On the rich eight branch spectrum of the oblique propagating longitudinal waves in partially spin polarized electron-positron-ion plasmas

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We consider the separate spin evolution of electrons and positrons in electron-positron and electron-positron-ion plasmas. We consider oblique propagating longitudinal waves in this systems. Working in a regime of high density $n_0 \sim 10^{27}$ cm^{-3} and high magnetic field $B_0 = 10^{10}$ G we report presence of the spin-electron acoustic waves and their dispersion dependencies. In electron-positron plasmas, similarly to the electron-ion plasmas, we find one spin-electron acoustic wave (SEAW) at propagation parallel or perpendicular to the external field and two spin-electron acoustic waves at the oblique propagation. At the parallel or perpendicular propagation of the longitudinal waves in electron-positron-ion plasmas we find four branches: the Langmuir wave, the positron-acoustic wave and pair of waves having spin nature, they are the SEAW and, as we called it, spin-electron-positron acoustic wave (SEPAW). At the oblique propagation we find eight longitudinal waves: the Langmuir wave, Trivelpiece-Gould wave, pair of positron-acoustic waves, pair of SEAWs, and pair of SEPAWs. Thus, for the first time, we report existence of the second positron-acoustic wave existing at the oblique propagation and existence of SEPAWs.

I. INTRODUCTION

The field of spin quantum plasmas has been rapidly growing over the last decade. Takabayasi [1] derived and analyzed the quantum hydrodynamic equations for a single spin-1/2 particle. The effects of electron spin on the plasma dynamics were first studied by Kuz'menkov at. al, [2, 3] in 2001. These authors have developed a method of explicit derivation of many-particle quantum hydrodynamic (QHD) equations. These equations were truncated to consist of the continuity equation, the Euler equation, the energy balance equation, the magnetic moment evolution equation for spin-1/2 quantum plasmas. The starting point of these derivation was the many-particle Pauli equation. These set of equations contain the effects of the spin-spin exchange interactions and Coulomb exchange interactions. Another form of derivation was recently suggested in Ref. [4]. A simplified form of QHD equations were considered in Refs. [5, 6].

The method of many particle QHDs was applied to study the eigenwave problem for spin-1/2 electron-ion plasmas with an account of the ion motion [7]. The dispersion relations of electrostatic and electromagnetic waves has been studied and found that the spin-plasma waves (found for electrons in Ref. [8]) exist in the vicinity of electron and ion cyclotron frequencies in the spectrum of waves propagating perpendicular to the external magnetic field, and the dispersion relation for self-consistent spin waves with a linear spectrum is also obtained in Ref. [7]. Further applications of the method of many particle QHDs have been given in Refs. [9, 10] which predicts a mechanism of instabilities which arises due to the interaction of neutron beam with electron-ion magnetized spin-1/2 quantum plasma, including instability of the spin-plasma waves. Further analysis of spin-plasma waves was presented in Refs. [11, 12]. Considering kinetics in the extended phase space suggested by Kagan in 1961 [13]-[15], where the spin or magnetic moment is considered along with the coordinate and momentum, the fine structure of the Berstein modes is found [16]. The fine structure is demonstrated on the example of the second mode. It arises due to the presence of the anomalous magnetic moment of electrons.

Recently, a set of QHD equations for charged spin-1/2 particles is derived from the Pauli equation in Ref. [17]. It forms separate spin evolution QHDs (SSE-QHDs) Which treats spin-up and spin-down electrons as two different fluids. It is essential if the populations of spin-up electrons and spin-down electrons in the presence of external magnetic field is different ($n_\uparrow \neq n_\downarrow$). This difference of populations of quantum states is responsible for difference of Fermi pressures of the spin-up and spin-down electrons. Revealing in a new type of a soundlike solution called the spin-electron acoustic wave (SEAW) [17], see some additional discussion below, after formula (2). Spin current evolution in terms of SSE-QHD was considered in Ref. [18].

In the framework of the hydrodynamic model and linear-response-function formalism the effects of spin polarization on the Langmuir and zero sound waves is investigated in Ref. [19]. It was found that spin polarization

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increases the coefficient of spatial dispersion of Langmuir waves. It has also shown that phase velocity of zero sound increases with increase of the degree of polarization.

