Calculating Masses of Pentaquarks Composed of Baryons and Mesons

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Abstract

In this paper, we consider an exotic baryon (pentaquark) as a bound state of two-particle systems composed of a baryon (nucleon) and a meson.We used a baryon - meson picture to reduce a complicated five-body problem to two simpler two-body problems. The homogeneous Lippmann-Schwinger integral equation is solved in configuration space by using Yukawa potential. We calculate the masses of pentaquarks $\theta_c(uudd\bar{c})$, $\theta_b(uudd\bar{b})$, $\theta_{bs}(buud\bar{s})$, and $\theta_{cs}(cuud\bar{s})$.

Keywords: Exotic baryon, pentaquark, binding energy, Cornell potential, Lippmann-Schwinger equation.

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1 Introduction

There are two types of hadrons, baryons and mesons. Baryons are equivalent to the bound states of three quarks, and mesons are known to be the bound states of a fghquark and an antiquark. However, QCD describes mesons and baryons even with a more intricate structure; there are anomalous mesons such as ggg, $q\bar{q}g$, $q\bar{q}q\bar{q}$, etc as well as exotic baryons like $qqqg$, $qqqq\bar{q}$, etc. Pentaquarks are baryons with at least four quarks and one antiquark. In exotic pentaquarks, the antiquark has a flavor different from the other four quarks.

Exotic hadrons containing at least three valence quarks are being studied fairly extensively in modern physics. Although there are hundreds of ordinary hadrons, exotic ones haven't been found stable yet. However, QCD does not reject their existence. Pentaquark θ^+, studied in photo production experiments [1, 2] is a prototype of exotic hadrons in light and strong quark sector. Theoretically, hadronic reactions contribute to θ^+ production more vividly than other types of reactions.

The quark model is commonly used to describe hadrons. In this model, mesons are described as $q\bar{q}$ and baryons as three-quark composite particles. In a more microscopic view, QCD usually serves to describe the strong interaction. According to Lipkin [3] and Gignoux et al [4], among pentaquarks, the five-quark anticharmed baryons of the $P^0 = [uud\bar{c}s]$ and $\bar{p} = [udd\bar{c}s]$ or similar anti-beauty baryons are the most bound.

A lot of experimental evidence on the existence of exotic hadrons has been found since 2003. Exotic hadrons' quantum numbers cannot be justified based on two- and three-quark bound states. Pentaquarks of $qqqq\bar{q}$ form are examples of exotic baryon states. Conjugation quantity of C charge is not an accurate quantum number for baryons, and all combinations of total spin J and parity P can exist. However, an exotic baryon combination can be readily identified by its electric charge Q and its strangeness S. Some evidence has been reported during the last few years. For example, the pentaquark θ^{++} was proven to exist in Hermes experiment in Humburge, Germany [5, 6].

For exotic baryons, we consider the following:

θ^+: The existence of this exotic baryon was predicted in chiral solution model [7]. It has an $S = +1$, $J^P = 1/2^+$ and $I = 0$. It is a narrow light-mass particle of 1540 MeV. These attributes initially made θ^+ a subject of experimented observation by LEPS [8] .The most suitable hadronic decay mode to identiy it is $\theta^+ \longrightarrow K^0 p$.

θ_c and θ_b: The existence of the bound exotic hadron θ_c was predicted through bound Skyrmion approach. This particle has a mass of 2650 MeV, quantum numbers $J^P = 1/2^+$, and $I = 0$. An experiment [9] showed a positive signal at a mass of about 3.1 GeV, but it wasn't confirmed later [10]. In strongly bound states, the decay mode $K^+\pi^-\pi^-p$ is easy to identify.

Likewise, the mass of θ_b with the same quantum numbers $J^P = 1/2^+$ and $I = 0$ was predicted to be 5207 MeV. The possible weak decay made $K^+\pi^-\pi^- + \pi^+ + p$.

