Exploring open-charm decay mode $\Lambda_c\bar{\Lambda}_c$ of charmonium-like state $Y(4630)$

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The structures of the X, Y, Z exotic states which recently are experimentally observed one after another, have composed a challenge to our understanding of the fundamental principles in the hadron physics and the relevant study will definitely shed light on the concerned physics. Generally the four-quark states might be in a molecular state or teraquark or their mixture. In this work, we adopt the suggestion that $Y(4630)$ is a charmonium-like tetraquark made of a diquark and an anti-diquark. If it is true, its favorable decay mode should be $Y(4360)$ decaying into an open-charm baryon pair, since such a transition occurs via strong interaction and is super-OZI-allowed. In this work, we calculate the decay width of $Y(4630) \to \Lambda_c^+\Lambda_c^-$ in the framework of the quark pair creation (QPC) model. The numerical results suggest that the $Y(4630)$ state is a radially excited state.

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I. INTRODUCTION

In 2007, the Belle collaboration reported that a $J^{PC} = 1^{--}$ resonance peak $Y(4630)$ with mass $M = 4634^{+9}_{-11}$ MeV and width $\Gamma = 92^{+41}_{-32}$ MeV appeared at the invariant mass spectra of the $e^+e^- \to \Lambda_c^+\Lambda_c^-$ channel [1].

Besides an interpretation that the observed $Y(4630)$ as the 5^3S_1 charmonium state [2, 3], there are many alternative suggestions for the observed peaks, for example, $Y(4360)$ was considered to be induced by a threshold effect instead of being a genuine resonance [4], then it was also interpreted as a molecular state made of $\psi(2S)$ and $f_0(980)$ by another theoretical physics group [5]. Among those proposals, the suggestion that $Y(4630)$ is a tetraquark state is more favorable [6, 7]. In Ref. [6], the $Y(4630)$ is identified as the ground state with its orbital angular momentum $L = 1$. It is noted that the mass and width of $Y(4630)$ are consistent within errors with those for the $Y(4660)$ state ($M = 4664 \pm 12$MeV, $\Gamma = 48 \pm 15$MeV), which is found in the invariant mass spectrum of $\psi(2S)\pi^+\pi^-$ by the Belle collaboration [8]. By understanding the $\Lambda_c^+\Lambda_c^-$ and $\psi(2S)\pi^+\pi^-$ spectra, Cotugno *et al.* suggested that the $Y(4630)$ and $Y(4660)$ could be the same tetraquark state, and is the first radial excitation of the $Y(4360)$ with $L = 1$ [7].

Inspired by the fact that the $Y(4630)$ predominantly decays into charmed baryon pair, one is tempted to conjecture this resonance as a tetraquark which is made of the diquark-antidiquark structure $[cq][\bar{c}\bar{q}]$, where q is a light quark either u or d and $[cq]$ resides in a color anti-triplet whereas $[\bar{c}\bar{q}]$ is in a color triplet, in later calculations we do not distinguish between u and d. The quark diagram for $Y(4630) \to \Lambda_c\bar{\Lambda}_c$ shows that the decay is an OZI super-allowed process. The corresponding reaction mechanism is that a quark-antiquark pair is created from the vacuum, then the quark and anti-quark would join the diquark and antidiquark respectively to constitute a baryon-anti-baryon pair. Moreover, $M_{\Lambda_c} + M_{\bar{\Lambda}_c}$ is only slightly below 4630 MeV, so that the suppression induced by the momentum matching which usually appears at similar processes, does not exist here. The decay width can be well calculated by using the quark pair creation (QPC) model.

In this work, we suppose that $Y(4360)$ is a tetraquark composed of the diquark-antidiquark structure suggested by Brodsky *et al.* [9]. In the tetraquark a diquark and an anti-diquark are bound together via the QCD confinement, but are separated by a substantial distance once they are created. This picture is somehow different from the picture about the tetraquark first proposed by Maiani *et al.* [6, 10], where the authors studied the tetraquark states in terms of spin structure in a Hamiltonian formalism [11]. Under this assignment, we study the strong decay of $Y(4630)$ by computing the decay width of $Y(4630) \to \Lambda_c^+\Lambda_c^-$ in the QPC model.

