The New SI and fundamental constants: different meanings assigned to the same data

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Abstract
This note discusses the role of fundamental constants in the proposed "New SI" formulation of the definition of the International System of Units, namely in the present official documents and in some relevant literature. The meaning assigned to their use is found substantially different even among the advocates of the proposal. Some reasons are discussed why it is urgent that this basic issue is clarified.

Introduction

At the 2014 meeting of the Conférence Générale des Poids et Measures (CGPM), Resolution 1 [1] concerning the International System of Units (SI) "encouraged continued effort" toward a suitable proposal "that could enable the planned revision of the SI to be adopted by the CGPM" in 2018. Drafts of the needed official documents to that effect have been provided by the Comité Consultatif des Unités (CCU) of the Bureau International des Poids et Measures (BIPM) [2], and they are available, together with an illustration, on the website of the BIPM [3]. However, some discrepancies in the present intention to use fundamental constants in the definition of the system of measurement units are found among the advocates of the new definition.

The present "SI consists of a set of base units, prefixes and derived units, as described in these pages. The SI base units are a choice of seven well-defined units, which by convention are regarded as dimensionally independent: the metre, the kilogram, the second, the ampere, the kelvin, the mole, and the candela. Derived units are formed by combining the base units according to the algebraic relations linking the corresponding quantities." [4].
"Les unités choisies doivent être accessibles à tous, supposées constantes dans le temps et l'espace, et faciles à réaliser avec une exactitude élevée" (from the official French text) [4].
Each of the base units is individually defined. The present definitions are of different types:

(a) Make use of an artefact: for mass "The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram";
(b) Make use of a physical state or condition: e.g., for temperature "The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water". Similarly for time, amount of substance and luminous intensity;
(c) Define a realisation of the unit, e.g. for length: "The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 45 8 of a second". Similar approach for the electrical current.

Actually, also the base unit types (a) and (b) indirectly define a 'realisation method' to be implemented with appropriate procedures, according to term 2.6 of VIM [6] (not to be confused with the "mises en pratique" [5] that are approximations of the definition).

The use of the fundamental constants in the New SI BIPM official documents

On the page "On the future revision of the SI" of the BIPM website, under the title "What", [3] one finds the following statements:
"… In brief, the [new] SI will be the system of units in which:

> - the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ is exactly 9 192 631 770 hertz,
> - [and so forth for each of the seven base units: the [specific constant: c, h, e, k, N_{A}, or K_{cd}] is exactly [numerical value] [SI unit],], …

… where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, …".
(the speed of light in vacuum, c, is often referred to as c_0; the Boltzmann constant, k, is often referred to as k_{B})
This can be called the '*group-definition*' [7] and looks a self-contained and sufficient new definition of the SI, as also acknowledged in [8], and it represents a turning point with respect to the present one. *The base units remain the same*, but the rational of the definition is completely different from the present one.

In fact, until now the core of the definitions of the base units consisted in the specification of a 'definitional method' for the realisation of each base unit: initially by using artefacts (e.g., for mass, length), then using also invariant states of the nature (e.g., for temperature); further, more recently, by adopting a "mise en pratique" of the definitions of the base units. As a consequence, any National Measurement Institute (NMI) wanting to realise a base unit in an independent way was required to realise the definitional procedure. [9] In some cases, that were a strong limitation, also considering the fact that new methods were developed in the mean time, often superior to the definitional one.

The addition of a *mise en pratique* [5] for each of the base units was adopted by the Comité International des Poids et Mesures (CIPM), an executive committee of the BIPM, to extend the flexibility of the NMI in realising the standards. However, though already a good advancement, these methods were constrained to a lower-rank status with respect to the current base unit definitions. [10]

Since more than a decade, a search for a more advanced solution accelerated, especially due to the impelling problems with the mass standard and with some electrical standards.
In the panorama of science, a single frame has basically emerged in the last decades, of which indirectly also metrology benefited: the experimental studies on the numerical values of fundamental constants of physics and chemistry, whose uncertainties has dramatically decreased in many cases, with a much larger number of experiments available and wider in methodologies than in the past. The main reason for these studies was not, at least initially, strictly metrological, though they used the superior tools of top metrology to advance in the check for the consistency of the basic models of the physical/chemical world.

