derivative and the square bracket around spacetime indices stands for complete anti-symmetrization. In (2.1c),  $\epsilon_{\kappa\lambda\mu\nu}$  is the Levi-Civita symbol.

In the action (2.1a),  $\mathcal{L}^M(q, \psi)$  is the Lagrange density of the standard-model matter fields  $\psi$ . The parameters of the matter action depend, in principle, on the vacuum variable  $q$ . Here, we neglect this  $q$  dependence of the standard-model parameters,  $\mathcal{L}^M = \mathcal{L}^M(\psi)$ . But, for the moment, we allow for a  $q$  dependence of the gravitational coupling,  $G = G(q)$ .

The variation of the action (2.1a) over the three-form gauge field  $A$  gives the generalized Maxwell equations for the four-form  $F$  field,

$$\nabla_\nu \left( \sqrt{-g} \frac{F^{\kappa\lambda\mu\nu}}{q} \left[ \frac{d\epsilon(q)}{dq} + \frac{R}{16\pi} \frac{dG^{-1}(q)}{dq} \right] \right) = 0. \quad (2.2)$$

In the spatially-flat ( $k = 0$ ) Friedmann–Robertson–Walker (FRW) universe this can be written as

$$\partial_t \left( \frac{d\epsilon(q)}{dq} - \frac{3}{8\pi} [\partial_t H + 2H^2] \frac{dG^{-1}(q)}{dq} \right) = 0, \quad (2.3)$$

for Ricci scalar  $R = -6(\partial_t H + 2H^2)$  according to our curvature conventions [3]. Solving (2.3) gives the integration constant  $\mu$ ,

$$\frac{d\epsilon(q)}{dq} - \frac{3}{8\pi} [\partial_t H + 2H^2] \frac{dG^{-1}(q)}{dq} = \mu. \quad (2.4)$$

Based on the thermodynamic discussion of Ref. [2], the integration constant  $\mu$  may be called the “chemical potential,” where the chemical potential  $\mu$  is conjugate to the conserved quantity  $q$  in flat spacetime.

### III. ENERGY EXCHANGE BETWEEN MATTER AND VACUUM

We are, now, interested in the quantum-dissipative exchange between vacuum and matter. In this case, the chemical potential  $\mu$  is no longer constant and can relax in the evolving universe. We replace Eq. (2.3) by

$$\partial_t \left( \frac{d\epsilon(q)}{dq} - \frac{3}{8\pi} [\partial_t H + 2H^2] \frac{dG^{-1}(q)}{dq} \right) = S, \quad (3.1)$$

where a possible *Ansatz* for the source term is given by

$$S = \Gamma_q (\partial_t q)^2 + \Gamma_H (\partial_t H)^2, \quad (3.2)$$

with decay constants  $\Gamma_q$  and  $\Gamma_H$ . The crucial question is if the system will approach the de-Sitter asymptote (with  $H = \text{const} > 0$ ) or the Minkowski vacuum (with  $H = 0$ ).

The Einstein equations are still valid and are obtained by variation of the action (2.1a) over the metric  $g_{\mu\nu}$ ,

$$\begin{aligned} & \frac{1}{8\pi G(q)} \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + \frac{1}{16\pi} q \frac{dG^{-1}(q)}{dq} R g_{\mu\nu} \\ & + \frac{1}{8\pi} \left( \nabla_\mu \nabla_\nu G^{-1}(q) - g_{\mu\nu} \square G^{-1}(q) \right) - \left( \epsilon(q) - q \frac{d\epsilon(q)}{dq} \right) g_{\mu\nu} + T_{\mu\nu}^{\text{M}} = 0, \end{aligned} \quad (3.3)$$

where  $\square$  is the invariant d’Alembertian and  $T_{\mu\nu}^{\text{M}}$  is the energy-momentum tensor of the matter fields (without dependence on  $q$  as discussed in Sec. II).

#### IV. CONSTANT- $G$ CASE

For the present article, it suffices to consider the simplest possible *Ansatz* for the function  $G(q)$ , namely a constant function,

$$G(q) = G_N, \quad (4.1)$$

with  $G_N$  Newton's gravitational constant. The generalized Einstein and Maxwell equations from Sec. III become

$$\frac{1}{8\pi G_N} \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = \rho_V(q) g_{\mu\nu} - T_{\mu\nu}^M, \quad (4.2)$$

$$-q \partial_t \left( \frac{d\epsilon(q)}{dq} \right) = \partial_t \rho_V(q) = -q S, \quad (4.3)$$

with the source term of the gravitational equation (4.2) containing the following vacuum energy density:

$$\rho_V(q) \equiv \epsilon(q) - q \frac{d\epsilon(q)}{dq}. \quad (4.4)$$

Note that the definition (4.4) also explains the first equality in (4.3).