The paper is organized as follows: after this introduction, we calculate the rate of $Y(4630)$ decaying into the $\Lambda_c^\pm$ pair in section. II A and perform a numerical analysis in Sec. II B. Sec. III is devoted to our discussion and conclusion.

II. THE $Y(4630) \to \Lambda_c\bar{\Lambda}_c$ STRONG DECAY

In this work, we use the two-body wave function for the diqaurk-antidiquark bound system $Y(4630)$, since the constituents (diquark and antidiquark) are treated as two point-like color sources which are in the same configuration of an ordinary $Q\bar{Q}$ meson.

The spin wave functions of a $Y(J^{PC} = 1^{--})$ state with $L = 1$ in the basis of $|S_{qc}, S_{\bar{q}\bar{c}}, S_{\text{total}}, L\rangle_{J=1}$ can be assigned in four distinct states as [6]

$$Y_1 = |0,0,0,1\rangle_1,$$

$$Y_2 = \frac{1}{\sqrt{2}}\left[|1,0,1,1\rangle_1 + |0,1,1,1\rangle_1\right],$$

$$Y_3 = |1,1,0,1\rangle_1,$$

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$$Y_4 \;=\; |1,1,2,1\rangle_1 .$$

In the following, we present all the details of calculating $Y(4630) \to \Lambda_c\bar{\Lambda}_c$ in the QPC model.

A. Implementation in the QPC model

The QPC model [12–18] has been widely applied to calculate the rates of Okubo-Zweig-Iizuka (OZI) allowed strong decays [19–31], which obviously compose the dominant contributions to the total widths of the concerned hadrons.

As indicated in the introduction, we suppose $Y(4630)$ as a tetraquark in the diquark-antidiquark structure, thus in our case, the decay of $Y(4630)$ is a fall-apart process where the diquark and antidiquark bound state is loosened by a quark-antiquark pair which is created in vacuum. Concretely, the quark and antiquark of the pair excited out from the vacuum would join the diquark and antidiquark respectively to compose a $\Lambda_c^+\Lambda_c^-$ pair, and the process is graphically shown in Fig. 1.

The quantum number of the created quark pair is 0^{++} [12, 13]. In the non-relativistic limit, the transition operator is expressed as

$$\begin{aligned} T \;=\; & -3\gamma \sum_m \langle 1\ m; 1\ -m|0\ 0\rangle \int \mathrm{d}\mathbf{k}_5\ \mathrm{d}\mathbf{k}_6 \delta^3(\mathbf{k}_5+\mathbf{k}_6) \\ & \times \mathcal{Y}_{1m}\left(\frac{\mathbf{k}_5-\mathbf{k}_6}{2}\right) \chi^{56}_{1,-m}\ \varphi^{56}_0\ \ \omega^{56}_0\ d^\dagger_{5i}(\mathbf{k}_5)\ b^\dagger_{6j}(\mathbf{k}_6)\, , \end{aligned} \quad (1)$$

where i and j are the SU(3)-color indices of the created quark and anti-quark. $\varphi^{56}_0 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $\omega^{56}_0 = \delta_{ij}$ are for flavor and color singlets, respectively. $\chi^{56}_{1,-m}$ is a spin triplet. Here the indices 5 and 6 distinguish between the quark and antiquark respectively as shown in Fig. 1. $\mathcal{Y}_{\ell m}(\mathbf{k}) \equiv |\mathbf{k}|^{\ell} Y_{\ell m}(\theta_k, \phi_k)$ denotes the ℓth solid harmonic polynomial. γ is a dimensionless constant for the strength of quark pair creation from vacuum and is fixed by fitting data.

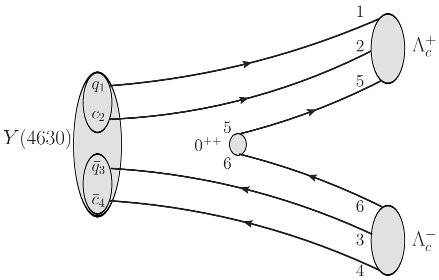

FIG. 1: The QPC mechanism for decay $Y(4630) \to \Lambda_c^+ + \Lambda_c^-$, we label the quark c and antiquark $\bar{c}$ with subscripts 2 and 4, as well understood. q stands for the light quark u/d.

