System-Wide Early Warning Signs of Instability in Stochastically Forced Power Systems

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***Abstract*—Prior research has shown that spectral decomposition of the reduced power flow Jacobian can yield participation factors that describe the extent to which particular buses contribute to particular mode shapes in a power system. Research has also shown that both variance and autocorrelation of time series voltage data tend to increase as a power system approaches a critical transition. This paper presents evidence suggesting that a system's participation factors predict the relative bus voltage variance values for all nodes in a system. As a result, these participation factors can be used to combine PMU data from various locations dispersed throughout a power network into a single, coherent measure of global stability. This paper first describes the method of computing the participation factors. Next, two methods for using these factors in conjunction with dynamic time series data are presented. The method is tested using a dynamic model of a 2383-bus test case. Results from these tests indicate that system wide cross-correlation and system wide weighted variance ratios can both be effective early warning signs of a looming transition.**

***Index Terms*—Power system stability, phasor measurement units, time series analysis, stochastic processes, autocorrelation, cross-correlation, critical slowing down, modal analysis.**

I. Introduction

As a result of the Arizona and Southern California Blackout of September 8, 2011, over 2.7 million customers lost power for a period of up to 12 hours. In the incident's official report [1], two sources were cited as the causes of the failure: inadequate operations planning and poor situational awareness. A high level of situational awareness is primarily achieved through constant monitoring of a system's (a) contingency resilience and (b) dynamic stability. Because power systems are frequently operated close to critical or bifurcation points (in order to optimize limited infrastructure), estimating the proximity to voltage collapse is an essential tool grid operators could use to gauge dynamic stability.

There is increasing evidence that as a dynamical system approaches a bifurcation, early warning signs (EWSs) of the looming transition appear in the statistical properties of the system's time series data. This fact has been evidenced in many complex systems, including ecological networks, financial markets, the human brain, and power systems [2], [3]. Researchers have even found that human depression onset can be predicted by these same statistical properties [4]. In the statistical physics literature this phenomenon is known as Critical Slowing Down (CSD) [5]. When stressed, systems experiencing CSD require longer periods to recover from stochastic perturbations. Specifically, CSD is evidenced as state variable signals begin to show increased variance, autocorrelation, and cross-correlation statistics [2].

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Real power systems are burdened with highly stochastic loads and an increasing level of renewable energy penetration. Consequently, researchers have begun to quantify the presence of CSD in large scale power system networks. Voltage collapse in such a system can be understood as a critial transition (via Saddle Node bifurcation) [6]. When close to such a transition, reference [7] has quantified increases in variance and autocorrelation in bus voltage. Similarly, reference [8] computes the power system state vector auto-correlation function to gauge collapse probability. Finally, variance and autocorrelation are measured in an unstable power system in [3] across many state variables. The results indicate that variance of bus voltages and autocorrelation of line currents show the most useful signals of CSD. Current angles, voltage angles, generator rotor angles, and generator speeds did not yield strong CSD signs capable of indicating proximity to a bifurcation.

Although typically a useful indicator, not all variables in a complex system exhibit CSD sufficiently early enough to be useful early warning signs [9]. For instance, reference [3] destabilized a simulated power system by over stressing all load buses. Signals were then collected from many nodes in this system, and certain nodes conclusively did not show early and strong CSD warning signs. In order to mitigate this problem, we employ power flow matrix modal analysis to determine which variables will show the strongest CSD indicators. By understanding which variables are the best dynamic instability indicators, we can make stability assessments which are highly representative of the entire system.

When performing power flow calculations, the presence of voltage collapse results in the Newton Raphson AC Power Flow equations failing to converge to a solution. Reference [10] shows that when the eigenvalues of the reduced power flow matrix are positive, the system is voltage stable. Formally, voltage stability implies that, for each bus in the the system, if reactive power is injected into a bus, voltage magnitude increases. If the system reaches a voltage collapse, the reduced power flow matrix becomes singular and at least one eigenvalue is driven to zero. Therefore, the eigenvalues can be a useful indicator of proximity to instability, but more importantly, through spectral decomposition of this matrix, the eigenvectors can be used to pinpoint how strongly voltages at different nodes contribute to the most unstable modes of system operation.