Should the base units be *dimensionally independent* to a sufficient degree—as initially assumed by the historical promoters of the MKSA system and then of the SI ("Les grandeurs de base sont, par convention, considérées comme indépendantes" Section 1.2 in [4])—their mutual consistency would intrinsically be respected, irrespective of the magnitude chosen for each of them.
Considering instead that already at present four of the seven base units are not dimensionally independent, and that the numerical values of the constants are determined by the magnitude of the

used measurement units, the feedback of the above results to the metrological field was that a corresponding degree of *metrological compatibility* [6] of the base units has been established. This degree has further been refined, with respect to the normal statistical analysis of the experimental data, by the studies of the CODATA Group on fundamental constants using the Least Squares Adjustment (LSA) method. [11, 12]

The new situation seemed therefore to offer the tools for a new definition combining a sounder basis for the definitions, by using only invariant states or constants of nature [3], with the possibility for the users of a more flexible implementation in realising the standards. In fact, this choice also avoids the indication of any definitional method.

It consists, as already indicated, in taking the 'best' numerical value of each constant today available as the correct one, and in stipulating it, i.e., in fixing it *exact by convention* (reference value)—thus also zeroing its non-uniqueness.
On the other hand, the group-definition of the SI, as spelled out above, does not tell anything explicit about the definition of individual base units or about their realisation.
For that reason, one finds in [3] the following complement to the group-definition: "The SI may *alternatively* be defined by statements that explicitly define seven *individual base* units: the second, metre, kilogram, ampere, kelvin, mole, and candela. These correspond to the seven base quantities time, length, mass, electric current, thermodynamic temperature, amount of substance, and luminous intensity. All other units are then obtained as products of powers of the seven base units, which involve no numerical factors; these are called coherent derived units" (emphases added).
One should consider that as the possible basis for an '*individual base unit'* definition.

The use of the fundamental constants in the New SI relevant literature

In a recent paper [13], one of the leaders of the CODATA Group on fundamental constants that advocates the New SI definition, illustrates a perspective that is common to other authors and is closer to the original intention of the physical Community. As shown in Table 1, the fundamental constants can be taken as representing base quantities different from the present ones.

Table 1. Constants, present and New SI base quantities and their units, base quantities in [13].

Constant [a]	Present dimensions	Present and New SI base quantity (BIPM [3])	Present and New SI base units (BIPM [3])	New SI base quantity (Newell [13])
$\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$	s^{-1}	Time	second	Frequency
c_0	$\mathrm{m\ s}^{-1}$	Length	metre	Velocity [b]
h	$\mathrm{J\ s = kg\ m^2\ s^{-1}}$	Mass	kilogram	Action
e	$\mathrm{C = A\ s}$	Electrical current	ampere	Electric charge
k_{B}	$\mathrm{J\ K^{-1} = kg\ m^2\ s^{-2}\ K^{-1}}$	Thermodynamic temperature	kelvin	Heat capacity
N_{A}	mol^{-1}	Amount of substance	mole	Amount of substance (reciprocal)
K_{cd}	$\mathrm{lm\ W = cd\ kg^{-1}\ m^{-2}\ s^3}$	Luminous intensity	candela	Luminous intensity

[a] Five are fundamental constants, in the first and last row they are 'constants' in the broad sense.
[b] Actually, it is "speed", a non-vectorial quantity.

This position is amply justified from the physics viewpoint, but, as clearly evident from the Table 1, it would imply a *change in the base units* too. That would be a very intricate affair, because the second, the metre, the kilogram, the ampere, the kelvin and the mol would become *derived* units, i.e., to be expressed in terms of the new base ones. E.g., for the simplest cases of the electrical current, the present unit would be A = C Hz; and, for the amount of substance, the old unit would be mol = ent^{-1} —using as new unit the term “ent” for ‘entity’ proposed by some authors.