Electron-positron (e-p) plasma is distinct because it consists of particles which have mass symmetry and anti charge symmetry. Naturally electron-positron plasmas are found in many astrophysical environments like early universe [21], in neutron star magnetosphere [22], [23]. Naturally existence of electron-positron plasma in compact stars has been investigated by applying a simplified model of a gravitationally collapsing or pulsating baryon core. It has shown that possible electric processes that lead to the production of electron-positron pairs in the boundary of a baryon core and calculate the number density of electron-positron pairs $n_{pair} = 10^{28}\ cm^{-3}$ [24]. The degenerate electron-positron plasmas with ions are believed to be found in compact astrophysical bodies like neutron stars and the inner layers of white dwarfs [25–27].

Physicists also trying to generate e–p plasmas in laboratories. In this context different schemes have been proposed for the laboratory generation. For example, in large-scale conventional accelerators, the possibility of recombining high-quality electron and positron beams via magnetic chicanes [28]. Pederson et. al, [29] have been presented plan for the creation and diagnosis of electron-positron plasmas in a stellarator, based on extrapolation of the results from the Columbia Non-neutral Torus stellarator, as well as recent developments in positron sources. Interaction of ultrashort laser pulses with gaseous or solid targets could lead to the generation of the optically thin e-p plasma with above solid state densities in the range of $(10^{23} - 10^{28})\ cm^{-3}$[30]. Recently, it has shown that, by using a compact laser-driven setup, ion-free electron–positron plasmas can be generated in the laboratory [31]. Their charge neutrality, density about $10^{16} cm^{-3}$ and small divergence finally open up the possibility of studying electron–positron plasmas in controlled laboratory experiments.

The wave propagation phenomenon in electron-positron plasma is different as in usual electron-ion plasma. Using a two-fluid model and a kinetic model, it has been observed that many wave phenomena like acoustic waves, whistler waves, Faraday rotation, lower hybrid waves and shear Alfven waves are absent in the nonrelativistic e-p plasmas [32, 33]. Most of applications related to the plasma wave phenomenon presented in above mentioned studies has focused on the electron–ion spin quantum plasmas. However, some applications were presented for electron-positron plasmas and electron-positron-ion plasmas for both spin-1/2 quantum plasma and spinless quantum plasma. For instance, the set of spin-1/2 QHD equations developed for electron-ion plasmas was applied for e-p plasmas by Brodin and Marklund [34]. They found new spin depended Alfvénic solitary structures obeying the Kortewegde Vries equation, where the nonlinearity is caused by spin effects. Mushtaq et. al [35], [36] studied the effects of quantum Bohm potential and spin corrections on the spectrum of magnetosonic waves in non- relativistic and relativistic degenerate electron-positron-ion (e-p-i) plasmas, where the relativistic effects are included in the Fermi pressure only. A hydrodynamic and kinetic models for spin-1/2 electron-positron quantum plasmas has been developed in Ref. [37] which incorporates the Coulomb, spin-spin, Darwin and annihilation interactions. There was concluded that the contributions of the annihilation interactions shifts the eigenfrequencies of the transverse electromagnetic plane polarized waves and transverse spin-plasma waves.

New longitudinal wave in the degenerate e-p-i spinless quantum plasma has been reported in Refs. [38]–[42]. It was called positron acoustic wave (PAW). For ultrarelativistic electrons and non-relativistic positrons the dispersion relation of PAWs in the intermediate wave range were obtained [40]. The nonlinear wave structure of large amplitude PAWs in e-p plasma with electron beam has been discussed in Ref. [38].

In the present work, we employ the separated spin evolution QHD for the e-p and e-p-i magnetized degenerate plasmas. At consideration of separate evolution of spin-up and spin-down electrons and positrons we discuss the oblique propagation of longitudinal waves. So, in the present work we calculate the spectrum of SEAWs, PAWs and predict spin-electron-positron acoustic waves.

II. ANALYTICAL MODEL

The SSE-QHDs developed in Refs. [17], [43] can be applied to the electron-positron plasmas and electron-positron-ion plasmas. Therefore, we present the continuity and Euler equations for each spin projection of each species.

In this paper we consider the evolution of the longitudinal waves in non-relativistic plasmas. Hence the spin evolution does not affects our results. The Fermi spin current (the thermal part of the spin current for the degenerate spin-1/2 fermions) obtained in Ref. [44] is not considered here. Explicit spin contribution in the spectrum of the Langmuir waves arising via the spin-orbit interaction is found in Ref. [45] (see formula (34) for describing propagation perpendicular to the external magnetic field). In our approximation we need the continuity and Euler equations for each subspecies.

