



Charge-dependent correlations from event-by-event anomalous hydrodynamics

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Abstract

We report on our recent attempt of quantitative modeling of the Chiral Magnetic Effect (CME) in heavy-ion collisions. We perform 3+1 dimensional anomalous hydrodynamic simulations on an event-by-event basis, with constitutive equations that contain the anomaly-induced effects. We also develop a model of the initial condition for the axial charge density that captures the statistical nature of random chirality imbalances created by the color flux tubes. Basing on the event-by-event hydrodynamic simulations for hundreds of thousands of collisions, we calculate the correlation functions that are measured in experiments, and discuss how the anomalous transport affects these observables.

Keywords: Chiral magnetic effect, Chiral anomaly, Heavy-ion collisions, Hydrodynamics

1. Introduction

The Chiral Magnetic Effect (CME) [1, 2, 3, 4] has received considerable attention in recent years, particularly in the context of heavy-ion collisions. The anomaly-induced transport effects like the CME are macroscopic and are incorporated into hydrodynamic equations giving rise to “anomalous hydrodynamics” [5]. Theoretically, the CME is expected to occur in heavy-ion collision experiments. The data reported by STAR [6, 7] and PHENIX [8] collaborations at RHIC and ALICE collaboration [9] at the LHC show a behavior consistent with the CME, but the quantitative understanding is still lacking. In order to reach a definitive conclusion, a reliable theoretical tool that can describe the charge-dependent observables is indispensable.

In this work [10], we quantitatively evaluate the observables to detect the anomalous transport, basing on event-by-event simulations of anomalous hydrodynamics. The observable of interest in this talk is a charge-dependent two-particle correlation [11],

$$\gamma_{\alpha\beta} = \langle \cos(\phi_\alpha^1 + \phi_\beta^2 - 2\Psi_{\text{RP}}) \rangle, \quad (1)$$

where ϕ_α^i is the azimuthal angle of i -th particle ($i = 1, 2$) with charge $\alpha \in \{+, -\}$, and Ψ_{RP} is the reaction plane angle for v_2 . Physical meaning of this observable is evident if we decompose $\gamma_{\alpha\beta}$ as

$$\gamma_{\alpha\beta} = \langle \cos(\phi_\alpha^1 - \Psi_{\text{RP}}) \cos(\phi_\beta^2 - \Psi_{\text{RP}}) \rangle - \langle \sin(\phi_\alpha^1 - \Psi_{\text{RP}}) \sin(\phi_\beta^2 - \Psi_{\text{RP}}) \rangle \equiv \langle v_1^\alpha v_1^\beta \rangle - \langle a_1^\alpha a_1^\beta \rangle, \quad (2)$$

where v_1^α (a_1^α) is the directed flow which is parallel (perpendicular) to Ψ_{RP} , respectively.

Let us see how a_1 's behave in the presence of anomalous effects. In off-central collisions, the magnetic fields perpendicular to Ψ_{RP} (on average) are created. If the CME occurs, a current should be generated along the magnetic field, which would result in finite a_1^+ and a_1^- . The direction of the current depends on the sign of the initial axial charge, which is basically random, so the signs of a_1 s are also random. However, the signs of a_1^+ and a_1^- tend to be opposite. Thus, the CME expectations are the following: (1) $\langle a_1^+ \rangle = \langle a_1^- \rangle = 0$, because the sign of initial axial charge is random; (2) $\langle (a_1^\alpha)^2 \rangle$ becomes larger in the presence of the CME currents; (3) $\langle a_1^+ a_1^- \rangle < 0$, which indicates the anti-correlation between a_1^+ and a_1^- .