Are the two approaches equivalent with each other?

The present ‘group definition’ reported in the BIPM website (see hereinbefore) is *not* the more exact translation of the idea behind Newell’s meaning (also shared by others), i.e. that the physical quantity closest to the meaning of the corresponding fundamental constant becomes the new base unit. However, the latter would fulfil anew the original intention of the SI founders, to have dimensionally independent base units (leaving aside the unchanged luminous intensity and its unit, the candela). This would also be appropriate in avoiding the use of inverse quantities, a logical difficulty.

However, in that case it would be wise to consider a full change of the name of the System of Measurement Units, from SI to another name, in order to avoid confusion, as it was done when changing from the CGS to the SI systems.

One can appreciate the advantages of the above possibility, by looking at the difficulties found by the CCU in implementing the wish in [3] to adopt also (or alternatively) single-unit definitions. The current CCU implementation of this indication is presently still the Draft of December 2013 of the 9th Brochure on the SI, Chapter 2.4, [2] where the typical wording for the proposed new definitions of each base unit [3] is as follows:
The “… magnitude [of the unit] is set by fixing the numerical value of the [single relevant constant] to be exactly [numerical value] when it is expressed in the SI unit for [kind-of-quantity] [new SI units]”; e.g., for length the constant is the speed of light in vacuum, c_0, the numerical value is 299 792 458, the kind-of-quantity is length, and the SI units are m s^{-1}.
For five base units the relevant constant is one of the fundamental ones listed in the group-definition of the SI (c_0 having already been used since 1983 for the metre), included in the CODATA analysis. For the second and the candela the fixed numerical value is that of an invariant physical state, *not* presently included in the CODATA analysis.
Why the above formulations are not satisfactory is clear: each of them only uses one of the relevant constants for the definition of some of the single base units. See below the base units expressed as combinations on the proposed constants (dimensional expressions), originating from Table 1, column 2:

$$\begin{aligned}
&[c_0/\Delta\nu\,(^{133}\mathrm{Cs})] = [\mathrm{metre}];\\
&[h\,\Delta\nu\,(^{133}\mathrm{Cs})/\,c_0^{2}] = [\mathrm{kilogram}];\\
&[h\,\Delta\nu\,(^{133}\mathrm{Cs})/k_\mathrm{B}] = [\mathrm{kelvin}];\\
&[e\,\Delta\nu\,(^{133}\mathrm{Cs})] = [\mathrm{ampere}];\\
&[1/N_\mathrm{A}] = [\mathrm{mole}].
\end{aligned} \tag{1}$$

(N_A depends only on the unit mol—but see [9]; $\Delta\nu\,(^{133}\mathrm{Cs})$ for the second, and K_cd for the candela are not “fundamental constants”)

The constant indicated in [2] is the ‘dominant’ one in the following sense illustrated here by using the original pre-stipulation uncertain numbers (CODATA 2010 data): for the kelvin, the contributions to the uncertainty of $\Delta\nu(^{133}\mathrm{Cs})$ and h are not significant ($u_\mathrm{tot} = 4.16\cdot10^{-7}$ taking them into account, compared with $4.15\cdot10^{-7}$ for k_B only). Similarly, considering the relevant constants, for the kilogram ($u_\mathrm{tot} = 3.01\cdot10^{-48}$ instead of $2.97\cdot10^{-48}$ for h only) and the ampere ($u_\mathrm{tot} = 3.23\cdot10^{-12}$

instead of $3.22\cdot10^{-12}$ for e only), a bit less for the candela ($u_{\text{tot}} = 1.68\cdot10^{-18}$ instead of $1.58\cdot10^{-18}$ for K_{cd} only) and for the metre ($u_{\text{tot}} = 9.64\cdot10^{-11}$ instead of $9.53\cdot10^{-11}$for c_0 only). However, it cannot be considered a sufficiently rational reason for the exclusive adoption in the spelling of the definitions of the 'dominant' constant. This issue will not be discussed further here. That difficulty is totally avoided in Newell's scenario, should new base units be adopted.
The above is one reason why the two scenarios are *not* equivalent.