The Einstein equations for the spatially-flat FRW universe give two Friedmann equations:

$$3H^2 = 8\pi G_N (\rho_V + \rho_M), \quad (4.5a)$$

$$2\partial_t H = -8\pi G_N (\rho_M + P_M). \quad (4.5b)$$

The evolution equations for vacuum and matter are

$$\partial_t \rho_V = -q S, \quad (4.6a)$$

$$\partial_t \rho_M = -3H (P_M + \rho_M) + q S. \quad (4.6b)$$

Now, consider matter with a constant equation-of-state parameter,

$$w_M \equiv \rho_M / P_M = \text{const.} \quad (4.7)$$

Then, the following two ordinary differential equations (ODEs) suffice to determine  $q(t)$  and  $H(t)$  in a spatially-flat FRW universe:

$$\partial_t \left( \frac{d\epsilon(q)}{dq} \right) = S, \quad (4.8a)$$

$$\frac{2}{1 + w_M} \partial_t H + 3H^2 = 8\pi G_N \rho_V(q), \quad (4.8b)$$

with  $\rho_V(q)$  from Eq. (4.4) and  $S$  from Eq. (3.2) or otherwise.

In fact, a companion paper [20] will focus on the source term (3.2), which keeps de Sitter spacetime stable, whereas the present paper looks for another type of source term which gives Minkowski spacetime. Most importantly, we do not wish to take some *ad hoc* source term but will try to get a term with a clear physical motivation.

## V. PARTICLE PRODUCTION AND BACKREACTION

Consider the production of massless particles (e.g., gravitons) by the curved spacetime of the spatially-flat FRW universe with appropriate boundary conditions on the matter fields and background [21–23]. Then, the increase of particle number density is given by [21]

$$\partial_t n_M \sim R^2 \sim (\partial_t H + 2H^2)^2, \quad (5.1a)$$

for Ricci scalar  $R = -6(\partial_t H + 2H^2)$ . The typical particle energy ( $E = \hbar\omega$ ) is determined by the Hubble expansion rate [22],

$$E_M \sim \hbar H, \quad (5.1b)$$

with  $\hbar$  temporarily reinstated. From (5.1a) and (5.1b), the standard adiabatic change of the particle energy density gets modified by a source term on the right-hand side,

$$\partial_t \rho_M + 3H(1 + w_M)\rho_M \sim \hbar H R^2, \quad (5.2)$$

where  $w_M$  equals 1/3 for the massless particles considered and, from now on,  $\hbar$  will again be set to 1 [admittedly, our display of  $\hbar$  is somewhat cavalier, as (5.1a) is also eminently quantum mechanical]. Introducing a dimensionless constant  $\tilde{\gamma}$ , the matter evolution equation reads

$$\partial_t \rho_M + 3H(1 + w_M)\rho_M = \tilde{\gamma} H R^2 \equiv S_M. \quad (5.3)$$

In a consistent classical description based on general relativity and diffeomorphism invariance, we must have (cf. Appendix E.1 of Ref. [29]) that the vacuum evolution equation has precisely the opposite source term compared to (5.3),

$$\partial_t \rho_V = -S_M = -\tilde{\gamma} H R^2 = -\gamma H (\partial_t H + 2H^2)^2, \quad (5.4)$$

with  $\gamma = 36\tilde{\gamma} \sim 1/(8\pi)$  for graviton production [21, 23]. Hence, the right-hand side of (5.4) can be interpreted as describing the *backreaction* of the particle production given by Eq. (5.2).

Let us end this section with two general remarks. First, the result (5.1a) does not apply to de Sitter spacetime. In fact, the result (5.1a) relies on being able to define an adiabatic vacuum, which is possible if the expansion rate vanishes asymptotically in the past and in the future [21–23]. For a massless scalar, the imaginary part of the effective action, calculated to quadratic order in the curvature, is given by the spacetime integral of Eq. (29) in Ref. [23]. For a spatially-flat ( $k = 0$ ) FRW universe, the spacetime integral reduces to an integral over the sum of the  $R^2$  term and the Gauss–Bonnet term. For asymptotically-flat  $k = 0$  FRW universes, the Gauss–Bonnet term integrates to zero, leaving the single  $R^2$  term.

