Collective fermion excitation in a warm massless Dirac system

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Abstract

We predict theoretically that there occurs not only a normal (quasi) fermion mode, but also a collective fermion mode, plasmino, in a warm 2D massless Dirac system, especially in a warm intrinsic graphene system. Results of Landau damping show that both fermion and plasmino are well defined modes. We find that there are sharp differences between the discussed system and the QCD/QED system. Firstly, the thermal mass is proportional to $\alpha_g^{3/4}T$ but not $\alpha_g T$. Secondly, under a finite momentum satisfying $0 < q < q_c$, the plasmino energy is not smaller but larger than the fermion energy. Thirdly, the fermion behaves as a "relativity particles" at large momentum and the plasmino exhibits an anormal dispersion at moderate momentum.

keywords: plasmino, massless Dirac system, Landau damping.

One of the hot topics of LHC and RHIC is the hot QCD or quark-gluon-plasma (QGP) [1]. QGP exists at extremely high temperature and/or high density and is difficult to detect, so an important way to study QGP is to study the collective modes of the system, for instance, collective bosonic mode, plasmon, and collective fermionic mode, plasmino.

Scientists have predicted that at high temperature and/or high density, there are two types of fermionic excitations. One is the well known normal (quasi)fermion branch, and the other is the collective excitation branch, plasmino or antiplasmino [2, 3]. One of the remarkable characteristic of plasmino is that its chirality is opposite to the ordinary (quasi) fermion (In this Letter we only focus on energy larger than zero; we do not distinguish between fermion/plasmino and quasifermion/antiplasmino). The excitation has been extensively investigated in many literatures, for instance, Refs. [4–7]. Meanwhile, there are still some debates on the mode; for instance, Ref. [8, 9] claimed non-existence of a temperature generated plasmino mode.

Experimenting on plasmino effects in QCD faces many difficulties. Therefore, one alternative way is to study plasmino in other condensed matter systems, for instance, in superconductors [10]. In this Letter we report our study on plasmino in a warm 2D massless Dirac system. We show that, although there are some similarities in the discussed system and QCD system, there are also many striking differences between these systems.

The most famous 2D Dirac system is the graphene system [11], the dispersion of which is $\epsilon_p = \pm v_F p$, where ϵ_p is the fermion energy with momentum p. In this Letter we use notations $\hbar = v_F = k_B = 1$. In an intrinsic warm massless Dirac system (that is, having no net charge), the fermion propagator is read as

$$iS_F(p^0,p) = iS_F^0(p^0,p) - 2\pi(p^0 + \alpha\cdot\mathbf{p})f_+(p)\delta(p^{02}-p^2), \quad (1)$$

where $iS_F^0(p) = \frac{i}{2}[\frac{1+\alpha\cdot\mathbf{p}/\epsilon_p}{p^0-\epsilon_p+i\eta} + \frac{1-\alpha\cdot\mathbf{p}/\epsilon_p}{p^0+\epsilon_p-i\eta}] = \frac{i(p^0+\alpha\cdot\mathbf{p})}{p^{02}-\epsilon_p^2+i\eta}$ is the fermion propagator without temperature correction, $f_+(p) = \frac{1}{1+e^{\beta p}}$ is the so-called Fermi-Dirac distribution function, and temperature $T = 1/\beta$.

According to Ref. [12], when $p_0 \gg p$, the potential between fermions reads as

$$\begin{aligned} V(p^0,p) &= \frac{2\pi\alpha_g}{p+p^2N_c\alpha_g v_c} \\ &\simeq \frac{2\pi\alpha_g}{q} - \frac{iN_c\alpha_g^2}{4q^0} + \frac{N_c\pi\alpha_g^2}{q^{02}}\times \\ &\quad (-\frac{N_cq\pi^2\alpha_g}{32} + \theta(T-p)T\ln 4), \quad (2) \end{aligned}$$

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where N_c is fermion degenerate ($N_c = 4$ in graphene) and $v_c = \frac{i\pi}{8q^0}(1+\frac{p^2}{2p^{02}}) - \frac{T\ln 2}{p^{02}}\theta(T-p)$. The first term in v_c is the ordinary vortex correction and the second term in v_c is attributed to the temperature correction. At finite temperature, the Coulomb interaction between fermions is suppressed by temperature correction, that is, the potential between fermions behaves as e^{r/r_D}, where r is the radius between fermions and $r_D \propto 1/T$ is the Debye radius. However, if r is not large, the temperature correction is not important and therefore can be neglected.

The Dirac fermion self-energy correction is depicted in Fig. 1.