2. Event-by-event anomalous hydrodynamic model for heavy-ion collisions

The model consists of three parts: (i) anomalous-hydro evolution, (ii) hadronization via Cooper-Frye formula, and (iii) calculation of the observables. For the hydro part, we solve the equations of motion for a dissipationless anomalous fluid, $\partial_\mu T^{\mu\nu} = eF^{\nu\lambda} j_\lambda$, $\partial_\mu j^\mu = 0$, $\partial_\mu j_5^\mu = -CE_\mu B^\mu$, where $C \equiv \frac{N_c}{2\pi^2} \sum_f q_f^2$ is the anomaly constant, $E^\mu \equiv F^{\mu\nu} u_\nu$, $B^\mu \equiv \tilde{F}^{\mu\nu} u_\nu$ with $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$. The energy-momentum tensor and currents are written as $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p\eta^{\mu\nu}$, $j^\mu = nu^\mu + \kappa_B B^\mu$, $j_5^\mu = n_5 u^\mu + \xi_B B^\mu$, where ε is the energy density, p is the hydrodynamic pressure, n and n_5 are electric and axial charge densities, $e\kappa_B \equiv C\mu[1 - \mu n/(\varepsilon + p)]$ and $e\xi_B \equiv C\mu[1 - \mu_5 n_5/(\varepsilon + p)]$ are transport coefficients for chiral magnetic/separation effects (CME/CSE), and $\eta^{\mu\nu} \equiv \text{diag}\{1, -1, -1, -1\}$ is the Minkowski metric. In this work, the electromagnetic fields are not dynamical and treated as background fields. As for the equation of state (EOS), we use that of an ideal gas of quarks and gluons.

Let us specify the electromagnetic field configurations used to get the results shown later. We take B_y to be (x -axis is chosen to be the reaction plane angle Ψ_{RP})

$$eB_y(\tau, \eta_s, \mathbf{x}_\perp) = eB_0 \frac{b}{2R} \exp\left(-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2} - \frac{\eta_s^2}{\sigma_{\eta_s}^2} - \frac{\tau}{\tau_B}\right), \quad (3)$$

where σ_x , σ_y , and σ_{η_s} are the widths of the field in x , y , and η_s (space-time rapidity) directions, τ_B is the duration time of the magnetic field, $R = 6.38$ fm is the radius of a gold nucleus, and b is the impact parameter. Other elements of \mathbf{B} and \mathbf{E} are set to zero. The widths are taken so that the fields are applied only in the region where matter exists as $\sigma_x = 0.8(R - \frac{b}{2})$, $\sigma_y = 0.8\sqrt{R^2 - (b/2)^2}$, and $\sigma_{\eta} = \sqrt{2}$. We set other parameters as $\tau_B = 3$ fm and $eB_0 = 0.5\text{GeV}^2$ in following calculations, which is equivalent to $eB_y(\tau_{\text{in}}, 0, \mathbf{0}) \sim (3m_\pi)^2$.

By solving the hydrodynamic equations, we obtain a particle distribution via the Cooper-Frye formula with freezeout temperature $T_{\text{fo}} = 160$ MeV. We produce the hadrons by the Monte-Carlo sampling based on that distribution. Thus, one random initial condition results in the particles in an event. We repeat this procedure many times and store the data of many events, that are later used to calculate the charge-dependent correlation functions. We calculate fluctuations of v_1 and a_1 separately, with the following expressions,

$$\langle (v_1^\alpha)^2 \rangle \equiv \left\langle \frac{1}{M P_2} \sum_{\langle i, j \rangle} \cos(\phi_i^\alpha - \Psi_{\text{RP}}) \cos(\phi_j^\alpha - \Psi_{\text{RP}}) \right\rangle, \quad \langle (a_1^\alpha)^2 \rangle \equiv \left\langle \frac{1}{M P_2} \sum_{\langle i, j \rangle} \sin(\phi_i^\alpha - \Psi_{\text{RP}}) \sin(\phi_j^\alpha - \Psi_{\text{RP}}) \right\rangle. \quad (4)$$

for the same-charge correlation, where M is the number of produced particles, $M P_2 = M(M - 1)$, $\sum_{\langle i, j \rangle}$ indicates the sum over all the pairs, and outer bracket means averaging over events. Similar expression is used for the opposite-charge correlation.

It is an important issue to estimate the amount of axial charges at the beginning of hydro evolutions. The major sources of the initial chiralities are color flux tubes in heavy-ion collisions. When two nuclei collide, numerous color flux tubes are spanned between them. The anomaly equation, $\partial_\mu j_5^\mu = C\mathbf{E}^a \cdot \mathbf{B}^a$, determines the rate of the axial charge generation, so the rate is determined by the value of $\mathbf{E}^a \cdot \mathbf{B}^a$. There is no preferred sign of $\mathbf{E}^a \cdot \mathbf{B}^a$ and it can be positive or negative for different color flux tubes.