In the dynamical picture of tetraquark, the (anti)diquark is considered to be a point-like color source, then the two-body wave function(meson-like) should be a good approximation to describe the inner structure of $Y(4630)$. Including the color ($\omega_Y^{(12)(34)}$), spin ($\chi_Y^{(12)(34)}$), flavor ($\varphi_Y^{(12)(34)}$) and the spatial ($\Psi_{n_Y L_Y M_{L_Y}}(\mathbf{p}_1, \mathbf{p}_2)$) parts, the wave function is written as

$$\begin{aligned} & \left|Y({n_Y}^{2S_Y+1}L_{Y\ J_Y M_{J_Y}})(\mathbf{K}_Y)\right\rangle \\ &= \sqrt{2E_Y} \sum_{M_{L_Y},M_{S_Y}} \langle L_Y M_{L_Y} S_Y M_{S_Y} | J_Y M_{J_Y}\rangle \\ &\quad \times \int \mathrm{d}\mathbf{p}_1 \mathrm{d}\mathbf{p}_2 \mathrm{d}\mathbf{k}_1 \mathrm{d}\mathbf{k}_2 \mathrm{d}\mathbf{k}_3 \mathrm{d}\mathbf{k}_4 \delta^3\left(\mathbf{K}_{\mathrm{Y}} - \mathbf{p}_1 - \mathbf{p}_2\right) \\ &\quad \times \delta^3\left(\mathbf{p}_1 - \mathbf{k}_1 - \mathbf{k}_2\right) \delta^3\left(\mathbf{p}_2 - \mathbf{k}_3 - \mathbf{k}_4\right) \Psi_{n_Y L_Y M_{L_Y}}\left(\mathbf{p}_1, \mathbf{p}_2\right) \\ &\quad \times \chi_{S_Y M_{S_Y}} \varphi_Y^{(12)(34)} \omega_Y^{(12)(34)} \left| \, q_1\left(\mathbf{k}_1\right)\ q_2\left(\mathbf{k}_2\right) \bar{q}_3\left(\mathbf{k}_3\right) \bar{q}_4\left(\mathbf{k}_4\right)\right\rangle , \end{aligned} \quad (2)$$

where we use the (super)subscript 1∼4 to mark the (anti)quark in the tetraquark as clearly shown in Fig 1. $\mathbf{K}_Y$ is the 3-momentum of $Y(4630)$, $\mathbf{p}_{1(2)}$ is the 3-momentum of the (anti)diquark, $\mathbf{k}_{1,2,3,4}$ are the 3-momentum of the (anti)quarks in $Y(4630)$ and the three delta functions respectively specify the 3-momentum combinations of the ingredients in the two separate groups: diquark and antidiquark and the combination of the 3-momenta of diquark-antidiquark in $Y(4630)$. $\mathbf{S}_Y = \mathbf{s}_{q_1} + \mathbf{s}_{q_2} + \mathbf{s}_{\bar{q}_3} + \mathbf{s}_{\bar{q}_4}$ is the total spin. $\mathbf{J}_Y = \mathbf{L}_Y + \mathbf{S}_Y$ denotes the total angular momentum of $Y(4360)$.

For the Λ_c baryon, we have

$$\begin{aligned} & \left|\Lambda_c({n_{\Lambda_c}}^{2S_{\Lambda_c}+1}L_{\Lambda_c\ J_{\Lambda_c} M_{J_{\Lambda_c}}})(\mathbf{K}_{\Lambda_c})\right\rangle \\ &= \sqrt{2E_{\Lambda_c}} \sum_{M_{L_{\Lambda_c}},M_{S_{\Lambda_c}}} \left\langle L_{\Lambda_c} M_{L_{\Lambda_c}} S_{\Lambda_c} M_{S_{\Lambda_c}} | J_{\Lambda_c} M_{J_{\Lambda_c}}\right\rangle \\ &\quad \times \int \mathrm{d}\mathbf{k}_1 \mathrm{d}\mathbf{k}_2 \mathrm{d}\mathbf{k}_3 \delta^3\left(\mathbf{K}_{\Lambda_c} - \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3\right) \\ &\quad \times \Psi_{n_{\Lambda_c} L_{\Lambda_c} M_{L_{\Lambda_c}}}\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3\right) \chi^{123}_{S_{\Lambda_c} M_{S_{\Lambda_c}}} \varphi^{123}_{\Lambda_c} \omega^{123}_{\Lambda_c} \\ &\quad \times \left| \, q_1\left(\mathbf{k}_1\right) q_2\left(\mathbf{k}_2\right) q_3\left(\mathbf{k}_3\right)\right\rangle . \end{aligned} \quad (3)$$