The continuity equation in the SSE-QHD arises as follows [17]

$$\partial_t n_{as} + \nabla(n_{as}\mathbf{v}_{as}) = (-1)^{i_s} T_{az}, \tag{1}$$

where $a = e, p$ for electrons and positrons correspondingly, $s = u, d$ for the spin-up and spin-down conditions of particles, n_{as} and $\mathbf{v}_{as}$ are the concentration and velocity field of particles of species a being in the spin state s, $T_{az} = \frac{\gamma}{\hbar}(B_x S_{ay} - B_y S_{ax})$ is the z-projection of spin torque, i_s: $i_u = 2$, $i_d = 1$, with the spin density projections S_{ax} and S_{ay}, each of them simultaneously describe evolution of the spin-up and spin-down particles of each

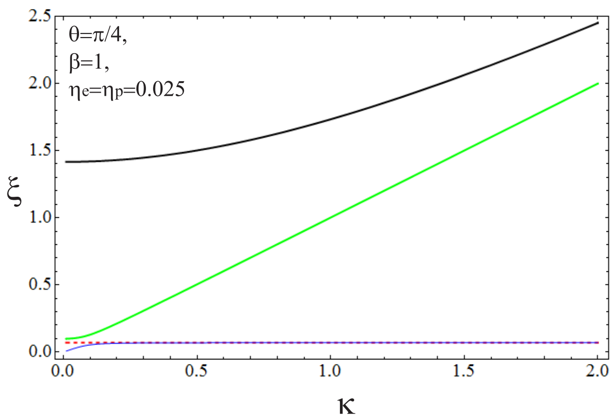

FIG. 1: (Color online) The figure shows the dispersion of the oblique propagating longitudinal waves in the electron-positron plasmas. It shows four waves. Upper branch describes the Langmuir wave. Second line from the bottom, which looks almost horizontal and depicted by the dashed line, presents the Trivelpiece-Gould wave. Two other waves are the lower and upper branches of the spin-electron acoustic waves. Details of the low frequency branches are shown in the next figure.

species. Therefore, functions S_{ax} and S_{ay} do not bear subindexes u and d. In this model the z-projection of the spin density S_{az} is not an independent variable, it is a combination of concentrations $S_{az} = n_{au} - n_{ad}$.

The time evolution of the velocity fields of all species of particles for each projection of spin $\mathbf{v}_{au}$ and $\mathbf{v}_{ad}$ is governed by the Euler equations [17]

$$mn_{as}(\partial_t + \mathbf{v}_{as}\nabla)\mathbf{v}_{as} + \nabla P_{as}$$

$$= q_a n_{as}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_{as}, \mathbf{B}]\right) + (-1)^{i_s}\gamma_a n_{as}\nabla B_z$$

$$+\frac{\gamma_a}{2}(S_{ax}\nabla B_x + S_{ay}\nabla B_y) + (-1)^{i_s} m(\widetilde{\mathbf{T}}_{az} - \mathbf{v}_{as}T_{az}), \quad (2)$$

with $P_{as} = (6\pi^2)^{\frac{2}{3}} n_{as}^{\frac{5}{3}}\hbar^2/5m$, $\widetilde{\mathbf{T}}_{az} = \frac{\gamma_a}{\hbar}(\mathbf{J}_{(M)ax}B_y - \mathbf{J}_{(M)ay}B_x)$, which is the torque current, where $\mathbf{J}_{(M)ax} = (\mathbf{v}_{au} + \mathbf{v}_{ad})S_{ax}/2$, and $\mathbf{J}_{(M)ay} = (\mathbf{v}_{au} + \mathbf{v}_{ad})S_{ay}/2$ are the convective parts of the spin current tensor. All species affect each other via the electric field: $\nabla\mathbf{E} = 4\pi\sum_{a,s} q_{as}n_{as}$ and $\nabla\times\mathbf{E} = 0$.

The SSE-QHDs was applied to two-dimensional electron gas in plane samples and nanotubes located in external magnetic fields [46]. It was found that in two-dimensional electron gas Langmuir wave replaced by the couple of hybrid waves by considering separate spin-up electrons and spin- down electrons evolution. One of them is the modified Langmuir wave and the other is SEAW. Surface SEAWs was considered in Ref. [47]. Linear interaction between surface SEAW and surface Langmuir wave (surface plasmons) is found in Ref. [47]. In

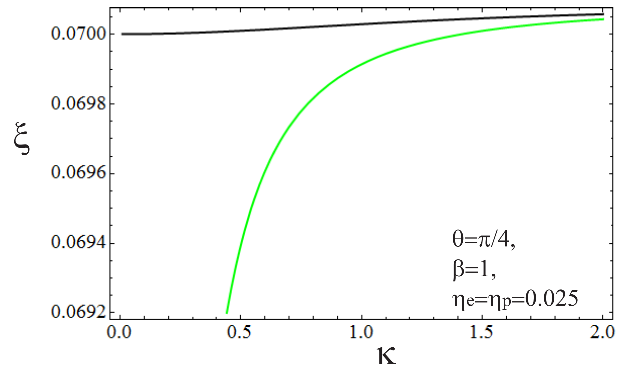

FIG. 2: (Color online) The figure shows the low frequency part of spectrum of oblique propagating longitudinal waves in e-p plasmas, where $\Sigma = 0.1$.

