$*$-ISOMORPHISM OF LEAVITT PATH ALGEBRAS OVER $\mathbb{Z}$

TOKE MEIER CARLSEN

ABSTRACT. We characterise when the Leavitt path algebras over $\mathbb{Z}$ of two arbitrary countable directed graphs are $*$-isomorphic. We also prove that any $*$-homomorphism between two Leavitt path algebras over $\mathbb{Z}$ maps the diagonal to the diagonal.

1. INTRODUCTION

Graph C^*-algebras were introduced in [11] and [12] as a generalisation of Cuntz-Kriger [9] and Cuntz algebras [8], and have since then attracted a lot of interest (see [18] and its references). It was later discovered that certain Leavitt algebras [13, 14, 15] could be considered as algebraic analogues of Cuntz algebras. This led to the introduction of *Leavitt path algebras* as algebraic analogues of graph C^*-algebras ([1] and [3]). Since then the connection between graph C^*-algebras and Leavitt path algebras has been thoroughly studied (see for example [2], [10], and [19]). Both the graph C^*-algebra and the Leavitt path algebra of a directed graph can be constructed from the *graph groupoid* of the graph (see [4], [5], [7], [12], [17], and [21]).

The purpose of this paper is to describe, in terms of the graph C^*-algebras and the graph groupoids, when the Leavitt path algebras over $\mathbb{Z}$ of two arbitrary countable directed graphs are $*$-isomorphic. This is done in Theorem 1 in Section 3. We also remark on how this is related to *orbit equivalence* of graphs (Remark 2), and prove that all projections in a Leavitt path algebra over $\mathbb{Z}$ belong to the *diagonal* of the Leavitt path algebra (Proposition 3). It follows as a corollary that any $*$-homomorphism between two Leavitt path algebras over $\mathbb{Z}$ maps the diagonal to the diagonal (Corollary 4).

2. DEFINITIONS AND NOTATION

We recall in this section the definition of a directed graph, as well as the definitions of the Leavitt path algebra, the graph C^*-algebra, and the graph groupoid of a graph; and introduce some notation. Most of this section is copied from [5].

A *directed graph* is a quadruple $E = (E^0, E^1, s, r)$ where E^0 and E^1 are sets, and s and r are maps from E^1 to E^0. A graph E is said to be *countable* if E^0 and E^1 are countable.

A *path* μ of length n in E is a sequence of edges $\mu = \mu_1 \dots \mu_n$ such that $r(\mu_i) = s(\mu_{i+1})$ for all $1 \leq i \leq n-1$. The set of paths of length n is denoted E^n. We denote by $|\mu|$ the length of μ. The range and source maps extend naturally to paths: $s(\mu) := s(\mu_1)$ and $r(\mu) := r(\mu_n)$. We regard the elements of E^0 as path of length 0, and for $v \in E^0$ we set $s(v) := r(v) := v$. For $v \in E^0$ and $n \in \mathbb{N}_0$ we denote by vE^n the set of paths of length

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n with source v. We define $E^* := \bigcup_{n\in\mathbb{N}_0} E^n$ to be the collection of all paths with finite length. We define $E^0_{\text{reg}} := \{v \in E^0 : vE^1 \text{ is finite and nonempty}\}$ and $E^0_{\text{sing}} := E^0 \setminus E^0_{\text{reg}}$. If $\mu = \mu_1\mu_2\cdots\mu_m, \nu = \nu_1\nu_2\cdots\nu_n \in E^*$ and $r(\mu) = s(\nu)$, then we let $\mu\nu$ denote the path $\mu_1\mu_2\cdots\mu_m\nu_1\nu_2\cdots\nu_n$. A *loop* (also called a *cycle*) in E is a path $\mu \in E^*$ such that $|\mu| \geq 1$ and $s(\mu) = r(\mu)$. An edge e is an *exit* to the loop μ if there exists i such that $s(e) = s(\mu_i)$ and $e \neq \mu_i$. A graph is said to satisfy *condition (L)* if every loop has an exit.