In order to incorporate this feature, we have made an extension to the so-called MC-Glauber model. For each binary collision, we assign ± 1 randomly. Each sign indicates to the sign of color $\mathbf{E}^a \cdot \mathbf{B}^a$ of the flux tube. Then, we initialize the axial chemical potential as

$$\mu_5(\mathbf{x}_T, \eta_s) = C_{\mu_5} f(\eta_s) \sum_{j=1}^{N_{\text{coll}}(\mathbf{x}_T)} X_j, \quad (5)$$

where X_j are the signs of $\mathbf{E}^a \cdot \mathbf{B}^a$ randomly assigned to binary collisions, and C_{μ_5} is a constant which expresses the typical strength of the μ_5 , and $f(\eta_s) = \exp\left[-\theta(|\eta_s| - \Delta\eta_s) \frac{(|\eta_s| - \Delta\eta_s)^2}{\sigma_\eta^2}\right]$. The sum is taken over the binary collisions happening on that point in the transverse plane. The strength of the color fields are of the order of the saturation scale. Taking this into account, we chose $C_{\mu_5} = 0.1$ GeV [10].

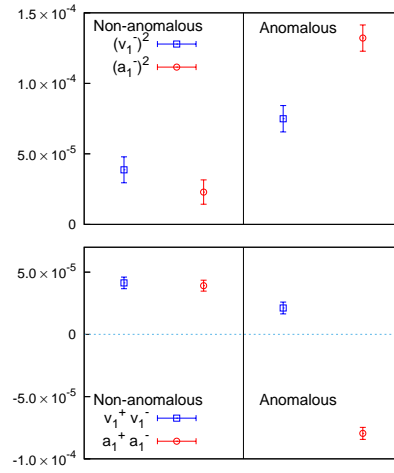


Fig. 1. The correlations $\langle (v_1^-)^2 \rangle$, $\langle (a_1^-)^2 \rangle$ (upper figure), $\langle v_1^+ v_1^- \rangle$, and $\langle a_1^+ a_1^- \rangle$ (lower figure) for anomalous and non-anomalous cases at $b = 7.2$ fm. Those quantities are calculated from the data of 10,000 events for both of the anomalous and non-anomalous cases.

3. Calculated observables

The values of the observables are shown in Fig. 1. The data from 10,000 events are used to calculate those observables for each of anomalous and non-anomalous case. Impact parameter is set to 7.2 fm. The upper figure of Fig. 1 shows the values of $\langle (v_1^-)^2 \rangle$ and $\langle (a_1^-)^2 \rangle$. In the left figure, anomalous transport effects are switched off (no CME and CSE). The plots in the right figure are from anomalous hydrodynamic simulations. In the non-anomalous case, the values of the fluctuations of v_1 and a_1 are similar. When we switch on the anomaly (right figure), $\langle (v_1^-)^2 \rangle$ goes up, and $\langle (a_1^-)^2 \rangle$ increases further. The large fluctuation of a_1 is in line with the qualitative expectation from the CME. The order of magnitude of $\gamma_{\alpha\alpha} = \langle (v_1^\alpha)^2 \rangle - \langle (a_1^\alpha)^2 \rangle$ is comparable to experimentally measured values.

In the lower figure of Fig. 1, we show the values of $\langle v_1^+ v_1^- \rangle$, and $\langle a_1^+ a_1^- \rangle$. In the absence of anomaly, they take similar positive values, but once we turn on the anomaly, $\langle a_1^+ a_1^- \rangle$ becomes negative. This is the indication of the anti-correlation between a_1^+ and a_1^- and is consistent with the CME expectations.

It has been discussed that the observed values of $\gamma_{\alpha\beta}$ might be reproduced by other effects unrelated to the CME, including transverse momentum conservation [12, 13], charge conservation [14], or cluster particle correlations [15]. Such effects are absent in the calculations here, because the particles are sampled based on the Cooper-Frye formula, which is one-particle distribution, whereas all of the background effects arise

from multi-particle correlations. Thus, the difference between anomalous and non-anomalous calculations purely originates from the CME and CSE. The contribution from the transverse momentum conservation in the CME signal is recently estimated in Ref. [16], in which the charge deformations are treated as linear perturbations on the bulk evolutions in 2+1D.

4. Conclusions and outlook

We reported the results of event-by-event simulations of an anomalous hydrodynamic model for heavy-ion collisions. We solved the hydrodynamic equations including anomalous transport effects (CME and CSE) in 3+1D, and calculated the values of observables. We also developed a model of the initial axial charges created from the color flux tubes. The calculated values of the observables indicate that this observable works as expected, and the order of magnitude is comparable to experimentally measured values.

The largest uncertainty arises from the choice of the life-time of the magnetic fields. The existence of conducting matter affects the duration of the magnetic fields. We thus have to solve the hydrodynamic equations together with the Maxwell equations – this work is deferred to the future.

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