θ_{cs}: It is the five-quark state with $J^P = 1/2^-$ and $I = 0$. In a quark model which includes color-spin interaction, it can be bound and despite its strong decay, it becomes stable [11]. The mass depending on the model parameters is predicted to be 2420 MeV. This state was traced in the Fermilab E791 experiment via $\phi\pi p$ mode [12] and $K^{*0}K^-p$ mode [13].

Lippman-Schwinger Equation for two-body bound states is solved in sec 1. We explain Gauss-Legendre method in sec 2. In sec 3, the procedure of the study is given and pentaquark masses are determined.

2 Lippman-Schwinger equation for two-body bound states

In this part, the binding energy of the entire system (pentaquark) is calculated by numerical solution of homogeneous Lippman-Schwinger equation for each subsystem of bound meson and baryon. Schrodinger equation for a two-body bound state with the potential V runs as the following integral equation [14]:

$$\mid \psi_b >= G_0 V \mid \psi_b > \tag{1}$$

G_0 is the propagator of a free particle. In configuration space, it turns out as:

$$\psi_b(r) = -m\sqrt{\pi/2}\int_0^\infty dr' r'^2 \int_{-1}^1 dx' \int_0^{2\pi} d\phi' \frac{exp(-\sqrt{m|E_b|}|r-r'|)}{|r-r'|} V(r')\psi_b(r') \tag{2}$$

where E_b stands for the binding energy of the two-body bound system (meson+baryon). The interaction potential considered locally, the wave function will be:

$$\psi_b(r) = \int_0^\infty dr^{'} \int_{-1}^1 dx^{'} M(r, r^{'}, x^{'})\psi_b(r^{'}) \tag{3}$$

where:

$$M(r, r^{'}, x^{'}) = -2\pi m\sqrt{\pi/2}\frac{exp((-\sqrt{m|E_b|})\sqrt{r^2 + \acute{r}^2 - 2r\acute{r}\acute{x}})}{\sqrt{r^2 + \acute{r}^2 - 2r\acute{r}\acute{x}}} V(\acute{r}^2) \tag{4}$$

Equation (4) is of the following eigenvalue form:

$$K(E_b)|\psi_b >= \lambda(E_b)|\psi_b > \tag{5}$$

$\lambda = 1$ is the highest positive eigenvalue. The eigenvalue equation is solved through iteration method (direct method) [15]. To discretize the integrals, Gauss-Legendre method [16] is employed. Gauss lattice points for $r, r\prime, x\prime$ are supposed to be 100 (The more the points, the more accurate the results, although this lowers the running speed of the program).

3 Gauss-Legendre Method

In Gauss-Legendre method, each integral of [-1,+1] interval is treated as:

$$\int_{-1}^{+1} f(x)dx = \sum_{i=1}^{n} w_i f(x_i) \tag{6}$$

where x_i are the roots of the type-one order-N Legendre function, and w_i are the functions of point weight. The following variable change is used to transfer the integralization interval of $r\prime$ from $[0, r_{max}]$ to $[-1, +1]$. If the integrals are discretized, then

$$r = r_{max}\frac{1+x}{2} \tag{7}$$

$$\psi_b(r) = -2\pi m\sqrt{\pi/2}\sum_{j=1}^{N_r'}\sum_{i=1}^{N_r'} W_{r_i}' W_{x_j}' {r_i'}^2 \frac{exp(-\sqrt{m|E_b|}\rho(r, r_i', x_j'))}{\rho(r, r_i', x_j')} V(r_i')\psi_b(r_i') \tag{8}$$

Equation (8) could be rewritten as:

$$\psi_b(r) = \sum_{i=1}^{N_{r'}} N(r, r_i^{'})\psi_b(r_i^{'}) \tag{9}$$

where

$$N(r, r_i') = -2\pi m\sqrt{\pi/2}\sum_{j=1}^{N_r'} W_{r_i}' W_{x_j}' {r_i'}^2 \frac{exp(-\sqrt{m|E_b|}\rho(r, r_i', x_j'))}{\rho(r, r_i', x_j')} V(r_i') \tag{10}$$

Matrix N is diagonized to find $\lambda = 1$ in the eigenvalue spectrum. The energy corresponding to $\lambda = 1$ will be the system's binding energy.