Second, it is well-known that the gravitational backreaction of quantum matter fields is a subtle problem, which is not completely solved [22, 29]. Our description is the simplest possible: keep the form of the energy-momentum tensor on the right-hand sides of Eqs. (4.5a) and (4.5b) and modify both the evolution equation of the matter component (5.3) and the evolution equation of the vacuum component (5.4). Other modifications are certainly to be expected, but our minimal description suffices for an exploratory study.

## VI. $Q$ -THEORY MODEL OF VACUUM-ENERGY DECAY

Henceforth, we use the same dimensionless variables as in Ref. [3], obtained by rescaling with appropriate powers of the Planck energy  $E_P \equiv \sqrt{\hbar c^5/G_N} \approx 1.22 \times 10^{19}$  GeV and denoted by lower-case letters. Specifically, we have the dimensionless time  $\tau$  and the dimensionless Hubble parameter  $h$ . The 4-form field strength (2.1b) gives rise to the scalar field  $q$  of mass dimension 2 and rescaling this scalar field  $q$  (denoted  $F$  in Ref. [3]) produces the dimensionless variable  $f$ . The overdot will denote differentiation with respect to  $\tau$  and the prime differentiation with respect to  $f$ .

We now present the  $q$ -theory equivalent of the source term found in Sec. V, which physically corresponds to the backreaction from the particle production by curved spacetime. In addition, we allow for a dimensionless cosmological constant  $\lambda \equiv \Lambda/(E_P)^4$  in the dimensionless energy density  $\epsilon(f)$ .

Specifically, the generalized Maxwell equation (4.8a) with the specific source term from (5.4) and the generalized Friedmann equation (4.8b) give

$$f \dot{f} \epsilon'' = \gamma h \left( \dot{h} + 2h^2 \right)^2, \quad (6.1a)$$

$$\frac{2}{1 + w_M} \dot{h} + 3 h^2 = 3 r_V, \quad (6.1b)$$

with

$$r_V(f) = \epsilon(f) - f \epsilon'(f), \quad (6.2a)$$

$$\epsilon(f) = \lambda + f^2 + 1/f^2, \quad (6.2b)$$

where the left-hand side of (6.1a) equals  $-\dot{r}_V$  according to (6.2a). The *Ansatz* (6.2b) has two important properties [other *Ansätze* for  $\epsilon(f)$  are certainly possible]: first, the corresponding values of  $r_V$  range over  $(-\infty, \infty)$  for  $q^2 \in (0, \infty)$  and any finite value of  $\lambda$ ; second, the corresponding vacuum compressibility [2]  $\chi \equiv (f^2 d^2 \epsilon / df^2)^{-1}$  is positive for any  $q^2 \in (0, \infty)$ .

The two ODEs (6.1) are to be solved simultaneously and the matter energy density is obtained from the two solutions  $\bar{f}(\tau)$  and  $\bar{h}(\tau)$  by

$$r_M = \bar{h}^2 - r_V[\bar{f}]. \quad (6.3)$$

The numerical solutions of the ODEs (6.1) will be presented in the next section.

Note that, strictly speaking, the ODEs (6.1) can be written solely in terms of  $r_V(\tau)$  and  $h(\tau)$ , without need of the  $q$ -type field  $f(\tau)$ . But this is only because of the special case considered,  $G(q) = \text{const}$ . For generic  $G(q)$ , the  $q$  field appears explicitly on the left-hand side of the generalized Maxwell equation (3.1). Moreover, precisely  $q$  theory in the four-form realization gives rise to the  $\rho_V$  evolution equation as a field equation, namely, the generalized Maxwell equation (2.3) for the classical theory. For these reasons, it is appropriate to speak of a  $q$ -theory model of vacuum-energy decay.

## VII. MINKOWSKI-VACUUM ATTRACTOR

From the ODEs (6.1a) and (6.1b) follows that the curves for  $r_V(\tau)$  and  $h(\tau)$  are monotonically decreasing, provided that  $r_M(\tau) = h(\tau)^2 - r_V[f(\tau)]$  is nonnegative for all values of  $\tau$ . Numerical solutions are given in Figs. 1 and 2 for  $\lambda = \pm 1$ , showing that the  $r_V = 0$  Minkowski vacuum is approached without need of fine-tuning. Similar numerical results have been obtained for  $\lambda = 0$ . Note that the decay-constant value used for these numerical results,  $\gamma = 1/10$ , is entirely realistic (see Sec. V).