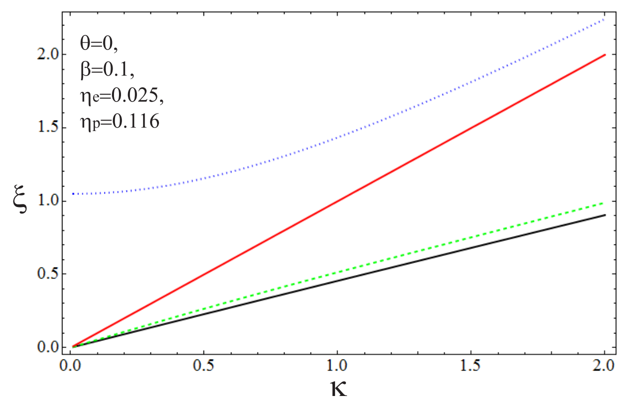

FIG. 3: (Color online) The figure shows the longitudinal waves in e-p-i plasmas propagating parallel to the external magnetic field. Ratio between the electron and positron concentrations is chosen to be $\beta = 0.1$. The upper branch shows the Langmuir wave dispersion. Three linear dependencies present SEAW, PAW and SEPAW.

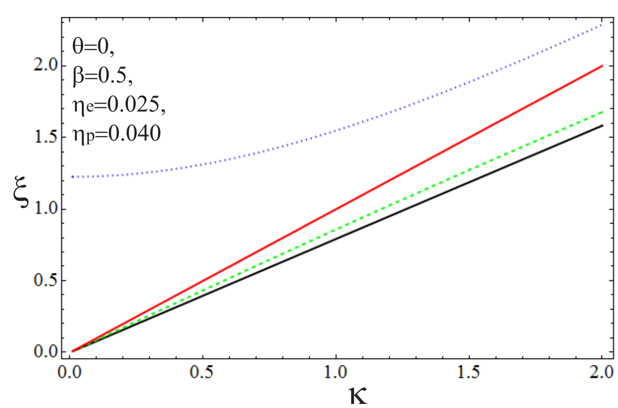

FIG. 4: (Color online) The figure shows the longitudinal waves in e-p-i plasmas propagating parallel to the external magnetic field for $\beta = 0.5$ and $\Sigma = 0.1$.

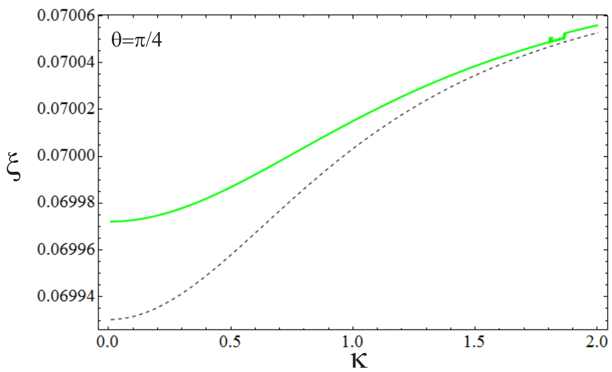

FIG. 5: (Color online) The figure shows the TrivelpieceGould wave in the e-p-i plasmas for the different ratios between concentrations of electrons and positrons. Upper (lower) line is constructed for $\beta = 0.5$ ($\beta = 0.1$) and $\Sigma = 0.1$.