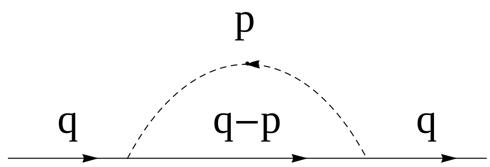

Figure 1: The Feynman Diagram of fermion self-energy correction.

The self-energy expression is in fact an integral,

$$\Sigma(q) = \int \frac{d^3p}{(2\pi)^3} iS_F(p'^0, \mathbf{p}')V(p^0, \mathbf{p})e^{ip^0\eta}, \tag{3}$$

where $(p^{0\prime}, \mathbf{p}') = (q^0 - p^0, \mathbf{q} - \mathbf{p})$. The integral can be decomposed into two parts. As studied by many literatures, for instance, Ref. [13], the integrand of the first one is temperature independent, $iS_F^0(p'^0, \mathbf{p}')V(p^0, \mathbf{p})e^{ip^0\eta}$, the effect of which is v_F renormalization at $p^0 \simeq v_F p$, and will be ignored in this paper.

Since we focus on the case $q^0 \gg q$, it is enough to consider only the second part. Therefore, we have

$$\begin{aligned}\Sigma &= \frac{1}{8\pi^2}\int d^2p f_+(p')[(1-\alpha\cdot\mathbf{p}'/p')V(q^0+p', \mathbf{p}) - \\ &\quad (1+\alpha\cdot\mathbf{p}'/p')V(q^0-p', \mathbf{p})]. \end{aligned} \tag{4}$$

One can further decompose the above expression into two parts, a scalar part, imaginary part and real one of which are even and odd functions of q^0 respectively and can be written as aq^0, and a spinor part, which is the even function of q^0 and can be written as $-b\alpha\cdot\mathbf{q}$. With the notation $x = q/T$ and $y = q^0/T$, we have,

$$\begin{aligned} a &\simeq \frac{3\pi N_c\alpha_g^2\zeta(3)}{16y^3}i + \frac{N_c^2\pi\alpha_g^3}{16y^4}d_1(x) - \frac{4N_c\alpha_g^2\ln 2}{\pi y^4}d_{s1}(x) \\ b &\simeq \frac{\alpha_g}{\pi}d_2(x) + \frac{N_c^2\pi\alpha_g^3}{64y^2}d_3(x) + \frac{N_c\alpha_g^2\ln 2}{\pi y^2}d_{s2}(x), \end{aligned} \tag{5}$$

where

$$\begin{aligned} d_1(x) &= \int \frac{x^4u^2v^2dudv}{(1+e^{vx})\sqrt{((1+v)^2-u^2)(u^2-(1-v)^2)}} \\ &\simeq 8.926 + 0.372x - 0.017x^2, \\ d_{s1}(x) &= \int \frac{x^3uv^2\theta(1/x-u)dudv}{(1+e^{vx})\sqrt{((1+v)^2-u^2)(u^2-(1-v)^2)}} \\ &\simeq \begin{cases} 0.169 + 0.025x^2, & x<1 \\ 0.193e^{(x-1)(1.63-0.955x)/(1.48+x)}, & x>1 \end{cases} \\ d_2(x) &= \int \frac{(1+v^2-u^2)dudv}{(1+e^{vx})\sqrt{((1+v)^2-u^2)(u^2-(1-v)^2)}} \\ &\simeq \ln(0.713 + \frac{1.11}{x} + 0.019x), \\ d_3(x) &= \int \frac{x^2u^2(u^2-1-v^2)dudv}{(1+e^{vx})\sqrt{((1+v)^2-u^2)(u^2-(1-v)^2)}} \\ &\simeq 1.33 - 0.218x + 0.012x^2, \\ d_{s2} &= \int \frac{(1+v^2-u^2)u\theta(1/x-u)dudv}{(1+e^{vx})\sqrt{((1+v)^2-u^2)(u^2-(1-v)^2)}} \\ &\simeq \begin{cases} 0.426 - 0.1x^2, & x<1 \\ 1.09e^{-1.2x}. & x>1 \end{cases} \end{aligned} \tag{6}$$

Notice that in the above integrals the domain of the integration is $v \in (0, \infty)$ and $u \in (|1-v|, 1+v)$. To demonstrate the coincidence of the integrals and their approximations, we show them in Fig. 2. We find that in the interesting region all the approximations coincide with the corresponding integrals very well.