The numerical results also establish the existence of the  $r_V = 0$  attractor for  $\lambda \in \{-1, 0, +1\}$  with a finite domain of boundary conditions  $\{h(1), f(1)\} = \{6 \pm 1, 0.6 \pm 0.2\}$ ; see Fig. 3. The actual domain of attraction for  $|\lambda| \leq 1$  may be larger than this rectangle. As regards the approach of the Minkowski vacuum, the asymptotic behavior from (6.1) is given by

$$h(\tau) \sim (6 \gamma \tau)^{-1/3}, \quad (7.1a)$$

$$r_V(\tau) \sim (6 \gamma \tau)^{-2/3}. \quad (7.1b)$$

Remark that the attractor behavior found here is qualitatively similar to the one of Dolgov theory [24, 25] shown numerically in Fig. 2 of Ref. [4] and proven mathematically in App. A of Ref. [26]. But the Dolgov theory as such ruins the standard Newtonian dynamics [27] and needs to be modified significantly [26, 28].

Three final remarks are in order. First, the asymptotic decay of the Hubble expansion parameter is slow, giving rise to an inflationary behavior (cf. Refs. [30–32]) of the particle horizon,

$$d_{\text{hor}}(\tau) \equiv d_1 + a(\tau) \int_1^\tau \frac{d\tau'}{a(\tau')} \sim \exp \left[ \frac{3}{2} \frac{(\tau^{2/3} - 1)}{(6 \gamma)^{1/3}} \right], \quad (7.2)$$

where  $d_1$  is the contribution from times before  $\tau = 1$  (a radiation-dominated universe with an initial singularity at  $\tau = 0$  gives  $d_1 = 2$ ) and  $a(\tau)$  is the expansion factor defined by  $h = \dot{a}/a$  for  $h(\tau)$  given by (7.1a). But there is the following caveat: this inflationary behavior only holds as long as the particle production is given by the  $R^2$  term in (5.2), which may not be the case forever [see the last paragraph of Sec. V].

Second, the same type of slow asymptotic decay,  $h(\tau) \propto \tau^{-1/3}$  and  $r_V(\tau) \propto \tau^{-2/3}$ , has also been found in a nonconstant- $G$  model with the same *Ansätze* for  $G(f)$  and  $\epsilon(f)$  as in Ref. [3], but now with an arbitrary cosmological constant  $\lambda$  added to  $\epsilon(f)$ .

Third, returning to the constant- $G$  model considered here, it needs to be emphasized that we remain within the framework of standard general relativity (with certain quantum effects of the matter fields included, as discussed in Sec. V). Moreover, there are essentially no free parameters in the equation system (6.1), as the decay constant  $\gamma$  has been calculated [21, 23] to be of order 1/10 for massless particles ( $w_M = 1/3$ ).

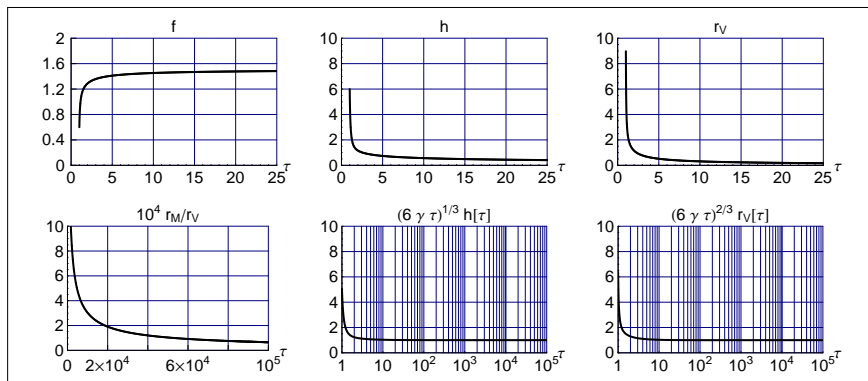


FIG. 1: Numerical solution of the ODEs (6.1) with auxiliary functions (6.2). The model parameters are  $\{\lambda, w_M, \gamma\} = \{1, 1/3, 1/10\}$  and the boundary conditions are  $\{h(1), f(1)\} = \{6, 3/5\}$ . The initial energy densities are  $\{r_V(1), r_M(1)\} = \{8.97333, 27.0267\}$ . The final value of the vacuum energy density is  $r_V(10^5) = 6.5 \times 10^{-4}$ .