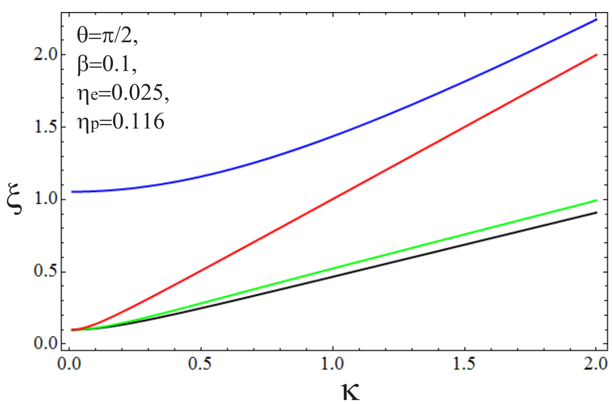

FIG. 6: (Color online) The figure shows the longitudinal waves in e-p-i plasmas propagating perpendicular to the external magnetic field for $\beta = 0.1$. As in Fig. 3 we have four branches: Langmuir wave (in other words the upper hybrid wave), SEAW, PAW and SEPAW. It shows the frequency square shift for all branches on Ω^2, with the dimensionless cyclotron frequency $\Sigma = 0.1$.

order to discuss the in-depth analysis of SEAW, which was predicted by method of SSE-QHDs, method of separate spin evolution quantum kinetics, which separately describes spin-up and spin-down electrons, was developed in Ref. [48]. By applying this method, the effects of SSE on the real dispersion and Landau damping of SEAW were addressed and real and imaginary parts of spectrums of ion-acoustic waves and zeroth sound have also been found. Nonlinear SEAWs in presence of the exchange interaction are considered in Ref. [49], where the existence of spin-electron acoustic soliton is demonstrated. Subsequently, in the Ref. [50] it has been demonstrated that the existence of SEAW leads to an explanation of the mechanism of the electron Cooper pair formation in the high temperature superconductors as result of electron-spelnon interaction (spelnon is the quanta of the SEAW). Moreover, SSE-QHD model applied to study the oblique propagation of longitudinal waves in magnetized spin-1/2 plasmas and found that instead of two well known waves (Langmuir wave and Trivelpiece–Gould wave) four wave solutions appeared in separate spin-up and spin-down degenerate magnetized plasma [43].

We deal with e-p plasmas and e-p-i plasmas located in an external magnetic field $\mathbf{B}_{ext} = B_0\mathbf{e}_z$ and study the dispersion of waves in these systems. We assume that electrons, positrons, and ions have non-zero uniform equilibrium concentrations. The equilibrium velocity fields of all species are equal to zero. For e-p plasmas concentrations of all electrons and all positrons are equal $n_{0e} = n_{0p}$. Their spin polarization are equal to each other as well $\eta_e = \eta_p$, where $\eta_a = 3\mu_B B_0/2\varepsilon_{Fa}$, with the Bohr magneton μ_B and Fermi energy $\varepsilon_{Fa} = (3\pi^2 n_{0a})^{\frac{2}{3}}\hbar^2/2m$ of species a. Thus, we have $n_{0eu} = n_{0pd}$ and $n_{0ed} = n_{0pu}$, where $n_{0eu} = n_{0e}(1-\eta_e)/2$, $n_{0ed} = n_{0e}(1+\eta_e)/2$, $n_{0pu} = n_{0p}(1+\eta_p)/2$, $n_{0pd} = n_{0p}(1-\eta_p)/2$. For the e-p-i plasmas we have $n_{0e} = n_{0p} + n_{0i}$. Consequently, the equilibrium concentrations of electrons and positrons are not equal each other. Therefore, their spin polarization $\eta_a \sim n_{0a}^{-2/3}$ are not equal each other either. Hence, parameters n_{0eu}, n_{0ed}, n_{0pu} and n_{0pd} are four independent parameters (one can use another set of parameters n_{0e}, n_{0p}, η_e, η_p). Relations between parameters depend on the external magnetic field. Next, we consider linear evolution of perturbations. For the oblique propagating plane waves $\mathbf{k} = \{k_x, 0, k_z\}$ we find the following dispersion equation:

$$\sum_{a=e,p}\left(\frac{\sin^2\theta}{\omega^2-\Omega^2}+\frac{\cos^2\theta}{\omega^2}\right)\Biggl[\frac{\omega_{Lau}^2}{1-(\frac{\sin^2\theta}{\omega^2-\Omega^2}+\frac{\cos^2\theta}{\omega^2})U_{au}^2k^2}$$

$$+\frac{\omega_{Lad}^2}{1-(\frac{\sin^2\theta}{\omega^2-\Omega^2}+\frac{\cos^2\theta}{\omega^2})U_{ad}^2k^2}\Biggr]=1, \qquad (3)$$

where $\omega_{Lau}^2 = 4\pi e^2 n_{0au}/m$, and $\omega_{Lad}^2 = 4\pi e^2 n_{0ad}/m$ are the Langmuir frequencies for the spin-up and spin-down particles of species a, and $\omega_{La}^2 = \omega_{Lau}^2 + \omega_{Lad}^2$, $\Omega = eB_0/mc$ is the cyclotron frequency, $k^2 = k_x^2 + k_z^2$, $U_{as}^2 = (6\pi^2 n_{0as})^{\frac{2}{3}}\hbar^2/3m^2$, θ is the angle between direction of wave propagation $\mathbf{k}$ and the external magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_z$.