For $\Lambda_c(\bar{\Lambda}_c)$, the (super)subscripts in the expressions correspond to the three constituent quarks(antiquarks), as shown in Fig. 1 and $\mathbf{K}_{\Lambda_c}$ is the 3-momentum of Λ_c. $\mathbf{S}_{\Lambda_c} = \mathbf{s}_{q_1} + \mathbf{s}_{q_2} + \mathbf{s}_{q_3}$ is the total spin of the three quarks in Λ_c and $\mathbf{J}_{\Lambda_c} = \mathbf{L}_{\Lambda_c} + \mathbf{S}_{\Lambda_c}$ denotes the total angular momentum of Λ_c.

The wave functions respect the normalization conditions

$$\langle Y(\mathbf{K}_Y)|Y(\mathbf{K}'_Y)\rangle \;=\; 2E_Y\ \delta^3(\mathbf{K}_Y - \mathbf{K}'_Y), \quad (4)$$

$$\langle \Lambda_c(\mathbf{K}_{\Lambda_c})|\Lambda_c(\mathbf{K}'_{\Lambda_c})\rangle \;=\; 2E_{\Lambda_c}\ \delta^3(\mathbf{K}_{\Lambda_c} - \mathbf{K}'_{\Lambda_c}). \quad (5)$$

For $Y(4630) \to \Lambda_c^+ + \Lambda_c^-$ process, the transition hadronic matrix element is written as

$$\langle \Lambda_c^+\Lambda_c^-|S|Y(4630)\rangle \;=\; I - i2\pi\delta(E_f - E_i)\langle \Lambda_c^+\Lambda_c^-|T|Y(4630)\rangle .$$

In the center of the mass frame of $Y(4630)$, $\mathbf{K}_Y = 0$ and $\mathbf{K}_{\Lambda_c^+} =$

$-\mathbf{K}_{\Lambda_c^-} = \mathbf{K}$. Then, we have

$$
\begin{aligned}
&\langle \Lambda_c^+ \Lambda_c^- | T | Y(4630) \rangle = -3\gamma \sqrt{8 E_Y E_{\Lambda_c^+} E_{\Lambda_c^-}} \\
&\quad \times \sum_{M_{L_Y}, M_{S_Y}, m} \; \sum_{M_{L_{\rho_{\Lambda_c^+}}}, M_{S_{\Lambda_c^+}}} \; \sum_{M_{L_{\rho_{\Lambda_c^-}}}, M_{S_{\Lambda_c^-}}} \langle 1\, m; 1\ -m |\, 0\, 0 \rangle \\
&\quad \times \langle s_5\ m_5; s_6\ m_6 |\ 1\ -m \rangle \langle L_Y M_{L_Y} S_Y M_{S_Y} | J_Y M_{J_Y} \rangle \\
&\quad \times \langle S_{12} M_{S_{12}} S_{34} M_{S_{34}} | S_Y M_{S_Y} \rangle \langle S_1 M_{S_1} S_2 M_{S_2} | S_{12} M_{S_{12}} \rangle \\
&\quad \times \langle S_3 M_{S_3} S_4 M_{S_4} | S_{34} M_{S_{34}} \rangle \langle J_{15} M_{J_{15}} S_2 M_{S_2} | J_{\Lambda_c^+} M_{J_{\Lambda_c^+}} \rangle \\
&\quad \times \langle L_{\rho_{\Lambda_c^+}} M_{L_{\rho_{\Lambda_c^+}}} L_{\lambda_{\Lambda_c^+}} M_{L_{\lambda_{\Lambda_c^+}}} | L_{\Lambda_c^+} M_{L_{\Lambda_c^+}} \rangle \langle L_{\Lambda_c^+} M_{L_{\Lambda_c^+}} S_{15} M_{S_{15}} | J_{15} M_{J_{15}} \rangle \\
&\quad \times \langle S_1 M_{S_1} S_5 M_{S_5} | S_{15} M_{S_{15}} \rangle \langle J_{36} M_{J_{36}} S_4 M_{S_4} | J_{\Lambda_c^-} M_{J_{\Lambda_c^-}} \rangle \\
&\quad \times \langle L_{\rho_{\Lambda_c^-}} M_{L_{\rho_{\Lambda_c^-}}} L_{\lambda_{\Lambda_c^-}} M_{L_{\lambda_{\Lambda_c^-}}} | L_{\Lambda_c^-} M_{L_{\Lambda_c^-}} \rangle \langle L_{\Lambda_c^-} M_{L_{\Lambda_c^-}} S_{36} M_{S_{36}} | J_{36} M_{J_{36}} \rangle \\
&\quad \times \langle S_3 M_{S_3} S_6 M_{S_6} | S_{36} M_{S_{36}} \rangle \langle \varphi_{\Lambda_c^+}^{1,5,2} \varphi_{\Lambda_c^-}^{3,6,4} | \varphi_Y^{(12)(34)} \varphi_0^{56} \rangle \\
&\quad \times \langle \omega_{\Lambda_c^+}^{1,5,2} \omega_{\Lambda_c^-}^{3,6,4} | \omega_Y^{(12)(34)} \omega_0^{56} \rangle I_{M_{L_{\Lambda_c^+}}, M_{L_{\Lambda_c^-}}}^{M_{L_Y}, m} (\mathbf{K})\,. \qquad (6)
\end{aligned}
$$

