Comment on : Scarcity of real discrete eigenvalues in non-analytic complex $\mathcal{PT}$ -symmetric potentials.Z.Ahmed ,Pramana J.Phys 73(2),323(2009).

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We notice contradictory statements of Z.Ahmed on $\mathcal{PT}$-symmetry potential $V(x) = \frac{x^2}{4} + igx|x|$ (g=2) .The curve (dash-dot) shows only one zero ,whose corresponding energy eigenvalue is 1.72. This graphical representation contradicts the discussion (presented below the graph),which claims number of real discrete states between $0.4 \leq g \leq 2.3$ are 3. As g=2 < g=2.3 ,one should get number of real discrete states 3 instad of 1. However, present calculation using matrix diagonalisation method reflects 3 .Secondly the statement on complex Harmonic oscillator $V_H = x^2 + ix$ (which entails complete real discrete spectrum), as a singular example is not acceptable ,as we notice many complex $\mathcal{PT}$ symmetric potentials can be constructed ,which would reflect infinite real discrete energy eigenvalues.

PACS 03.65.Ge,11.30.Er.

Key words- $\mathcal{PT}$ symmetry,Non-Hermitian Hamiltonian,Real Eigenvalues , Matrix diagonalisation method .

In a paper Z.Ahmed[1] has made contradictory statement on non-analytic study of $\mathcal{PT}$ symmetric Hamiltonian

$$H = p^2 + \frac{x^2}{4} + igx|x| \tag{1}$$

in order to reflect limited real discrete eigenvalues for some selected values of parameter g.For g=2 ,author has plotted the graph(dash-dot) which reflects only one zero and the corresponding reported energy eigenvalue is 1.72.However the discussion (below the graph ,after Eq(7))clearly reflects that number of discrete real energy eigenvalues are 3; when $2.3 \geq g \geq 0.4$ .As g=2 is ¡ g=2.3 one should get 3 number of 3 real eigenstates ,instead of 1 as reported.In fact this contradictory statement urges any reader to scan the paper [1] to cross check the result .In order to cross - check ,we use matrix diagonalisation method (MDM)[2].In eigenvalue relation using MDM, we solve the equation

$$H|\Psi >= E|\Psi > \tag{2}$$

where

$$|\Psi >= \sum_m A_m|\phi_m > \tag{3}$$

and

$$(H = p^2 + x^2)|\phi_m >= (2m+1)|\phi_m > \tag{4}$$

We address the above controversial point in Table-I.

Table -I : Real eigenvalues of V(x)= $\frac{x^2}{4} + 2ix|x|$**.**

Quantum no	**Matrix size(N=700,900)**	**Previous [1]**	**Fernandez [3]**
0	**1.720 8**	**1.720**	**1.720 857 958**
1	**6.579**		**6.579 362 154**
2	**7.397**		**7.398 126 125**

Hence one will believe that author has not taken enough care in presenting the correct eigenvalues for g=2.However his discussion is correct as it is in conformity with the present calculation reflected in Table-I so also with the computed values of Fernandez [3].Apart from this Ahmed [1] has considered other $\mathcal{PT}$ -symmetric potentials in which the reported results are correct . Secondly Ahmed [1] feels that only a singular example on $\mathcal{PT}$ symmetric system is the simple Harmonic Oscillator

$$H = p^2 + x^2 + ix \tag{5}$$

possible ,whose entire spectrum is real.In fact ,one can construct many complex $\mathcal{PT}$ symmetry systems ,whose entire spectrum are real. In order

to reflect the same ,we present a new Hamiltonian as

$$H = p^2 + \frac{x^2}{4} + e^{-2ix|x|} \tag{6}$$

In table -II,we reproduce few even state eigenvalues using MDM [2] as given below.

Table -II :Real eigenvalues of $V(x) = \frac{x^2}{4} + e^{-2ix|x|}$.

Quantum number of even state	Matrix size(N=700,900)
0	0.881 8
2	2.736 0
4	4.609 4
6	6.514 2
8	8.481 5
10	10.510 7

In conclusion ,we suggest author should emphasize on the above aspect and suitably rectify the same .Last but not the least ,we suggest author should go through the work of Davies [5] on simple Harmonic Oscillator and proceed with his interest on $\mathcal{PT}$ symmetry .

Acknowledgement

Author thanks Prof .F.M.Fernandez for cross checking of results in the first table.

References

[1] Z.Ahmed ,: Scarcity of real discrete eigenvalues in non-analytic complex $\mathcal{PT}$ - symmetric potetials ,Pramana.J.Phys73(2) ,323 (2009).

[2] B.Rath,P.Mallick and P.K.Samal: A Study of Spectral Instability in $V(x) = ix^3$ Through Internal Perturbation :Breakdown of Unbroken $\mathcal{PT}$ Symmetry .The African Rev.Phys.10:0007,(2015).

[3] F.M.Fernandez (private communication) .

[4] D.Krejcirik,P.Siegl,M.Tater and J.Viola ,:Pseudospectra in non-Hermitian quantum mechanics. arXiv:1402.1082v1 (2014).

[5] E.B.Davies,:Pseudo-spectra ,the harmonic oscillator and complex resonances . Proc.R.Sco.Lond .A 455,585(1999).