For the electron-positron plasmas we have $n_{0e} = n_{0p}$, $n_{0eu} = n_{0pd}$, $n_{0ed} = n_{0pu}$. Hence, equation (3) in this regime is an equation of the fourth degree relatively ω^2 for the oblique propagation. At the propagation parallel or perpendicular to the external magnetic field equation (3) simplifies to equation of the second degree.

In the regime of e-p-i plasmas all four parameters n_{0eu}, n_{0ed}, n_{0pu} and n_{0pd} are different from each other. As the result equation (3) is an equation of eight degree relatively ω^2 for the oblique propagating waves. In the regimes of the parallel or perpendicular propagation it simplifies to the equation of fourth degree relatively ω^2.

III. NUMERICAL ANALYSIS

In our numerical analysis we use a single value of the electron concentration $n_{0e} \equiv n_0 = 10^{27}$ cm^{-3}. We have different regimes of the positron concentrations. To measure the concentrations of positrons in units of the equilibrium concentration of electrons we introduce parameter $\beta = n_{0p}/n_{0e}$. For the electron-positron plasmas we have $\beta = 1$. For the electron-positron-ion plasmas we have $\beta \in (0, 1)$. We consider two values of β for electron-positron-ion plasmas. They are $\beta = 0.5$ and $\beta = 0.1$.

Changing β at the fixed number of electrons n_{0e} we change the full concentration of light particles in the system. It changes the effective Langmuir frequency, which is the frequency of Langmuir wave at $k \to 0$, $\omega^2_{L,eff} = 4\pi e^2(n_{0e} + n_{0p})/m = 4\pi e^2 n_{0e}(1 + \beta)/m$. It creates difference in the behavior of the Langmuir wave spectrum on different figures. It is well known result which is not affect the effects caused by the spin polarization considered in this paper.

For presentation of numerical results we use the following dimensionless variables: the dimensionless frequency $\xi = \omega/\omega_{Le}$, the dimensionless cyclotron frequency $\Sigma = \Omega_e/\omega_{Le}$, and the dimensionless wave vector $\kappa = v_{Fe}k/3\omega_{Le}$.

A. Electron-positron plasmas

We start our analysis with relatively simple case of e-p plasmas ($\beta = 1$). Due to the equal concentrations of electrons and positrons they have same spin polarization. Due to difference of the sign of their electric charges the numbers of spin-up electrons and spin-down positrons are equal to each other and these subspecies moves in phase. Same picture we have for spin-down electrons and spin-up positrons.

At the propagation of waves parallel or perpendicular to the external field we find two wave solutions: the Langmuir wave and SEAW, with the properties similar to the properties of these waves in e-i plasmas described in [17].

Considering the oblique propagation of longitudinal waves in e-p plasmas we obtain Fig. 1 showing four waves. This regime demonstrates existence of two SEAWs, similarly to the e-i plasmas considered in Ref. [43]. Hence, Fig. 1 shows the Langmuir wave, the upper SEAW, the Trivelpiece-Gould wave, and the lower SEAW correspondingly, in order of the decrease of their frequency. Relative behavior of the lower SEAW and the Trivelpiece-Gould wave is shown in Fig. 2. We see that they do not have any overlapping. The SEAW has smaller frequencies for all physically possible wave vectors.

To summarize this subsection we report of existence of two SEAWs in the e-p plasmas. We also report increase of the Langmuir wave frequency due to spin polarization entering spectrum via the Fermi pressure.

B. Electron-positron-ion plasmas

Describing longitudinal waves in e-p-i plasmas, we start with propagation of waves parallel to the external magnetic field. Fig. 3 (Fig. 4) shows results for $\beta = 0.1$ ($\beta = 0.5$).

In the beginning of Sect. III we have described the mechanism of shift of the Langmuir wave dispersion dependence. Same effect reveals itself in the spectrum of Trivelpiece-Gould wave, as it is depicted in Fig. 5. First of all we see presence of four branches in both cases. Comparing these results with the well-known results and results found in Refs. [17], [43] we make the following conclusions. The upper dispersion branch belongs to the Langmuir wave. One of these linear branches describes the SEAW as it follows from the previous subsection and Ref. [17]. One of two other branches is the PAW found in Refs. [38], [39], which exists due to different concentrations of electrons and positrons. In addition to three earlier found wave solutions we obtain an extra solution.