The spatial integral $I_{M_{L_{\Lambda_c^+}}, M_{L_{\Lambda_c^-}}}^{M_{L_Y}, m} (\mathbf{K})$ manifests an overlap between the spacial parts of the initial state (include the created light quark pair) and the final state, and is explicitly expressed as

$$
\begin{aligned}
I_{M_{L_{\Lambda_c^+}}, M_{L_{\Lambda_c^-}}}^{M_{L_Y}, m} (\mathbf{K}) = &\int \mathrm{d}\mathbf{p}_1 \mathrm{d}\mathbf{p}_2 \mathrm{d}\mathbf{k}_1 \mathrm{d}\mathbf{k}_2 \mathrm{d}\mathbf{k}_3 \mathrm{d}\mathbf{k}_4 \mathrm{d}\mathbf{k}_5 \mathrm{d}\mathbf{k}_6 \\
&\times \delta^3(\mathbf{p}_1 + \mathbf{p}_2) \delta^3(\mathbf{p}_1 - \mathbf{k}_1 - \mathbf{k}_2) \delta^2(\mathbf{p}_2 - \mathbf{k}_3 - \mathbf{k}_4) \\
&\times \delta^3(\mathbf{K}_{\Lambda_c^+} - \mathbf{k}_1 - \mathbf{k}_5 - \mathbf{k}_2) \delta^3(\mathbf{K}_{\Lambda_c^-} - \mathbf{k}_3 - \mathbf{k}_4 - \mathbf{k}_6) \\
&\times \delta^3(\mathbf{k}_5 + \mathbf{k}_6) \\
&\times \Psi^*_{n_{\Lambda_c^+} L_{\Lambda_c^+} M_{L_{\Lambda_c^+}}} (\mathbf{k}_1, \mathbf{k}_5, \mathbf{k}_2) \Psi^*_{n_{\Lambda_c^-} L_{\Lambda_c^-} M_{L_{\Lambda_c^-}}} (\mathbf{k}_3, \mathbf{k}_6, \mathbf{k}_4) \\
&\times \Psi_{n_Y L_Y M_{L_Y}} (\frac{\mathbf{p}_1 - \mathbf{p}_2}{2}) \mathcal{Y}_{1m} \left(\frac{\mathbf{k}_5 - \mathbf{k}_6}{2} \right). \qquad (7)
\end{aligned}
$$

In this work, following the literature in this field, we employ the simple harmonic oscillator (SHO) wavefunctions to stand for the spacial parts of the two-body wave functions of $Y(4630)$ and the charmed baryons as well. Their explicit forms are collected in the appendix.