An *infinite path* in E is an infinite sequence $x_1x_2\ldots$ of edges in E such that $r(e_i) = s(e_{i+1})$ for all i. We let E^∞ be the set of all infinite paths in E. The source map extends to E^∞ in the obvious way. We let $|x| = \infty$ for $x \in E^\infty$. The *boundary path space* of E is the space

$$\partial E := E^\infty \cup \{\mu \in E^* : r(\mu) \in E^0_{\text{sing}}\}.$$

If $\mu = \mu_1\mu_2\cdots\mu_m \in E^*$, $x = x_1x_2\cdots \in E^\infty$ and $r(\mu) = s(x)$, then we let μx denote the infinite path $\mu_1\mu_2\cdots\mu_m x_1x_2\cdots \in E^\infty$.

For $\mu \in E^*$, the *cylinder set* of μ is the set

$$Z(\mu) := \{\mu x \in \partial E : x \in r(\mu)\partial E\},$$

where $r(\mu)\partial E := \{x \in \partial E : r(\mu) = s(x)\}$. Given $\mu \in E^*$ and a finite subset $F \subseteq r(\mu)E^1$ we define

$$Z(\mu \setminus F) := Z(\mu) \setminus \left(\bigcup_{e\in F} Z(\mu e)\right).$$

The boundary path space ∂E is a locally compact Hausdorff space with the topology given by the basis $\{Z(\mu \setminus F) : \mu \in E^*,\ F \text{ is a finite subset of } r(\mu)E^1\}$, and each such $Z(\mu \setminus F)$ is compact and open (see [20, Theorem 2.1 and Theorem 2.2]).

The *graph C^*-algebra* of a directed graph E is the universal C^*-algebra $C^*(E)$ generated by mutually orthogonal projections $\{p_v : v \in E^0\}$ and partial isometries $\{s_e : e \in E^1\}$ satisfying

(CK1) $s_e^*s_e = p_{r(e)}$ for all $e \in E^1$;
(CK2) $s_es_e^* \leq p_{s(e)}$ for all $e \in E^1$;
(CK3) $p_v = \displaystyle\sum_{e\in vE^1} s_es_e^*$ for all $v \in E^0_{\text{reg}}$.

If $\mu = \mu_1\cdots\mu_n \in E^n$ and $n \geq 2$, then we let $s_\mu := s_{\mu_1}\cdots s_{\mu_n}$. Likewise, we let $s_v := p_v$ if $v \in E^0$. Then $\operatorname{span}\{s_\mu s_\nu^* : \mu,\nu \in E^*,\ r(\mu) = r(\nu)\}$ is dense in $C^*(E)$. We define $\mathscr{D}(E)$ to be the closure in $C^*(E)$ of $\operatorname{span}\{s_\mu s_\mu^* : \mu \in E^*\}$. Then $\mathscr{D}(E)$ is an abelian C^*-subalgebra of $C^(E)$, and it is isomorphic to the C^*-algebra $C_0(\partial E)$. We furthermore have that $\mathscr{D}(E)$ is a maximal abelian sub-algebra of $C^*(E)$ if and only if E satisfies condition (L) (see [16, Example 3.3]).

Let E be a directed graph and R a commutative ring with a unit. The *Leavitt path algebra* of E over R is the universal R-algebra $L_R(E)$ generated by pairwise orthogonal idempotents $\{v : v \in E^0\}$ and elements $\{e, e^* : e \in E^1\}$ satisfying

(LP1) $e^*f = 0$ if $e \neq f$;
(LP2) $e^*e = r(e)$;

(LP3) $s(e)e = e = er(e)$;
(LP4) $e^*s(e) = e^* = r(e)e^*$;
(LP5) $v = \sum_{e \in vE^1} ee^*$ if $v \in E^0_{\text{reg}}$.