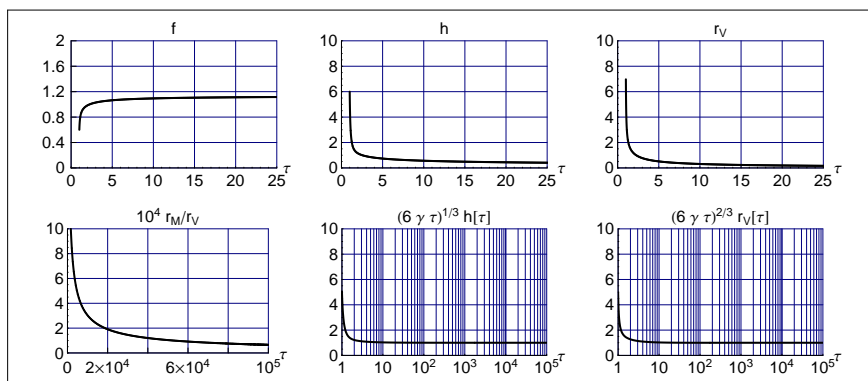


FIG. 2: Same parameters and boundary conditions as in Fig. 1, except for a different value of the cosmological constant,  $\lambda = -1$ . The initial energy densities are  $\{r_V(1), r_M(1)\} = \{6.97333, 29.0267\}$ . The final value of the vacuum energy density is  $r_V(10^5) = 6.5 \times 10^{-4}$ .

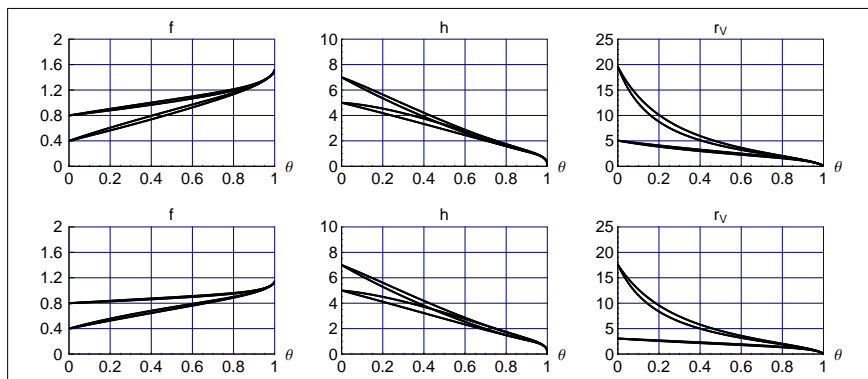


FIG. 3: Numerical solutions of the ODEs (6.1) with auxiliary functions (6.2) for  $\lambda = 1$  (top row) and  $\lambda = -1$  (bottom row). Further model parameters are  $\{w_M, \gamma\} = \{1/3, 1/10\}$ . The functions  $f, h$  and  $r_V[f]$  are plotted versus the compactified time coordinate  $\theta \equiv (\tau - 1)/(\tau + \tau_{\text{mid}} - 2)$  with  $\tau_{\text{mid}} = 11/10$ . Four sets of boundary conditions at  $\theta = 0$  are used:  $\{h(0), f(0)\} = \{6 \pm 1, 3/5 \pm 1/5\}$ .



## VIII. CONCLUSION

In this article, we have again addressed the cosmological constant problem, which can be formulated as follows: how can it be that the vacuum state does not have an effective cosmological constant  $\Lambda$  (or vacuum energy density  $\rho_V = \Lambda$  and vacuum pressure  $P_V = -\Lambda$ ) with an energy scale of the order of the known energy scales of elementary particle physics? A particular adjustment-type solution of the cosmological constant problem involves so-called  $q$  fields, which are scalars composites of higher-spin fields (for example,  $q$  as a composite from a gauge field  $A_{\mu\nu\rho}$  and the metric  $g_{\mu\nu}$ ).

By considering the energy exchange between this  $q$  field and massless particles produced by the spacetime curvature, we have found that an arbitrary Planck-scale cosmological constant  $\Lambda$  can be cancelled by the  $q$ -field dynamics without fine-tuning. As mentioned previously, this cancellation occurs within the realm of standard general relativity.

The Minkowski vacuum with  $\rho_V = -P_V = 0$  appears as an attractor of the dynamical equations. As the approach to the vacuum is rather slow, there occurs an inflationary behavior of the particle horizon, provided the nature of the particle production does not change. Indeed, the outstanding question is the microscopic origin of the particle-production feedback on the vacuum energy density.

The  $q$ -theory framework serves as the proper tool for studying physical processes related to the quantum vacuum. It describes, in particular, the relaxation of the vacuum energy density (cosmological constant) as the backreaction of the deep vacuum to different types of perturbations, such as the Big Bang, inflation, cosmological phase transition, and vacuum instability in gravitational or other backgrounds.

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