With the transition amplitude given in Eq. (6), the matrix element can be rewritten in terms of the helicity amplitude $\mathcal{M}^{M_{J_Y} M_{J_{\Lambda_c^+}} M_{J_{\Lambda_c^-}}}$ as

$$
\langle \Lambda_c^+ \Lambda_c^- | T | Y(4630) \rangle = \delta^3(\mathbf{K}_{\Lambda_c^+} + \mathbf{K}_{\Lambda_c^-} - \mathbf{K}_Y) \mathcal{M}^{M_{J_Y} M_{J_{\Lambda_c^+}} M_{J_{\Lambda_c^-}}}. \qquad (8)
$$

The decay width of $Y(4630) \to \Lambda_c^+ \Lambda_c^-$ is then

$$
\Gamma = \pi^2 \frac{|\mathbf{K}|}{M_Y^2} \frac{1}{2J_Y + 1} \sum_{M_{J_{M_Y}}, M_{J_{M_{\Lambda_c^+}}}, M_{J_{M_{\Lambda_c^-}}}} \left| \mathcal{M}^{M_{J_Y} M_{J_{\Lambda_c^+}} M_{J_{\Lambda_c^-}}} \right|^2,
$$

where $|\mathbf{K}|$, as aforementioned, is the 3-momentum of the daughter mesons in the center of mass frame.

B. Numerical results

We take $\gamma = 13.4$ following Ref. [19] and the β values for the S-wave charmed baryons can be fixed by fitting the mass spectra when the potential model is employed [21], thus $\beta_\rho = 0.6$ GeV and $\beta_\lambda = 0.6$ GeV.

Since there are still existing some ambiguities about the inner structure of $Y(4630)$, we calculate the decay width in two possible cases: assuming (1) $Y(4630)$ as the ground state with radial quantum number $n_r = 1$ and (2) the first radial excitation with $n_r = 2$ assignments.

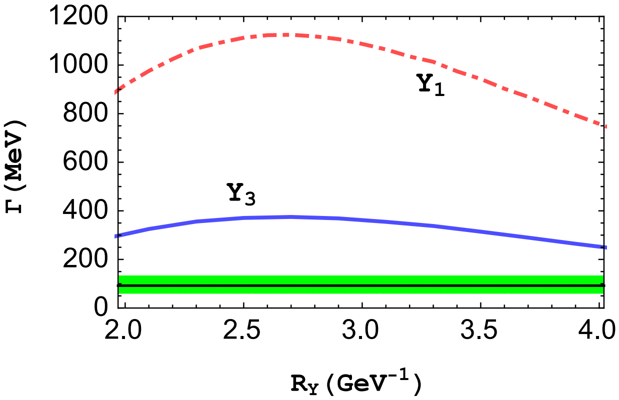

FIG. 2: The total decay width of $Y(4630)$ with $n_r = 1$ and Y_1, Y_3 spin states assignment. Here black line and the green band correspond to the central value and error range of 1σ for the total width of $Y(4630)$ measured by the Belle collaboration ($\Gamma = 92^{+41}_{-32}$ MeV) [1].

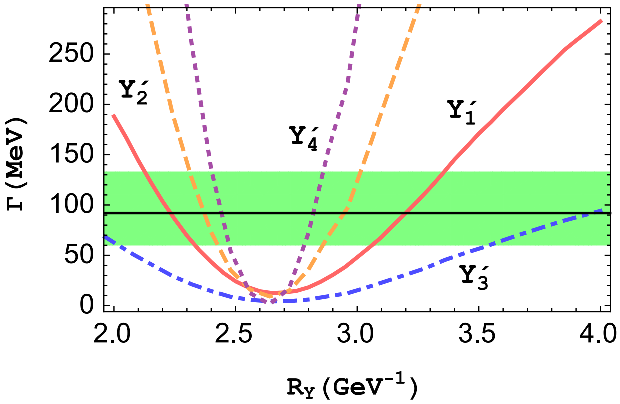

FIG. 3: Dependence of the predicted partial width of $Y(4630) \to \Lambda_c \bar{\Lambda}_c$ where $Y(4630)$ is supposed to be in $n_r = 2$ state, the Belle data are shown in the plot for a comparison. The black line and the green band correspond to the central value and error for the total width of $Y(4630)$ measured by the Belle collaboration ($\Gamma = 92^{+41}_{-32}$ MeV). The red, orange-dashed, purple-dotted and blue-dashed-dotted curves correspond to the four different spin assignments Y_1, Y_2, Y_3, Y_4, respectively. Here, we use the prime to distinguish Y states discussed in this figure.

