A hypothesis on neutrino helicity

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Abstract

It is firmly established by experimental results that neutrinos are almost 100% longitudinally polarized and left-handed. It is also confirmed by neutrino oscillation experiments that neutrinos have tiny but non-zero masses. Since the helicity is not a Lorentz invariant quantity for massive particles, neutrinos can not be strictly left-handed. On the other hand, it is generally assumed that ultrarelativistic massive fermions can be described well enough by the Weyl equations. We discuss the validity of this assumption and propose a new hypothesis according to which neutrinos can be described by pure helicity states although they are not massless.

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I. INTRODUCTION

In the original version of the Standard Model of particle physics neutrinos are accepted to be massless. Hence, they are described by pure helicity states which is consistent with the results obtained from experiments [1]. On the other hand, neutrino oscillation experiments point out an extension of the Standard Model. In the minimal extension of the Standard Model with massive neutrinos, flavor and mass eigenstates do not coincide.[1] Flavor eigenstates can be written as a superposition of mass eigenstates through the mixing equation $\nu_{\ell L} = \sum_{i=1}^{3} U_{\ell i}\ \nu_{iL}$ where $U_{\ell i}$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix element [2]. Since a flavor eigenstate is a mixture of different mass eigenstates each has a different rest frame, the rest frame of a flavor neutrino is somewhat uncertain. However, the notion of spin is closely related to the rest frame of the particle. The spin four-vector for a fermion

$$s^{\mu} = \left(\frac{\vec{p} \cdot \vec{s'}}{m}, \vec{s'} + \frac{\vec{p} \cdot \vec{s'}}{m(E+m)} \vec{p} \right) \tag{1}$$

is obtained by a Lorentz boost of $(s^{\mu})_{RF} = (0, \vec{s'})$ from the rest frame of the particle [3]. Therefore it is reasonable to accept that the notion of spin is undefined for a flavor neutrino. As far as we know, this problem has been skipped in the literature possibly because neutrino masses are extremely small ($m_{1,2,3} \lesssim 1\ eV$), and hence it is a very good approximation to use expressions obtained in the limit $m_{\nu_i} \rightarrow 0$. On the contrary, in the next section we will discuss some counter evidences obtained from spin dependent neutrino cross section that show this approximation may not be accurate enough.

II. A HYPOTHESIS ON NEUTRINO HELICITY AND SOME EVIDENCES FROM SPIN DEPENDENT CROSS SECTION

In his famous 1939 paper Wigner investigated unitary representations of the Poincaré group and classified particles according to their internal space-time symmetries [4]. One of the important criteria used in Wigner's classification is the existence of the rest frame of a particle. There is no frame of reference in which a massless particle such as a photon

[1] In this paper only Dirac neutrinos have been considered.

is at rest. Hence the little group for a massless particle is $E(2)$-like. On the other hand, a massive particle has a rest frame and in this frame we can rotate its spin three-vector without changing the momentum. Its little group is then $O(3)$-like. We assert the following hypothesis which makes a flavor neutrino 100% longitudinally polarized: The rest frame of a flavor neutrino is uncertain and this uncertainty makes its rest frame undefined. Since there is no frame of reference in which a flavor neutrino is at rest, its little group is no more $O(3)$-like. It should be classified together with massless particles described by $E(2)$-like little group, although it has a non-zero mass and does not propagate at the speed of light.

It is generally assumed that ultrarelativistic massive fermions can be described well enough by the Weyl equations. Hence neutrinos are accepted to be completely longitudinally polarized and pure helicity states for the neutrino fields are used in the cross section calculations. Let us perform cross section calculations for neutrinos with a general spin orientation and probe the validity of this assumption. We will consider a simple particular process, namely polarized neutrino production via electron capture, where much of the computation can be done easily. This process can be written at the quark level as $e^- u \to \nu_i d$, where ν_i represents a neutrino in the mass eigenstate. The process $e^- u \to \nu_i d$ is described by a t-channel W exchange diagram. Spin dependent amplitude for the process is given by[2]

$$M = \frac{G_F}{\sqrt{2}} U_{ei} U_{ud} \left[\bar{u}(p_{\nu_i}, s'_{\nu_i}) \hat{\Sigma}(s_{\nu_i}) \gamma^\mu (1-\gamma_5) u(p_e, s'_e) \right] \left[\bar{u}(p_d, s'_d) \gamma_\mu (1-\gamma_5) u(p_u, s'_u) \right] \qquad (2)$$