Through a combination of modal analysis and CSD theory, this paper seeks to identify and evaluate new statistical early warning signs of voltage instability. This is accomplished by weighting and filtering real time dynamic data (bus voltages) with participation factors derived from the decomposition of a static matrix composed of algebraic equations. Section II of this paper outlines the mathematical methods for forming and decomposing the reduced power flow Jacobian along with calculating system wide cross-correlation increases and CSD induced variance changes. Section III illustrates these methods by presenting simulated results from the 2383-bus Polish system. Finally, our conclusions are presented in Section IV.

II. Mathematical Methods for Modal Analysis and CSD Statistical Calculations

This section presents a method for using a spectral decomposition of the reduced power flow Jacobian to identify and weight variables that will most clearly show evidence of CSD. (Further information on this spectral decomposition method can be found in [10]). Using this analysis, unstable modes can be identified in the system, and participation factors can pinpoint exactly which nodes are the most unstable. Next, the participation factors are used to divide the system into two groups and system wide bus voltage cross-correlation is examined. Finally, we introduce a stability metric which is based on comparing the variance increases of the most unstable nodes to the variance increases of the rest of the system.

A. Reduced Power Flow Matrix Construction and Decomposition

The standard power flow Jacobian matrix, based on the linearization of steady state power system equations, is given by:

$$\begin{array}{c}\Delta P \\ \Delta Q\end{array} = \left[\begin{array}{cc} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV}\end{array}\right]\left[\begin{array}{c}\Delta\theta \\ \Delta V\end{array}\right] \qquad (1)$$

In order to perform V-Q sensitivity analysis (an important aspect of voltage stability analysis), we assume that the incremental change in real power ΔP is equal to 0. In this way, we can study how incremental changes in injected reactive power affect system voltages. Setting $\Delta P = 0$ and rearranging terms to remove $\Delta\theta$, the expression for the reduced Jacobian becomes:

$$\Delta Q = \left[J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{PV}\right] \Delta V = [J_R]\,\Delta V \qquad (2)$$

Assuming the system is voltage stable, the matrix J_R can be assumed non singular and written as the product of its right eigenvector matrix ξ, its left eigenvector matrix η, and its diagonal eigenvalue matrix Λ, such that:

$$J_R = \xi \Lambda \eta \qquad (3)$$

Finally, these eigenvectors can be used to define bus participation factors. Normalized participation factors describe how much (in a unitless percentage value) each bus contributes to each mode of the power system. Accordingly, each participation factor corresponds to the i^{th} mode and the k^{th} bus.

$$P_{ki} = \xi_{ki} \eta_{ik} \qquad (4)$$

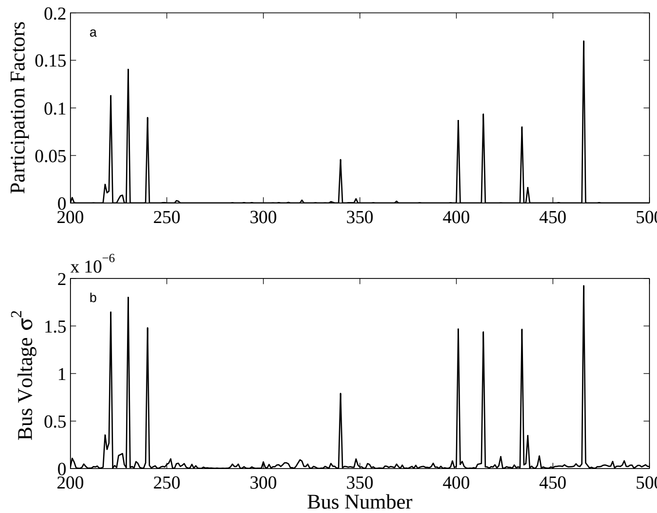

Figure 1. Shown are test results for buses 200 through 500 from the loaded 2383 bus system (see Sec. III-A for system description). Bus voltage variances at each node (panel (a)) are shown to have their relative magnitudes directly predicted by the participation factors for the most unstable mode (panel (b)).