We first compute the decay width of $Y(4630) \to \Lambda_c^+ \Lambda_c^-$ with $n_r = 1$. In Fig. 2, dependence of the calculated width Γ_Y on R_Y

which is within a reasonable range is plotted, where R_Y is the R-value in the $Y(4630)$ SHO wave function. It is noted that the predicted width for the Y_1 assignment is far away from the experimental data $\Gamma = 92^{+41}_{-32}$ MeV and the curve associated with the Y_3 assignment is slightly closer to the measured width, but still out the error tolerance. For other two spin states, the declination of the predicted values from the data is even larger than that for the Y_1 assignment, so that in the figure we do not show them after all. Therefore the data do not favor $Y(4630)$ to be a ground state with $n_r = 1$.

Then instead, assigning the $Y(4630)$ as the first radial excitation state of $Y(4360)$, we perform the numerical analysis and show the results in Fig. 3 with all the four spin assignments listed before.

In Fig. 3, the minima of the calculated decay width show up at $R_Y \sim 2.7$ GeV^{-1} which is due to the node structure of the radial wave function of $Y(4630)$.

Taking Y_1 state as an example, the calculated total decay width is consistent with the experimental data as long as R_Y lies in a range of $2.15 \sim 2.35$ GeV^{-1} and/or another range of $3.05 \sim 3.40$ GeV^{-1}, in the plot one can find the calculated value marked by a red curve does coincide with the data and the error band of 1σ marked by the green color was given by the Bell collaboration.

For the other three spin assignments, Y_2, Y_3, Y_4, the situation is similar to the Y_1 case, namely, there exist certain R_Y ranges to fit the data.

Thus, our numerical results indicate that the measured strong decay width of $Y(4630)$ (even though at present the accurate measurements are not available yet, the order of magnitude has been confirmed, i.e., the total width should be roughly determined) manifests that the assignment of $Y(4630)$ should be in the P-state of the first radial excitation of $Y(4360)$. Moreover, one could expect the physical $Y(4630)$ to be a linear combination of all the four spin states. As discussed above, we suggest that $Y(4630)$ is a tetraquark made of diquark-antidiquark, we argue that its main decay channel should be $Y(4630) \to \Lambda_c\bar{\Lambda}_c$, thus its partial width should be of the same order as the total width.

III. CONCLUSION AND DISCUSSION

To evaluate the hadronic matrix elements which are governed by the non-perturbative QCD, phenomenological models are needed. For the OZI-allowed strong decays, the QPC model, flux tube model, QCD sum rules and lattice QCD, etc. have been successfully used to estimate the decay rates, even though except the lattice calculation non of them can be directly derived from quantum field theory. We are assured that all of those models have certain reasonability and they are in parallel somehow. In this work, we employed the QPC model to study the strong decay of $Y(4630) \to \Lambda_c\bar{\Lambda}_c$.

First we assume that $Y(4630)$ is a tetraquark which is a bound state of a diquark and an anti-diquark. Because of the composition, it would favorably decay into $\Lambda_c\bar{\Lambda}_c$ pair, therefore we have reason to believe that the partial width is of the same order of magnitude as the total width which has been measured. There could be different quantum structures for the bound states, we try to assign it with different radial quantum numbers and spin assignments and then calculate the decay width of $Y(4630) \to \Lambda_c^+\Lambda_c^-$ in all possible cases. The numerical results show that one can hardly gain a decay width Γ_Y that agrees with the experimental data if we assign $Y(4630)$ as the ground state with $n_r = 1$.

Whereas for the case of $n_r = 2$, within certain ranges of the parameter R_Y, we can obtain the the partial decay width lying within the error band of the Belle data for all the four spin configurations. Thus, our analysis provides a strong support to the postulate that $Y(4630)$ is a radial excited state of the diquark-antidiquark bound state. This picture is the same as the conclusion made by the authors of Ref. [7], where $Y(4630)$ is identified as the radial excitations of the $Y(4360)$.

Besides the dominant $Y(4630) \to \Lambda_c^+\Lambda_c^-$ decay mode, the process $Y(4630) \to \psi(2S)\pi^+\pi^-$ is another possible decay channel of the $Y(4630)$ where a quark-antiquark rearrangement is required, concretely, a light quark q and a heavy antiquark $\bar{c}$ from different clusters are exchanged to form a charmonium state and light quark pair, then the light pair turns into two pions by exciting a quark-antiquark pair from the vacuum, as shown in Fig. 4. In that case, as the quark leaves the original cluster (either color antitriplet or triplet) it must overcome a potential barrier, so obviously, this process is suppressed compared to the direct fall apart decay. Observation of this mode is still missing, so far. In fact, such quark exchange mechanism was investigated by some authors for meson decays [33–35], but since it is completely induced by the non-perturbative QCD effect, the estimate is by no means accurate, and at the best can be valid to the order of magnitude.