4 Procedure

We consider a pentaquark as a bound state of a two-particle system formed by a baryon and a meson. The hadronic molecular structure consists of a baryon and a meson (Figure 1). Table 1 presents the structure of five hadrons.

Figure 1: Pentaquark

Interaction potential has an essential role in solving the eigenvalue (5). Different potentials have been introduced for meson-baryon interaction. Yukawa potential (screened coulomb potential) is proposed as one of the appropriate ones [17, 18]. This study deals with exotic baryon states created by a meson and a nucleon. The π exchange potential is among the most prominent meson exchange forces. π is the lightest hadron that can be exchanged between meson and nucleon. Therefore, we consider only π exchange, and ρ and ω meson exchange will be elaborated on in our subsequent works. OPEP is of the following form[17]:

$$V_\pi(r) = (\vec{I}_N.\vec{I}_H)(2S_{12}V_T(r) + 4\vec{S}_N.\vec{S}_H)V_c(r) \quad (11)$$

$$= (I^2 - I_N^2 - I_H^2)(K^2 - S_N^2 - S_l^2)V_c(r) \quad (12)$$

where the central part of the potential is:

$$V_c(r) = \frac{g_H g_A}{2\pi f_\pi^2}(m_\pi^2)\frac{e^{-mr}}{3r} \quad (13)$$

where, f_π is the pion decay constant and $f_\pi = 135MeV$, m_π is the pion mass and $m_\pi = 138MeV$, the axial coupling constant is $g_A = 13.0215$, the heavy-meson coupling constant is $g_H = g_\pi = 0.59$, then

$$V_c(r) = g'^2\frac{e^{-mr}}{r} \quad (14)$$

and tensor part is:

$$V_T(r) = \frac{g_H g_A}{2\pi f_\pi^2}(m_\pi^2)\frac{e^{-mr}}{6r}(\frac{3}{r^2} + \frac{3}{r} + 1) \quad (15)$$

I is the total isospin of meson-nucleon system and

$$S_{12} \equiv 4[3(\overrightarrow{S}_N.\widehat{r})(\overrightarrow{S}_N.\widehat{r}) - \overrightarrow{S}_N.(\overrightarrow{S}_N)] \tag{16}$$

where

$$\frac{g_H g_A}{2\pi f_\pi^2}(m_\pi^2)\frac{1}{3} = g'^2 = 0.4259 \tag{17}$$

Inserting I_N(nucleon isospin), I_H (meson isospin), S_N (nucleon spin), S_l (the lightest quark's spin in the meson), and $K = S_N + S_l$ into the potential, we obtain the potential's constant co-efficient (c) for different pentaquarks in $I = 0$ state (Table 2).

$$V_c(r) = cV_c'(r) \tag{18}$$

then

$$V_c(r) = cV_c'(r) = cg'^2\frac{e^{-mr}}{r} = -g^2\frac{e^{-mr}}{r} \tag{19}$$

We assume r axe along the direction of $\widehat{z}$ too. Pentaquark binding energy is defined as the energy used when breaking a pentaquark into its components, i.e. meson and baryon, so it is negative. Pentaquark mass is calculated according to equation (5)

$$M(pentaquark) = m_{meson} + m_{baryon} + E_b \tag{20}$$

In order to find the binding energy (E_b), first Mont-Carlo approach is employed to solve Lippman-Schwinger equation for the two-body system. In this approach, the kernel is diagonized and the eigenvalue spectrum is identified (spin-spin interaction in the potential and spin splitting are ignored). According to equation (5), the eigenvalue $\lambda = 1$ indicates an appropriate self-consistent wave function. The energy corresponding with this wave function will be the E_b. The data required include reduced mass of mesons and nucleons, proposed binding energy, potential co-efficients, and r-cutoff=10 fm. The masses of hadrons used in the study are presented in Table 3.