where G_F is the Fermi constant, U_{ei} is the PMNS matrix element, U_{ud} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element and $\hat{\Sigma}(s_{\nu_i}) \equiv \frac{1}{2}(1 + \gamma_5 \gamma_\mu s^\mu_{\nu_i})$ is the covariant spin projection operator for the neutrino. In the rest frame of the neutrino the spin four-vector is $(s^\mu_{\nu_i})_{RF} = (0, \vec{s}_{\nu_i})$. In an arbitrary reference frame the spin four-vector can be obtained by a Lorentz boost from the rest frame. In a reference frame where the neutrino has a momentum $\vec{p}$ and energy E its spin four-vector can be defined by Eq.(1) with $m = m_{\nu_i}$ and $\vec{s'} = \vec{s}_{\nu_i}$. When we square the amplitude and sum over fermion spins the projection $\hat{\Sigma}(s_{\nu_i}) u(p_{\nu_i}, s'_{\nu_i}) = \delta_{s'_{\nu_i}, s_{\nu_i}} u(p_{\nu_i}, s_{\nu_i})$ ensures that the sum over s'_{ν_i} yields just one term with $s'_{\nu_i} = s_{\nu_i}$ [3]. The spin-summed squared amplitude is calculated to be

$$\sum_{s'_{\nu_i}, s'_e, s'_d, s'_u} |M|^2 = 64 G_F^2 |U_{ei}|^2 |U_{ud}|^2 \left[(p_e \cdot p_u)(p_{\nu_i} \cdot p_d) - m_{\nu_i} (p_e \cdot p_u)(s_{\nu_i} \cdot p_d) \right]. \qquad (3)$$

[2] We assume that the W propagator can be approximated as $\frac{(g_{\mu\nu} - q_\mu q_\nu / m_w^2)}{q^2 - m_w^2} \approx -\frac{g_{\mu\nu}}{m_w^2}$.

It describes polarized neutrinos with spin four-vector s_{ν_i} but unpolarized electrons, u and d quarks. At first glance, it seems as if spin dependent term in Eq.(3) vanishes in the $m_{\nu_i} \to 0$ limit. But spin four-vector contains terms inversely proportional to m_{ν_i}. Therefore first we should perform Lorentz scalar products and then examine its zero-mass limit. After Lorentz scalar products are performed, the spin-summed squared amplitude can be written as

$$\sum_{s'_{\nu_i},s'_e,s'_d,s'_u} |M|^2 = 64G_F^2|U_{ei}|^2|U_{ud}|^2(E_eE_u - \vec{p}_e \cdot \vec{p}_u)(E_{\nu_i}E_d - E_d(\vec{s}_{\nu_i} \cdot \vec{p}_{\nu_i}) + \vec{p}_{\nu_i} \cdot \vec{p}_d \left(\frac{\vec{s}_{\nu_i} \cdot \vec{p}_{\nu_i}}{E_{\nu_i} + m_{\nu_i}} - 1\right) + m_{\nu_i}\vec{s}_{\nu_i} \cdot \vec{p}_d). \quad (4)$$

We observe from Eq.(4) that the term $\vec{s}_{\nu_i} \cdot \vec{p}_{\nu_i}$ does not completely vanish in the $m_{\nu_i} \to 0$ limit. Therefore it doesn't matter how small it is, if the neutrino has a nonzero mass then the cross section depends on its spin orientation. In the center-of-momentum frame zero-mass limit of the squared amplitude becomes

$$\lim_{m_{\nu_i}\to 0} \sum_{s'_{\nu_i},s'_e,s'_d,s'_u} |M|^2 = 64G_F^2|U_{ei}|^2|U_{ud}|^2(E_eE_u + |\vec{p}_e||\vec{p}_u|) \times (E_d + |\vec{p}_d|)(E_{\nu_i} - \vec{s}_{\nu_i} \cdot \vec{p}_{\nu_i}). \quad (5)$$

We see from the above expression that the squared amplitude and hence the cross section takes its largest value when the neutrino is left-handed ($\vec{s}_{\nu_i} = -\frac{\vec{p}_{\nu_i}}{|\vec{p}_{\nu_i}|}$) and zero when the neutrino is right-handed ($\vec{s}_{\nu_i} = +\frac{\vec{p}_{\nu_i}}{|\vec{p}_{\nu_i}|}$). If the spin three-vector is perpendicular to the direction of neutrino momentum then the cross section is half of the cross section for left-handed neutrino. In opposition to expectations these results indicate that the transverse component of the spin three-vector of a fermion does not vanish in the zero-mass limit.[3] This result contradicts to the generally accepted fact that when the speed of a particle approaches the speed of light its spin vector lays down on the momentum direction. Is it indeed possible that this generally accepted fact is fallacious? We should examine what the relativistic quantum mechanics says about the zero-mass limit of a spinor describing a general spin orientation.