There are many different ways to use these eigenvalues and eigenvectors to judge proximity to voltage collapse. For instance, Gao et al. suggest using the smallest eigenvalue of J_R to gauge proximity to bifurcation. Such stability analysis, though, is based solely on the decomposition of a model based static matrix and is highly limited in nature, as outlined by M. Pal in the discussion section of [10]. On the other hand, detecting CSD in a time series is a purely data driven stability assessment, but it can be difficult to understand which nodes will show the strongest EWSs [3]. Therefore, the novel approach outlined in this paper relies on using static decomposition results to weight and interpret incoming dynamic data.

Ultimately, we are concerned with the system's most unstable mode of operation. This is the mode which will correspond to the smallest eigenvalue of the system (the one closest to zero). After thoroughly testing this method on multiple test systems, we have shown that the modal participation factors corresponding to the smallest eigenvalue of J_R directly predict the relative bus voltage variances from buses across the system. Fig. 1 shows an example of this fascinating result using data collected from the loaded 2383 bus test system.

Participation factors of the most unstable mode also identify the node voltages which, as the system is overloaded, begin to diverge away from 1 per unit in magnitude and drift towards 0. These are the nodes which are primarily responsible for non convergent power flow equations. Interestingly, as PQ buses in the system are increasingly loaded, the recalculated participation factors do not change drastically. This is equivalent to saying that the *modal shapes* do not change significantly. This is a useful result, since real power flow models are only updated every few minutes.

B. System Wide Cross-Correlation

CSD theory predicts that signals from a system approaching a critical transition will begin to show high auto-correlation ($R(\Delta t)$). This can be due to the system's reduced ability to respond to high frequency fluctuations [11], but the system also beings to return to the equilibrium state more slowly after perturbations [12]. This has been verified in a number of papers in the power system literature, but as predicted in [2], zero lag cross-correlation ($P_{X_1X_2}(0)$) of two state variable signals $X_1(t)$ and $X_2(t)$ is another potentially useful early warning sign. As stated by Scheffer et al., there is a "general tendency toward increased spatial coherence" as a critical event approaches.

Voltage cross-correlation is inherently high for nodes in close proximity to each other, but we have found that highly unstable nodes (as predicted by the participation factors) also show very high cross-correlation, even when the system is relatively far from a transition. Based on this premise, we have devised a method of dividing a system into two sections, weighting and combining each section's voltages, and then testing for increases in cross-correlation. Diving the system and weighting the voltages is based entirely on the calculated participation factor values. One section of the divided system contains the most unstable nodes (and all surrounding nodes), and the second section contains the rest of the system nodes. As loading increases, the stable and unstable nodes begin to exhibit increasing cross-correlation.

Given a vector of eigenvalues $\underline{l}$ (corresponding to the the diagonal entries of Λ) for the reduced Jacobian J_R, the the smallest eigenvalue can be determined, with its index corresponding to i:

$$\lambda^i_{min} = \min(\underline{l}) \tag{5}$$

Now the participation factors for this mode, the i^{th} and most unstable mode, can be chosen. Dividing the system in two sections requires knowledge of the system topology, but we ultimately want to group buses with the largest participation factors with their surrounding nodes. For simplicity of notation, we will renumber the system nodes such that buses 1 through N are in one small, unstable group, and buses $N+1$ through K are in the second larger, more stable group, where there are a total of K buses in the system. For a sequence of $T+1$ bus voltage magnitude measurements, at each time step t, the mean of the voltage signal (μ_{V_k}) is subtracted, and the residual voltage is weighted by that node's participation factor and summed with all of the other weighted residual voltages. Two aggregate vectors are computed:

$$\underline{X_1}(t) = \sum_{k=1}^{N} (V_k(t) - \mu_{V_k})P_{ki} \quad \forall t \in (0,T) \tag{6}$$

$$\underline{X_2}(t) = \sum_{k=N+1}^{K} (V_k(t) - \mu_{V_k})P_{ki} \quad \forall t \in (0,T) \tag{7}$$