In the e-p-i plasmas we find four subspecies with different spin polarizations $(1+\eta_e)/2$, $(1-\eta_e)/2$, $(1-\eta_p)/2 = (1 + \eta_e\beta^{-\frac{2}{3}})/2$, and $(1 + \eta_p)/2 = (1 - \eta_e\beta^{-\frac{2}{3}})/2$ instead of two existing in e-i or e-p plasmas. Therefore, we have found reacher spectrum of the spin-electron acoustic excitations.

Since new wave arises in the e-p-i plasmas due to the account of the SSE we call it the spin-electron-positron acoustic wave (SEPAW). The spectrum changes at the change of positron number. As we mention above, it changes the full number of the light particles. However, it changes the spin polarization of positrons depending on the Fermi energy of positrons. Therefore, the change of β affects the SEPAW.

We calculate spectrum of longitudinal waves propagating perpendicular to the external field as well. We see that it increases square of all waves on Ω^2 as it follows from Fig. 6. The frequencies of acoustic waves tend to zero at $k \to 0$ in the regime of parallel propagation. Being shifted in the regime of the perpendicular propagation we have $\omega \to \Omega$ at $k \to 0$. It is different for the Langmuir wave. In the regime of parallel propagation we find $\omega^2 \to (1+\beta)\omega^2_{Le}$ at $k \to 0$, while for the perpendicular propagation we obtain $\omega^2 \to (1+\beta)\omega^2_{Le}+\Omega^2$ at $k \to 0$. Therefore, the shift of frequency $\omega(k \to 0)$ is smaller than Ω. In our case, for $\Omega = \Sigma\omega_{Le} \approx 0.1\omega_{Le}$. It is hardly visible in Fig. 6, since $\omega \simeq \omega_{Le}[\sqrt{(1+\beta)}+\Sigma^2/2\sqrt{(1+\beta)}] = \omega_{Le}[1.05 + 0.01]$.

The upper linear branch in Figs. (3), (4), (6) presents the SEAW similar to wave in the e-i plasmas. The middle linear branch presents the PAWs, where spin effects increases frequency of the PAW. The lower branch is the SEPAW, which is the SEAW in the subsystem of positrons. It is located lower than SEAW in the electron subsystem due to the smallar concentration of positrons in compare with the concentration of electrons $\beta < 1$.

As it is directly follows from the dispersion equation (3) number of the dispersion branches doubles at the oblique propagation. Thus, we have eight longitudinal waves, which reduces to four branches at the parallel and perpendicular propagations. It is well-known that in e-i plasmas at the oblique wave propagation we find Trivelpiece-Gould and Langmuir waves. The e-i plasmas with the SSE picture is more interesting. Instead of the Langmuir wave and the SEAW one can find: the Langmuir wave, the Trivelpiece-Gould wave, the lower SEAW and the upper SEAW. If we forget about spin separation and consider e-p-i plasmas one can find the PAW [38], [39] at the parallel or perpendicular propagation. At the transition to the oblique regime we expect to find the Langmuir wave, the Trivelpiece-Gould wave, and two PAWs. To the best of our knowledge the second PAW existing at the oblique propagation has not been reported in literature. Hence, in this paper, we report the existence of this wave.

Main subject of this paper is the e-p-i plasmas with the account of the SSE. Therefore, we obtain: the Langmuir wave, the Trivelpiece-Gould wave, the pair of PAWs, lower and upper SEAWs, and reported for the first time, a pair of SEPAWs. The spectrum of all these waves is shown in Fig. 7 for $\beta = 0.1$. We see that the decrease of the positron concentration causes shifts of dispersion dependencies of PAWs and SEPAWs into area of larger frequencies, while the dispersion dependencies of SEAWs do not show any visible changes. Consequently the dispersion dependencies of SEAW, PAW, and SEPAW becomes closer. It is hard to distinguish the low frequency part of the spectrum in Fig. 7. Hence, we present Fig. 8, where the low frequency part of the spectrum is depicted. Fig. 8 contains the Trivelpiece-Gould wave and lower branches of the spin-electron acoustic, positron acoustic, and spin-electron-positron acoustic waves.