5 Results and discussion

In this paper, we solved Lippman-Schwinger equation for pentaquark systems. We managed to obtain the binding energy and used it to calculate the masses of these systems. The pentaquark is considered as the bound state of a baryon and heavy meson. We used a baryon - meson picture to reduce a complicated five-body problem to two simpler two-body problems. In Table 4, we have listed our numerical results for masses of pentaquark systems, and pentaquark masses are compared with the results obtained in [20, 21]. Our results are in good agreement with the results derived from complicated relativistic methods and can be a good replacement for them.

Our method is appropriate for investigating tetraquark systems too. We study four-body systems consisting of diquarkantidiquark, and we analyze diquarkantidiquark in the framework of a two-body (pseudo-point) problem. We solve LippmanSchwinger equation numerically for charm diquarkantidiquark systems and nd the eigenvalues to calculate the binding energies and masses of heavy tetraquarks with hidden charms [22].

Table 1:Hadronic molecular structure of pentaquarks

Hadronic structure (meson+nucleon)	pentaquark
θ^+ $(uudd\bar{s})$	uud / udd , $d\bar{s}$ / $u\bar{s}$
θ_c $(uudd\bar{c})$	uud / udd , $d\bar{c}$ / $u\bar{c}$
θ_b $(uuddb)$	uud / udd ,db / ub
θ_{cs} $(cuud\bar{s})$	uud,$c\bar{s}$
θ_{bs} $(buud\bar{s})$	uud ,$b\bar{s}$

Table 2: Potential's constant co-efficient in I=0

Pentaquark	c	g^2
$\theta_c(uudd\bar{c})$	-3/4	0.3194
θ_b $(uuddb)$	-3/4	0.3194
θ_{cs} $(cuud\bar{s})$	-3/8	0.1597
θ_{bs} $(buud\bar{s})$	-3/8	0.1597

Table 3: Hadron masses[19]

Hadronic	Mass(MeV/c^2)
p(uud)	938.272
n(udd)	939.5653
$k^+(u\bar{s})$	439.677
$k^0(d\bar{s})$	497.614
$\bar{D}^0(u\bar{c})$	1864.63
$\bar{D}(d\bar{c})$	1869.60
$D_s^0(c\bar{s})$	1968.47
$B^0(d\bar{b})$	5279.50
$B_{\bar{s}}(b\bar{s})$	5366.6

Table 4: Masses (MeV/c^2) of pentaquark

Pentaquark	hadronic structure	M	Mass in [20, 21]
θ_c $(uudd\bar{c})$	$uud/d\bar{c}$	2646.54	2650
	$udd/u\bar{c}$	2646.554	
θ_b $(uudd\bar{b})$	$uud/d\bar{b}$	5196.732	5207
	$udd/u\bar{b}$	5196.719	
θ_{cs} $(cuud\bar{s})$	$uud/c\bar{s}$	2427.474	2420
θ_{bs} $(buud\bar{s})$	$uud/b\bar{s}$	5752.0743	5750

6 Conclusions

In this paper, we investigated pentaquark systems as hadronic composites of a meson and a nucleon. We used the Yukawa potential and solved Lippman Schwinger equation for systems consisting of mesons and baryons and obtained the masses of Pentaquarks in $I = 0$ state. Our results are in good agreement with previous research. According to our method, the solution of five-body systems is reduced to the solution of two-body systems without taking into account the relativistic corrections. We would like to claim that although this method is not a precise solution of the five-body system, its important advantage is the reduction of the complicated five-body problem to a two-body problem. In our next research, we consider the Tensor part of Yukawa potential, too.

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