In the present BIPM's scenario, as said, the 'group definition' of the New SI does not tell anything explicit about (do not spell out) the definition of the individual base units. The literal meaning of the wording of a *single-unit* definition derived from the group-definition, should in fact be spelled out as follows: *According to the new definition, the magnitude of each of the base units is 'such that' the chosen stipulated numerical value(s) of the relevant constant(s) apply*. A tautology, in practice.

From there, a second basic difference between the two scenarios derives.
Basically, the BIPM's New SI definition is explicitly based on the following reasoning:
(i) With the dramatic decrease of the experimental uncertainty of these constants (or others that might be possibly later preferred, like m_{u} [14]), the scientific community, is in the position to consider having today a sufficiently accurate check of the degree of mutual consistency of the present base units, basically thanks to the CODATA studies;
(ii) This is possible because, in the experimental determination of the numerical values of multidimensional constants, the use of different methods where the base units play a different role would reveal inconsistencies in the set of units, should metrologically incompatible results be obtained—something probably occurring at present for electrical quantities [15]. Actually, also the numerical value of constants that are in itself not multidimensional, like N_{A}, is often obtained through algebraic combination of other multidimensional constants, e.g., in that case $N_{\text{A}} = R\ k_{\text{B}}^{-1}$. The cross-combination of different constants of the selected group (see Eq. 1) increases the confidence in the consistency check.
(iii) Consequently, it is considered appropriate to retain those numerical values, and stipulate them as exact, which implicitly means retaining the *present* state-of-the-art degree of consistency of the *present* base units. In fact, it would be quite difficult, and indirect, to transfer this property to a different set of base units—apart from the obvious discomfort for the users.

An understandable consequence of this choice is that mutual consistency becomes *the single and only criterion* established by the new definition of the *same* base units presently used: *the magnitude of a base unit is* such that *its realisation provides a numerical value that is compatible with the set of stipulated numerical values of the relevant constants of the group*—actually it should mean '*exactly* the stipulated value(s)', but this cannot be proved, being the realisations experimental. However, instead of considering the multidimensionality of the constants as a problem to avoid, this choice uses it by privileging the resulting possibility of a sound consistency check. This choice is not compatible with Newell's one: the difference is the same that exists between a cause (the magnitude of the *present* base units determining the numerical values of the constants) and an effect (*these* same resulting numerical values of the constants be taken, as a group, to fix the magnitude of the set of the *same* base units).

Notice, however, that the above BIPM concept is *not* the same that is presently spelled out in [3] (and that is often claimed for), where the constants appear to be chosen for 'fundamental' physical reasons as in Newell's approach: indeed, the constants are not explicitly used in the definition for their physical meaning, instead—*repetita juvant*—for their multidimensional property that allows them to be fundamental for a sound consistency check of the set of base units.

In addition, the definition does not indicate any specific method or procedure to ensure the fulfilment of the above criterion, because the latter does not refer to any specific definition of individual base constant: any set of methods or procedures will be appropriate until they have the potential to fulfil the criterion. In all instances, they are not anymore the responsibility of the CIPM to be promulgated, but of the Comité International des Poids et Measures (CIPM) to later approve their choice delegated to the individual Comités Consultatifs.

Further considerations

The New SI proposal has been considered like a Kuhn's "scientific revolution" [16]. However, no achievement can be considered the "ultimate" one, as wished in the title of [7]: no achievement nor model can be considered 'definitive', no decision can be 'once and for all'. A Kuhn's revolution is only the next; there is not a final one. One should stay with Popper's principle of falsifiability as a necessary condition for the scientific frame. In metrology, a decision in 2018 could even be considered 'sufficient forever' for its regulatory purpose. In 'general' science, on the contrary, it never does.

At least, the scientists and the users should be correctly informed in a comprehensive and consistent way about the intended meaning of having based a New SI on fundamental constants, and on what the claim is based that the new definition basically does not imply any change for the users' standards.

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