Interestingly, the experimental data on the $Y(4630) \to \Lambda_c^+\Lambda_c^-$ and $Y(4660) \to \psi(2S)\pi^+\pi^-$ has been re-analyzed by the authors of [7] and they found that the two observations $Y(4630)$ and $Y(4660)$ are probably the same tetraquark state. Also they concluded the double baryon decay mode $\Lambda_c^+\Lambda_c^-$ is strongly preferred with a ratio $\mathcal{BR}(Y \to \Lambda_c^+\Lambda_c^-)/\mathcal{BR}(Y \to \psi(2S)\pi^+\pi^-) = 25 \pm 7$ at 90% C.L., which is consistent with our estimate.

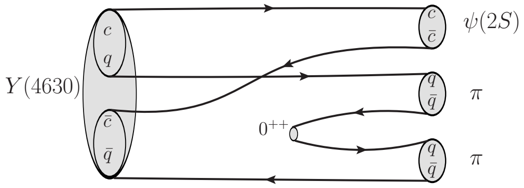

FIG. 4: The process of the tetraquark state decay into $\psi(2S)\pi^+\pi^-$.

However, we can also expect to observe $Y(4630)$ decay to, say, $p\bar{p}$, $D\bar{D}$ etc. channels which are supposed to be realized via re-scattering processes: $Y(4630) \to \Lambda_c\bar{\Lambda}_c \to p\bar{p}$; $D\bar{D}$, $\pi\pi$... and we will calculate the corresponding rates under this tetraquark postulate in our following work.

Thus, we are looking forward to getting more information from the BelleII, LHCb experiments, especially we will pay more attention to $Y(4630) \to \psi(2S)\pi^+\pi^-$ mode which may shed more light on the structure of $Y(4630)$. In particular, we suspect if there is a mixing between the tetraquark and molecular states to result in $Y(4630)$ and $Y(4660)$, it would be an interesting picture. Indeed in the near future, with the accumulated data at various accelerators, our understanding on the XYZ states will be improved and the observations of new states are expected.

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Appendix: wave functions

In this work, we employ the SHO wave functions for the baryons and $Y(4630)$ as the input wave functions. For the decay channels of interest, we need a P-wave two-body wave function for the $Y(4630)$ and the S-wave function for the charmed baryons.

For charmed baryons in S-wave [30]

$$\Psi_{n_r=1,l=0}(\mathbf{p}_\rho,\mathbf{p}_\lambda) = 3^{3/4}\left(\frac{1}{\pi\beta_\rho^2}\right)^{3/4}\left(\frac{1}{\pi\beta_\lambda^2}\right)^{3/4}\exp\left(-\frac{{\mathbf{p}_\rho}^2}{2\beta_\rho^2}-\frac{{\mathbf{p}_\lambda}^2}{2\beta_\lambda^2}\right), \tag{A.1}$$

and for the two-body P-wave function [32]

$$\Psi_{n_r=1,l=1}(\mathbf{k}) = -i2\sqrt{\frac{2}{3}}\frac{R^{5/2}}{\pi^{1/4}}\mathcal{Y}_{1m}(\mathbf{k})\exp\left(-\frac{R^2\mathbf{k}^2}{2}\right), \tag{A.2}$$

$$\Psi_{n_r=2,l=1}(\mathbf{k}) = i\frac{2}{\sqrt{15}}\frac{R^{5/2}}{\pi^{1/4}}(5-2\mathbf{k}^2R^2)\mathcal{Y}_{1m}(\mathbf{k})\exp\left(-\frac{R^2\mathbf{k}^2}{2}\right), \tag{A.3}$$

where $\mathcal{Y}_{1m}(\mathbf{k}) = \sqrt{3/(4\pi)}\boldsymbol{\epsilon}_{-m}\cdot\mathbf{k}$ is the solid harmonic polynomial, with $\epsilon_{\pm1} = (\pm1/\sqrt{2}, -i/\sqrt{2}, 0)$ and $\epsilon_0 = (0,0,1)$.

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