If $\mu = \mu_1 \cdots \mu_n \in E^n$ and $n \geq 2$, then we let μ be the element $\mu_1 \cdots \mu_n \in L_R(E)$. Then $L_R(E) = \text{span}\{\mu\nu^* : \mu, \nu \in E^*,\ r(\mu) = r(\nu)\}$. We define $D_R(E) := \text{span}\{\mu\mu^* : \mu \in E^*\}$. Then $D_R(E)$ is an abelian subalgebra of $L_R(E)$, and it is maximal abelian if and only if E satisfies condition (L) (see [6, Proposition 3.14 and Theorem 3.22]). If R is a a subring of $\mathbb{C}$ that is closed under complex conjugation, then $\mu\nu^* \mapsto \nu\mu^*$ extends to a conjugate linear involution on $L_R(E)$, i.e. $L_R(E)$ is a $*$-algebra. There is an injective $*$-homomorphism $\iota_{L_R(E)} \to C^*(E)$ mapping v to p_v and e to s_e for $v \in E^0$ and $e \in E^1$ (see [19, Theorem 7.3]).

For $n \in \mathbb{N}_0$, let $\partial E^{\geq n} := \{x \in \partial E : |x| \geq n\}$. Then $\partial E^{\geq n} = \cup_{\mu \in E^n} Z(\mu)$ is an open subset of ∂E. We define the *shift map* on E to be the map $\sigma_E : \partial E^{\geq 1} \to \partial E$ given by $\sigma_E(x_1x_2x_3\cdots) = x_2x_3\cdots$ for $x_1x_2x_3\cdots \in \partial E^{\geq 2}$ and $\sigma_E(e) = r(e)$ for $e \in \partial E \cap E^1$. For $n \geq 1$, we let σ_E^n be the n-fold composition of σ_E with itself. We let σ_E^0 denote the identity map on ∂E. Then σ_E^n is a local homeomorphism for all $n \in \mathbb{N}$. When we write $\sigma_E^n(x)$, we implicitly assume that $x \in \partial E^{\geq n}$.

The *graph groupoid* of a countable directed graph is the locally compact, Hausdorff, étale topological groupoid

$$\mathscr{G}_E = \{(x, m-n, y) : x, y \in \partial E,\ m, n \in \mathbb{N}_0, \text{ and } \sigma^m(x) = \sigma^n(y)\},$$

with product $(x,k,y)(w,l,z) := (x,k+l,z)$ if $y = w$ and undefined otherwise, and inverse given by $(x,k,y)^{-1} := (y,-k,x)$. The topology of $\mathscr{G}_E$ is generated by subsets of the form $Z(U,m,n,V) := \{(x,k,y) \in \mathscr{G}_E : x \in U,\ k = m-n,\ y \in V,\ \sigma_E^m(x) = \sigma_E^n(y)\}$ where $m, n \in \mathbb{N}_0$, U is an open subset of $\partial E^{\geq m}$ such that the restriction of σ_E^m to U is injective, and V is an open subset of $\partial E^{\geq n}$ such that the restriction of σ_E^n to V is injective, and $\sigma_E^m(U) = \sigma_E^n(V)$. The map $x \mapsto (x,0,x)$ is a homeomorphism from ∂E to the unit space $\mathscr{G}_E^0$ of $\mathscr{G}_E$. There is a $*$-isomorphism from the C^*-algebra of $\mathscr{G}_E$ to $C^*(E)$ that maps $C_0(\mathscr{G}_E^0)$ onto $\mathscr{D}(E)$ (see [5, Proposition 2.2] and [12, Proposition 4.1]), and a $*$-isomorphism from the Steinberg algebra $A_R(\mathscr{G}_E)$ of $\mathscr{G}_E$ to $L_R(E)$ that maps $\text{span}_R\{1_{Z(Z(\mu),0,0,Z(\mu))} : \mu \in E^*\}$ onto $D_R(E)$ (see [4, Theorem 2.2] and [7, Example 3.2]).