Finally, 0 lag cross-correlation can be computed between $\underline{X_1}(t)$ and $\underline{X_2}(t)$:

$$P_{X_1X_2}(0) = \frac{\frac{1}{T}\sum_{t=1}^{T} (\underline{X_1}(t) - \mu_{\underline{X1}}) \cdot (\underline{X_2}(t) - \mu_{\underline{X2}})}{\sigma_{\underline{X1}} \cdot \sigma_{\underline{X2}}} \tag{8}$$

where μ and σ are the signals' mean and standard deviation. As system load increases, it will be shown that, in a sufficiently large system, the cross-correlation between two such vectors constantly increases as it approaches the upper limit of unity.

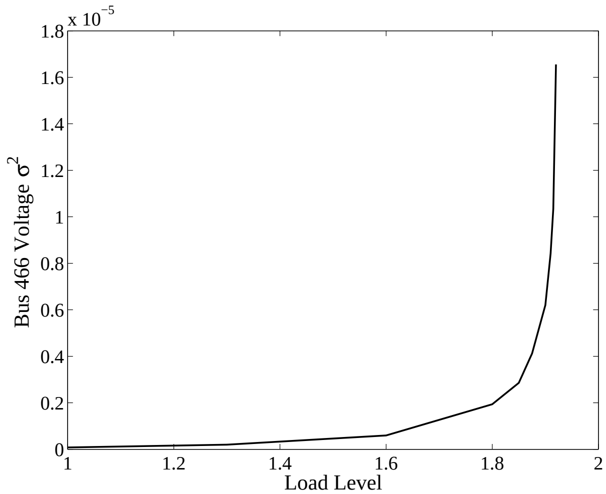

Figure 2. Bus 466 has the most unstable voltage. Clearly, its voltage variance increase is extremely dramatic, rising by a factor of over 200 (from nominal load to system destabilizing load). The majority of this increase occurs immediately before the bifurcation.

C. Using Variance Ratios as an EWS

CSD theory also predicts that the variance (σ^2) of signals will begin to increase as the system approaches a critical event [3]. Not all variables will show an extreme increase in variance, and not all increases will be sufficiently early to serve as an effective EWS. Even when monitoring the most unstable node of a system though, its increase in variance can be so dramatic (especially directly before the system has reached a critical point), it can be challenging to have an unambiguously clear measure of bifurcation proximity. Fig. 2 shows an example of this point. Participation factors can once again be helpful in determining how to monitor dynamic data.

As indicted previously, participation factors of the most unstable nodes amazingly serve as values indicating the relative bus voltage variance strengths. Therefore, as the system is increasingly loaded, the most unstable nodes will begin to have larger and larger participation factors as their relative variance strengths grow relative to other, more stable nodes. Fig. 3 shows an example of this for the 2383 bus system. As the system is loaded, the relative strength of the most unstable node's participation increases almost linearly, but when the critical transition approaches, the participation begins to limb more steeply.

Using the results predicted by the participation factor evolution, system wide variance strength ratios are shown to be an interesting and useful EWS. To develop a metric based on

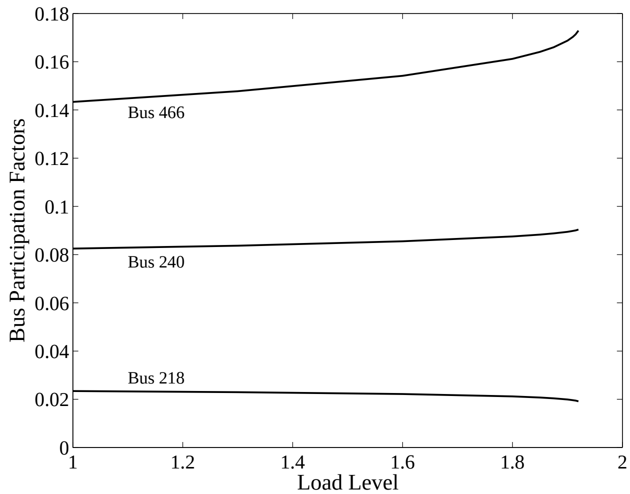

Figure 3. Depicted are the evolutions of three different nodal participation factors. As the system is increasingly loaded (right up to bifurcation), bus 466 (the most unstable bus) begins to see a sharp increase in participation to the instability. Bus 240 (the 5th most unstable bus) sees a very slight increase, while bus 218 (the 10th most unstable bus) begins to see a decrease.