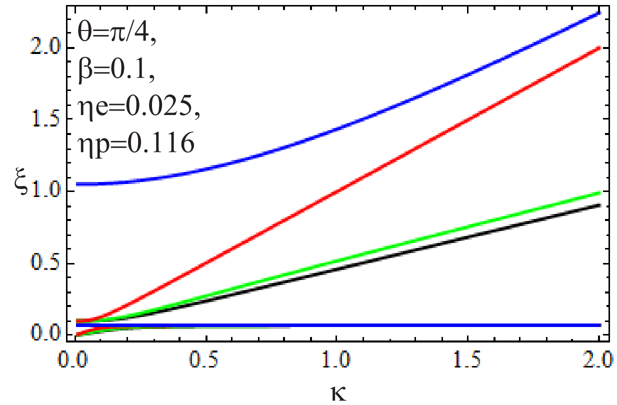

FIG. 7: (Color online) The figure shows the oblique propagating longitudinal waves in the e-p-i plasmas. In this regime we find eight branches described in the text. Figure is constructed for $\beta = 0.1$ and $\Sigma = 0.1$.

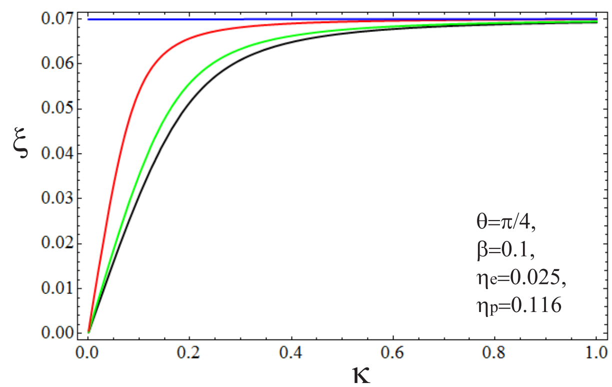

FIG. 8: (Color online) The figure shows details of the low frequency part of spectrum of the oblique propagating longitudinal waves for $\beta = 0.1$ and $\Sigma = 0.1$.

C. Area of applicability of obtained results

Let us consider the following thought experiment in context of analysis of propagation of waves parallel to the external magnetic field.

Keeping a fixed number of electrons we can add a number of positrons and take away same number of ions to keep the quasi-neutrality of the system. This imaginary quasicontinuous process we could find changes in the spectrum of the SEAW, along with the discussed above changes of the Langmuir wave spectrum. In addition to these changes we expect to find two other waves: the PAW and the SEAW. However, these two waves are predicted for plasmas with the degenerate electrons and positrons.

If we consider a temperature regime with the degenerate electrons $T \ll T_{Fe}$, and with the both subspecies of electrons are degenerate either $T \ll T_{Feu}, T_{Fed}$ (it requires relatively small spin polarization for the finite temperatures T), we find that positrons, for a small number of them, a non-degenerate $T \sim T_{Fp}$ or $T \gg T_{Fp}$. Here we have used the Fermi temperatures for the electrons T_{Fe}, spin-up electrons T_{Feu}, spin-down electrons T_{Fed}, and positrons T_{Fp}. In this regime, our model presented by equation (1) and (2) does not work. A strong collision damping in the system of positrons might destroy the PAW and SEPAW. Increasing number of positrons to reach conditions for the degenerate positron gas we enter area of applicability of our results.

For $n_{0e} = 10^{27}$ cm^{-3} and $n_{0p} = 0.1n_{0e} = 10^{26}$ cm^{-3}, at $B_0 = 10^{10}$ G, we have $\eta_e = 0.025$ and $\eta_p = 0.116$. It gives $T_{Fes}(1 \pm \eta_e)^{\frac{2}{3}} T_{Fe} \approx T_{Fe} = 3.47 \times 10^7$ K and $T_{Fps} = \{0.23, 0.21\} T_{Fe} \approx 0.2T_{Fe} = 0.7 \times 10^7$ K. Hence, our results can be used for the plasmas with temperatures below 10^6 K. At larger concentrations similar results can be found for larger temperatures.

IV. CONCLUSION

Separate spin evolution in systems with the partial spin polarization has revealed itself in existence of new waves.

Thereby, we have found the SEAWs in the e-p plasmas. One SEAW exists at the parallel and perpendicular propagation. Two branches exist at the oblique propagation. Their appearance is related to different Fermi pressure for the spin-up and spin-down electrons and positrons. For e-p-i plasmas we have demonstrated existence of pair of SEAWs, pair of PAWs, and pair of SEPAWs along with the Langmuir and Trivelpiece–Gould waves, at the oblique propagation. Three of them have been found for the first time: pair of SEPAWs and second (upper) PAW. These eight branches reduces to four branches at the parallel and perpendicular propagation. These branches are Langmuir wave, SEAW, PAW and SEPAW. The SEPAW has been reported for the first time.

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