3. THE RESULT

Theorem 1. *Let E and F be countable directed graphs. Then the following are equivalent.*

1. *The Leavitt path algebras $L_{\mathbb{Z}}(E)$ and $L_{\mathbb{Z}}(F)$ of E and F over $\mathbb{Z}$ are $*$-isomorphic.*
2. *There is a $*$-isomorphism $\pi : L_{\mathbb{Z}}(E) \to L_{\mathbb{Z}}(F)$ such that $\pi(D_{\mathbb{Z}}(E)) = D_{\mathbb{Z}}(F)$.*
3. *There is a $*$-isomorphism $\phi : C^*(E) \to C^*(F)$ such that $\phi(\mathscr{D}(E)) = \mathscr{D}(F)$*
4. *The graph groupoids $\mathscr{G}_E$ and $\mathscr{G}_F$ are isomorphic as topological groupoids.*
5. *There is a $*$-isomorphism $\pi : L_{\mathbb{Z}}(E) \to L_{\mathbb{Z}}(F)$ and a homeomorphism $\kappa : E^\infty \to F^\infty$ such that $\pi(d)(y) = d(\kappa^{-1}(y))$ for $y \in F^\infty$ and $d \in D(E)$.*

Remark 2. It follows from [5] that the following two conditions are equivalent and implied by (3) and (4).

(6) The pseudogroups $\mathscr{P}_E$ and $\mathscr{P}_F$ introduced in [5, Section 3] are isomorphic.
(7) E and F are orbit equivalent as in [5, Definition 3.1].

It also follows from [5] that if E and F both satisfy condition (L), then (6) and (7) imply (3) and (4). Thus, if E and F both satisfy condition (L), then (1)–(7) are all equivalent.

As in [10], we say that $p \in L_{\mathbb{Z}}(E)$ is a *projection* if $p = p^* = p^2$. For the proof of Theorem 1 we need the following generalisation of [10, Theorem 5.6].

Proposition 3. *Let E be a directed graph. If $p \in L_{\mathbb{Z}}(E)$ is a projection, then $p \in D_{\mathbb{Z}}(E)$.*

Proof. This proof is inspired by the proof of [10, Proposition 4.2] which is due to Chris Smith.

For $\mu, \nu \in E^*$, we shall write $\mu \leq \nu$ to indicate that there is an $\eta \in E^*$ such that $\mu\eta = \nu$, and $\mu < \nu$ to indicate that $\mu \leq \nu$ and $\mu \neq \nu$.

Since $L_{\mathbb{Z}}(E) = \operatorname{span}_{\mathbb{Z}}\{\alpha\beta^* : \alpha, \beta \in E^*\}$, it follows that there are finite subsets A, B of E^* and a family $(\lambda_{(\alpha,\beta)})_{(\alpha,\beta)\in A\times B}$ of integers such that

$$p = \sum_{(\alpha,\beta)\in A\times B} \lambda_{(\alpha,\beta)}\alpha\beta^*.$$

By repeatedly replacing $\alpha\beta^*$ by $\sum_{e\in r(\alpha)E^1} \alpha ee^*\beta^*$ if necessary, we can assume that there is a k such that $B \subseteq E^k \cup \{\mu \in E^* : |\mu| < k \text{ and } r(\mu) \in E^0_{\text{sing}}\}$. We can also, by letting some of the $\lambda_{(\alpha,\beta)}$s be 0 if necessary, assume that $B \subseteq A$. We have that $\alpha\beta^* = 0$ unless $r(\alpha) = r(\beta)$. For $\beta \in B$, let $A_\beta := \{\alpha \in A : r(\alpha) = r(\beta)\}$. We shall also assume that if $\beta \in B$, then there is a least one $\alpha \in A_\beta$ such that $\lambda_{(\alpha,\beta)} \neq 0$ (otherwise we just remove β from B). We claim that $\lambda_{(\alpha,\beta)} = 0$ for all $(\alpha,\beta) \in A \times B$ with $\alpha \in A_\beta \setminus \{\beta\}$, and that $\lambda_{(\alpha,\beta)} = (-1)^{m_\beta}$ for all $\beta \in B$ where m_β is the number of β's in B such that $\beta' < \beta$.