Let us construct the spinors describing a general spin orientation and examine their behavior in the zero-mass limit. Assume that S defined by spatial axes x-y-z and time t is

[3] When we use the notation $m \to 0$ or the phrase "zero-mass limit" we mean an infinitesimal mass but not equal to zero.

the rest frame of a spin-1/2 particle with mass m. In the rest frame of the particle we can safely use Pauli spinors to define its spin. Let q-axis be the spin quantization axis in the z-x plane which makes an angle ϕ with respect to z-axis. Then the non-relativistic 2×2 spin matrix is $\hat{S} = \frac{1}{2}(\sin\phi\ \hat{\sigma}_x + \cos\phi\ \hat{\sigma}_z)$ where $\hat{\sigma}_x, \hat{\sigma}_y$ and $\hat{\sigma}_z$ are Pauli spin matrices. The eigenvectors of the spin matrix are

$$\chi_+ = \begin{pmatrix} \cos\phi/2 \\ \sin\phi/2 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} -\sin\phi/2 \\ \cos\phi/2 \end{pmatrix}. \tag{6}$$

Here χ_+ and χ_- are the eigenvectors which correspond to eigenvalues $\lambda = +1$ and $\lambda = -1$ respectively, i.e., $\hat{S}\chi_+ = \chi_+$ and $\hat{S}\chi_- = -\chi_-$. We have so far considered only 2×1 matrix representations. In the Weyl representation, 4×1 Dirac spinors for a spin-1/2 particle can be written as [5]

$$u(p) = \begin{pmatrix} \phi_R(p) \\ \phi_L(p) \end{pmatrix} \tag{7}$$

where $\phi_R(p)$ and $\phi_L(p)$ are 2×1 spinors and subscripts "R" and "L" represents chirality. In the rest frame of the particle the equality $\phi_R(0) = \phi_L(0)$ holds. $\phi_R(0)$ and $\phi_L(0)$ can be considered as the eigenvectors of the 2×2 spin matrix. The spinor $u(p)$ for a particle with four-momentum $p^\mu = (E, \vec{p})$ can be obtained via Lorentz boost from rest spinor $u(0)$. Suppose that S' frame is moving along the negative z-axis with relative speed v with respect to S. If we choose $\phi_R(0) = \phi_L(0) = \chi_+$ and perform a Lorentz boost into S' frame then we obtain a Dirac spinor which describes a particle of four-momentum $p^\mu = (E, 0, 0, p_z)$ and four-spin given in Eq.(1) where $\vec{s'} = \sin\phi\ \hat{x} + \cos\phi\ \hat{z}$ is the unit vector on the spin quantization axis q in the rest frame of the particle. The spinor obtained in this way describes a "spin up" state. Similarly if we choose $\phi_R(0) = \phi_L(0) = \chi_-$ and perform a Lorentz boost, then we obtain the "spin down" spinor.[4] After some straightforward calculations, these "spin up" ($\uparrow$) and "spin down" ($\downarrow$) spinors are found to be

$$u^{(\uparrow)}(p, s) = \cos\left(\frac{\phi}{2}\right) u^{(R)}(p, s_*) + \sin\left(\frac{\phi}{2}\right) u^{(L)}(p, s_*) \tag{8}$$

$$u^{(\downarrow)}(p, s) = \cos\left(\frac{\phi}{2}\right) u^{(L)}(p, s_*) - \sin\left(\frac{\phi}{2}\right) u^{(R)}(p, s_*) \tag{9}$$

[4] Hereafter we will use the notation $u(p, s)$ instead of $u(p)$ and sometimes use the superscripts $\uparrow$ and $\downarrow$ for spin up and spin down and the superscripts R and L for right-handed and left-handed.

where $u^{(R)}(p, s_*)$ and $u^{(L)}(p, s_*)$ represent right-handed and left-handed spinors, i.e., $\vec{s'_*} = \lambda\frac{\vec{p}}{|\vec{p}|}$, $\lambda = +1(-1)$ for right-handed (left-handed). We observe from Eqs. (8) and (9) that "spin up" and "spin down" spinors can be written as a superposition of right- and left-handed spinors. We now come to a controversial point. One may assume that the angle ϕ converges to zero due to relativistic aberration when the relative speed approaches the speed of light. We claim that this assumption is fallacious because ϕ is the angle measured in the frame in which the particle is at rest. In analogy with the term proper time we can call it "proper angle". Then, coefficients $\cos\left(\frac{\phi}{2}\right)$ and $\sin\left(\frac{\phi}{2}\right)$ do not depend on the energy or the mass of the particle. Hence, these "spin up" and "spin down" spinors do not approach one type of helicity state (R or L) when the mass approaches zero. They are always given by the same superposition of right- and left-handed spinors. For the special case $\phi = \pi/2$, spin three-vector $\vec{s'}$ becomes perpendicular to the direction of momentum. In this case, $u^{(\uparrow)}(p, s)$ can be considered as a mixed state composed of helicity states where each helicity state has equal probability. (The similar thing is also true for $u^{(\downarrow)}(p, s)$.) This mixed state remains intact for every value of the mass greater than, but not equal to zero, i.e., $m > 0$. Since the helicity and chirality states coincide in the ultrarelativistic limit we can deduce that the spinors $u^{(\uparrow)}(p, s)$ and $u^{(\downarrow)}(p, s)$ do not converge to a pure helicity or a chirality state.