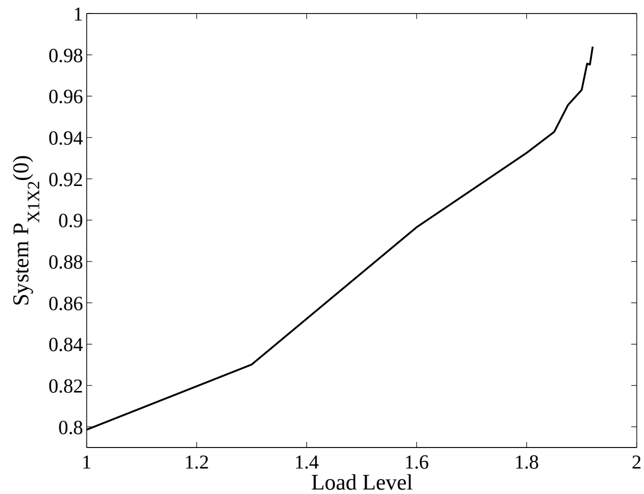

Figure 4. The system wide cross-correlation for increased system loading is shown here. As the system approaches a critical transition, combined bus voltages from around the system begin to swing together very tightly.

this result, we once again split the system into two groups: a small, highly unstable group, and a larger, more stable group (as predicted by the participation factors). We weight all bus voltage variances by the magnitude of their respective participation factors, and then we sum the variances in each group. The ratio of these weighted variance sums is given by Φ. As done previously, we will renumber the system nodes such that buses 1 through N are in one small, unstable group, and buses $N+1$ through K are in the second larger, more stable group, where there are a total of K buses in the system. As before, P_{ki} refers to the participation factor of the i^{th} mode and the k^{th} bus.

$$\Phi = \frac{\sum_{k=1}^{N} \sigma_k^2 \cdot P_{ki}}{\sum_{k=N+1}^{K} \sigma_k^2 \cdot P_{ki}} \tag{9}$$

III. Experimental Results

This section applies the methods of Sec. II to calculate the cross-correlation and variance ratio increases in the 2383 bus system. The system configuration and load noise assumptions are outlined in III-A, while III-B and III-C outline the specific test results.

A. Polish Test Case System Overview

In order to test our methods, we used simulated data from the 2383-bus Polish test system. This network contains 327 synchronous generators (each equipped with with type I turbine governors for frequency control and type II AVRs for voltage regulation). There are 322 shunt loads (all connected to generators buses) and 1503 active and reactive loads spread throughout the system. In order to push the system towards voltage collapse, we employed a simple uniform loading of all loads (except for those attached to generator buses). This method is justified in [13]. After running a power flow on the system, a time domain simulation was performed. Half of the loaded nodes were modeled as voltage controlled loads, while the other half were modeled as frequency controlled loads. Parameters controlling the voltage controlled loads were modeled after the Nordic Test System in [14], while parameters controlling the frequency controlled loads were modeled after the 39 bus test system described in [3].

During the time domain simulation, stochastic noise was injected into the loads at each step. The differential algebraic equations modeling the power system are given by:

$$\dot{x} = f(\underline{x}, \underline{y}) \tag{10}$$

$$0 = g(\underline{x}, \underline{y}, \underline{u}) \tag{11}$$

where f, g represent the differential and algebraic equations governing the system, $\underline{x}$, $\underline{y}$ are the differential and algebraic variables of these equations, and $\underline{u}$ represents the injected stochastic load noise. Load fluctuations $\underline{u}$ follow a mean-reverting Ornstein-Uhlenbeck process:

$$\underline{\dot{u}} = -E\underline{u} + \underline{\xi} \tag{12}$$

where E is a diagonal matrix whose diagonal entries equal the inverse correlation times t_{corr}^{-1} of load fluctuations and $\underline{\xi}$ is the vector of zero-mean independent Gaussian random variables. A further description of our noise model can be found in Sec. II A of [3]. Our noise correlation times, though, are chosen differently: to each t_{corr} we assign a random value from a bounded log-normal distribution of mean one.