Let $B' = \{\beta \in B : \lambda_{(\alpha,\beta)} = 0 \text{ for all } \alpha \in A_\beta \setminus \{\beta\} \text{ and } \lambda_{(\beta,\beta)} = (-1)^{m_\beta}\}$, and suppose $B' \neq B$. Choose $\beta \in B \setminus B'$ such that $\beta' < \beta$ for no $\beta' \in B \setminus B'$. Let

$$F_\beta = \{e \in r(\beta)E^1 : \beta e \leq \beta' \text{ for some } \beta' \in B \setminus \{\beta\}\}$$

and

$$\gamma_\beta = \beta - \beta \sum_{e\in F_\beta} ee^*$$

($F_\beta = \emptyset$ and $\gamma_\beta = \beta$ unless $|\beta| < k$ and $r(\beta)E^1$ is infinite). Then $\gamma_\beta^*\beta' = 0$ for $\beta' \in B$ unless $\beta' \leq \beta$.

Since $p = p^*p$, it follows that

(a) $$\gamma_\beta^* p \gamma_\beta = \gamma_\beta^* p^* p \gamma_\beta.$$

Recall that $L_{\mathbb{Z}}(E)$ is $\mathbb{Z}$-graded. The degree 0 part of the left-hand side of (a) is

$$\text{(b)} \qquad \sum_{\beta'\in B^{\leq\beta}} \lambda_{(\beta',\beta')}\left(r(\beta)-\sum_{e\in F_\beta} ee^*\right)$$

where $B^{\leq\beta} := \{\beta' \in B : \beta' \leq \beta\}$, and the degree 0 part of the right-hand side of (a) is

$$\text{(c)} \quad \left(\left(\sum_{\beta'\in B^{<\beta}} \lambda_{(\beta',\beta')}\right)^2 + 2\sum_{\beta'\in B^{<\beta}} \lambda_{(\beta',\beta')}\lambda_{(\beta,\beta)} + \sum_{\alpha\in A_\beta} \lambda^2_{(\alpha,\beta)}\right)\left(r(\beta)-\sum_{e\in F_\beta} ee^*\right)$$

where $B^{<\beta} := \{\beta' \in B : \beta' < \beta\}$ (we are using here that $\lambda_{(\alpha,\beta')} = 0$ for $\beta' \in B^{<\beta}$ and $\alpha \in A\setminus\{\beta'\}$).

Suppose m_β is even. Then $\sum_{\beta'\in B^{<\beta}} \lambda_{(\beta',\beta')} = 0$ (because $\lambda_{(\beta',\beta')} = (-1)^{m'_\beta}$ for each $\beta' \in B^{<\beta}$). Since (b) = (c), it follows that $\lambda_{(\beta,\beta)} = \sum_{\alpha\in A_\beta} \lambda^2_{(\alpha,\beta)}$. The fact that the $\lambda_{(\beta,\beta)}$s are integers, means that we must have that $\lambda_{(\alpha,\beta)} = 0$ for $\alpha \in A_\beta \setminus \{\beta\}$ and $\lambda_{(\beta,\beta)} = 1$ (recall that $\lambda_{(\alpha,\beta)} \neq 0$ for at least one $\alpha \in A_\beta$), but this contradicts the assumption that $\beta \notin B'$.