We conclude from the above discussion that although the helicity states converge to the chirality eigenstates when $m \to 0$, the free solutions of Dirac equation describing a general spin orientation *do not continuously* converge to chirality eigenstates. Here the phrase "*do not continuously*" is used to indicate the discontinuity at $m = 0$. We have shown that when the mass parameter is in the open interval $(0, \infty)$ general spin states and chirality eigenstates are disjointed. But in the point $m = 0$ the solution describing a general spin orientation jumps to a pure helicity state. This behavior seems contradictory to the fact that the $E(2)$-like little group is the Lorentz-boosted $O(3)$-like ($SU(2)$-like) little group for massive particles in the zero-mass limit [6–8]. However, the group contraction is established by relating the generators of these groups and hence it points out a local isomorphism between $SU(2)$ and $E(2)$ in the limit $m \to 0$. A general spin orientation can be obtained by a *finite* rotation from the direction in which the Lorentz boost is performed. Therefore it is controversial to conclude from a local isomorphism that the transverse component of the spin three-vector vanishes in the zero-mass limit. One may observe from Eq.(1) that when the speed of a particle approaches the speed of light, spatial component of its spin four-vector lays down on the momentum direction. Such a relativistic aberration is indeed true but the spatial component of the spin four-vector does not represent the spin orientation of the moving fermion. The true spin three-vector should be the one which is attached on the moving frame and coincides with the spin quantization axis. Therefore it should be: $\vec{s}-\vec{v}t = \vec{s'} - \frac{\vec{p}\cdot\vec{s'}}{E(E+m)}\vec{p}$. It corresponds to an imaginary ruler on the moving frame that coincides with the spin quantization axis. It is obvious that a ruler which is oriented transverse to the direction of Lorentz boost remains unchanged after the Lorentz transformation.

The argument discussed above might be found controversial by some readers since it contradicts to commonly accepted assumption that ultrarelativistic fermions can be described by the Weyl equations with a high accuracy. But we find these evidences serious enough to present to the physics community.

Experimental results confirm the fact that neutrinos are almost 100% longitudinally polarized and left-handed. Based on the above discussion about the zero-mass limit of a fermion we conclude that the assumption that neutrinos are almost completely longitudinally polarized is an *ad hoc* hypothesis. It is not founded on quantum field theory or relativistic quantum mechanics. On the other hand, the new hypothesis presented in this paper can explain these experimental results.

III. CONCLUSIONS

Massive particles which do not have a rest frame were not considered in the Wigner's work [4]. On the other hand, quantum mechanics makes such a peculiar case possible. Based on the discussion about neutrino spin we conclude that a modification or an extension of the Wigner's work is necessary. This modification is probably related to a more deeper problem, which is the unification of quantum mechanics with special relativity. It is generally believed that the unification of quantum mechanics with special relativity has been completed. Their offspring is the quantum field theory. Nevertheless, a new hypothesis and its evidences discussed in this paper raise some doubt on the completeness of this unification.

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[1] M. Goldhaber, L. Grodzins, and A. W. Sunyar, *Helicity of Neutrinos*, Phys. Rev. **109**, 1015 (1958).

[2] S. Bilenky, *Introduction to the Physics of Massive and Mixed Neutrinos* (Springer, 2010).

[3] W. Greiner and J. Reinhardt, *Quantum Electrodynamics*, (Springer, 1994).

[4] E. P. Wigner, *On Unitary Representations of the Inhomogeneous Lorentz Group*, Ann. Math. **40**, 149 (1939).

[5] L. H. Ryder, *Quantum Field Theory*, (Cambridge University Press, Second Ed. 1996).

[6] Y. S. Kim and E. P. Wigner, *Cylindrical Group and Massless Particles*, J. Math. Phys. **28**, 1175 (1987).

[7] Y. S. Kim and E. P. Wigner, *Space-time Geometry of Relativistic Particles*, J. Math. Phys. **31**, 55 (1990).

[8] Y. S. Kim and M. E. Noz, *Theory and Applications of the Poincare Group*, (D. Reidel Publishing Company, 1986).