In order to push the system towards a critical transition (voltage collapse), the system was repetitively simulated for ten distinct loading factors b, ranging from $b = 1$ up to $b = 1.92$. Voltage collapse would occur when the load factor increased past $b = 1.923$. For each load factor, we ran ten 120s simulations, computed statistics on the collected data, and then calculated averages of the statistics.

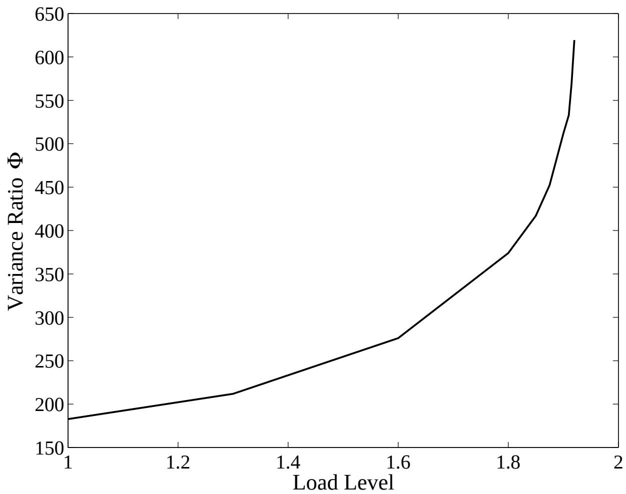

Figure 5. The ratio of the sum of weighted voltage variances for two groups of buses is plotted against load factor values.

B. Evidence of System Cross-Correlation

In order to test system cross-correlation, the method described in Sec. II-B was implemented on the 2383 bus system. After analyzing the participation factors and topology of the system, we chose to put nodes 180 through 470 (roughly 12% of system nodes) into one group while the remaining nodes were placed into a second group. Next, the voltages were combined according to the aforementioned methods and cross-correlation was computed for each load value.

Fig. 4 shows that the cross correlation between the voltages of the system appears to increase linearly through most of the system loading. Just before the bifurcation point, the cross correlation begins increasing drastically as it approaches unity.

C. Evidence of Increasing Variance Ratios

In order to test system wide variance ratios, the method described in Sec. II-C was implemented on the 2383 bus system. As in Sec. III-B, we chose to put nodes 180 through 470 into one group while the remaining nodes were placed into a second group. The ratio Φ of the summed weighted voltage variances of both groups was tracked at the system was increasingly loaded.

Fig. 5 shows that as the loading begins to approach $b = 1.923$, the variance ratio begins increasing drastically. The increase, though, begins to grow sufficiently early (well before bifurcation), making it a useful EWS. In total, the variance ratio increases only by a factor of 3.4. Comparatively, the variance in Fig. 2 increases by a factor of over 200.

IV. Conclusions

This paper presents evidence that participation factors from a spectral decomposition of the power flow Jacobian can be used to design methods for combining sychrophasor measurements to produce system-wide indicators of instability in power systems. This method uses model-based information from the power flow Jacobian, which can be updated every few minutes through the SCADA network, along with high sample-rate voltage magnitude measurements, which can be collected from synchronized phasor measurement systems deployed throughout the system. This combination of power flow results and dynamic real time data is used to develop two different global stability metrics. In particular, we find that the cross-correlation of combined weighted voltage measurements is a particularly useful early warning sign of proximity to instability. When the correlation of these signals approaches unity, bifurcation is near. System wide weighted variance ratios are also shown to be an effective EWS. Because these variance ratios begin to increase far from the bifurcation point, grid operators would have ample time to respond to these signals and take appropriate mitigating actions, such as to re-dispatch generation or (as a last resort) reduce demand.

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