If m_β is uneven, then $\sum_{\beta'\in B^{<\beta}} \lambda_{(\beta',\beta')} = 1$, so it follows from the equality of (b) and (c) that $1+2\lambda_{(\beta,\beta)} + \sum_{\alpha_\beta\in A} \lambda^2_{(\alpha,\beta)} = 1+\lambda_{(\beta,\beta)}$ from which we deduce that $\lambda_{(\alpha,\beta)} = 0$ for $\alpha \in A_\beta \setminus \{\beta\}$ and $\lambda_{(\beta,\beta)} = -1$, and thus that $\beta \in B'$. So we also reach a contradiction in this case.

We conclude that we must have that $B' = B$, and thus that $\lambda_{(\alpha,\beta)} = 0$ for all $(\alpha,\beta) \in A\times B$ with $\alpha \in A_\beta \setminus \{\beta\}$. Since $\alpha\beta^* = 0$ for $\alpha \notin A_\beta$, it follows that $p = \sum_{\beta\in B} \lambda_{(\beta,\beta)}\beta\beta^* \in D_{\mathbb{Z}}(E)$. ☐

Corollary 4. *Let E and F be directed graphs and $\pi : L_{\mathbb{Z}}(E) \to L_{\mathbb{Z}}(F)$ a $*$-homomorphism. Then $\pi(D_{\mathbb{Z}}(E)) \subseteq D_{\mathbb{Z}}(F)$.*

Proof. The proof is similar to the proof of [10, Proposition 6.1]. Let $\mu \in E^*$. Then $\pi(\mu\mu^*)$ is a projection, so it follows from Proposition 3 that $\pi(\mu\mu^*) \in D_{\mathbb{Z}}(F)$. Since $D_{\mathbb{Z}}(E) = \operatorname{span}_{\mathbb{Z}}\{\mu\mu^* : \mu \in E^*\}$, it follows that $\pi(D_{\mathbb{Z}}(E)) \subseteq D_{\mathbb{Z}}(F)$. ☐

Proof of Theorem 1. It is obvious that (5) implies (1). The implication $(1) \implies (2)$ follows directly from Corollary 4. The equivalence of (3) and (4) is proved in [5].

Next, we shall prove that $(2) \implies (3)$. We shall closely follow the proof of [10, Lemma 3.5]. Let $\pi : L_{\mathbb{Z}}(E) \to L_{\mathbb{Z}}(F)$ be a $*$-isomorphism such that $\pi(D_{\mathbb{Z}}(E)) = D_{\mathbb{Z}}(F)$. As in the proof of [2, Theorem 4.4], π extends to a $*$-isomorphism $\phi : C^*(E) \to C^*(F)$ satisfying $\phi \circ \iota_{L_{\mathbb{Z}}(E)} = \iota_{L_{\mathbb{Z}}(F)} \circ \pi$. If $\mu \in E^*$, then

$$\phi(s_\mu s^*_\mu) = \phi(\iota_{L_{\mathbb{Z}}(E)}(\mu\mu^*)) = \iota_{L_{\mathbb{Z}}(F)}(\pi(\mu\mu^*)) \in \iota_{L_{\mathbb{Z}}(F)}(D_{\mathbb{Z}}(F)) \subseteq \mathscr{D}(F).$$

Since $\mathscr{D}(E)$ is generated by $\{s_\mu s^*_\mu : \mu \in E^*\}$, it follows that $\phi(\mathscr{D}(E)) \subseteq \mathscr{D}(F)$. That $\phi(\mathscr{D}(F)) \subseteq \mathscr{D}(E)$ follows in a similarly way. Thus $\phi(\mathscr{D}(E)) = \mathscr{D}(F)$.

Finally the proof of $(2) \implies (1)$ in [4, Theorem 5.3] also works when E and F are not row-finite or have sinks, so this gives us $(4) \implies (5)$. ☐

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UNIVERSITY OF THE FAROE ISLANDS, NÁTTÚRUVÍSINDADEILDIN, NÓATÚN 3, FO-100 TÓRSHAVN, FAROE ISLANDS

E-mail address: toke.carlsen@gmail.com