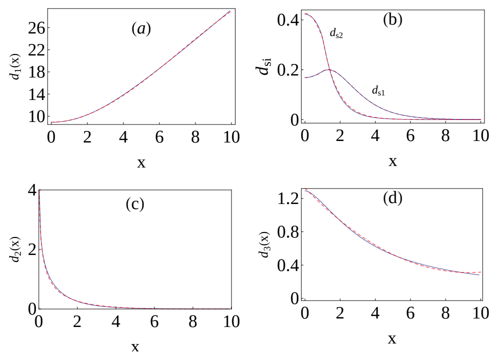

Figure 2: Integrals and corresponding approximations. Solid curves and dashed curves correspond integrals and corresponding approximation respectively of d_1 (a), d_{s1} and d_{s2} (b), d_2 (c) and d_3 (d).

In the RPA, the full fermion propagator is,

$$S_{FRPA}^{-1}(q) = S_F^{-1}(q) - \Sigma(q). \tag{7}$$

Or, it can be written as

$$S_{FRPA}^{-1}(q^0, \mathbf{q}) = (1-a)q^0 - (1-b)\alpha\cdot\mathbf{q}. \tag{7'}$$

The fermion propagator has twice as many as normal zero-temperature fermion propagators, analogous to the results in Refs. [2–4]. In noticing that both real parts of $1-a$ and $1-b$ are even functions of q^0, one finds that the expression $\frac{q^0}{T} = \frac{1-b}{1-a}\frac{q}{T}$ gives the both the fermion dispersion relation, $\epsilon_p = q^0 > 0$ and the (anti) plasmino dispersion relation, $\epsilon_h = -q^0 > 0$.

The fact that a is a complex function (b is a real function) leads to Landau damping both for fermion mode and for plasmino mode. This means that both the width (or the inverse lifetime) of the fermion mode, γ_p, and that of the plasmino mode, γ_h, are nonzero. However, from the expression of a, both γ_p/ϵ_p and γ_h/ϵ_h, caused by Landau damping, are suppressed by $|T/\epsilon_p|^3$ and $|T/\epsilon_h|^3$. If $\epsilon_p, \epsilon_h > T$ (we shall see in Fig. 4 that this is really the picture), the Landau damping is not important and the excitation modes are well defined. (The energies and widths of the excitation mode can be depicted by the Briet-Wigner approximation). In this case we can at first ignore the imaginary part in a to compute the excitation energies and then replace q^0 by ϵ_p or ϵ_h respectively in computing the exciton width in the term $\frac{3\pi N_c \alpha_g^2 \zeta(3)}{16 y^3}$.

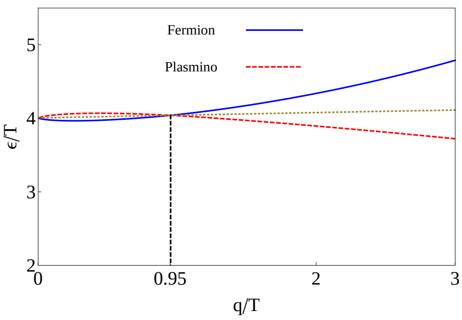

Figure 3: Dispersion relations of fermion and plasmino. The solid curve is the dispersion of fermion while the dashed one is the dispersion of antiplasmino. The dotted curve between solid and dashed curves is the expression $N_c^{1/4}\alpha_g^{1/2}(\frac{N_c\pi\alpha}{16}d_1(x) - \frac{4\ln 2}{\pi}d_{s1}(x))^{1/4}$. The vertical line shows the position of q_c.

Choosing a suspended graphene, that is, $\alpha_g = 2.1$ and $N_c = 4$, we present in Fig. 3 our results of energy dispersions of fermion excitation and plasmino excitation. (Notice that as pointed out by Ref. [12], the predicted semimetal-insulator transition has not yet been observed in experiments in zero magnetic field). We find that there are similarities and several striking differences between the results of our system and the ones of QCD/QED [3, 4, 7].

There are some similarities. First, in the larger $x = q/T$ region, the plasmino energy is always below the fermion energy, analogous to QCD and QED [2, 3, 7]. Second, the plasmino and fermion are the same at $q = 0$. In our system the coincident energy at $q = 0$ is around $\epsilon_0 \simeq 1.15 N_c^{1/2}\alpha_g^{3/4}(1 - 0.13/(N_c\alpha_g))^{1/4}T \simeq 4T$, which is also proportional to temperature T.

The most important fact is, however, that our results are significantly different to those of QCD/QED systems. Firstly, for ordinary $N_c = 4$ and $\alpha_g \sim 2$, *i.e.*, $0.13/(N_c\alpha_g) \ll 1$, $\epsilon_0 \propto N_c^{1/2}\alpha_g^{3/4}$ (Note that in a QED/QCD system $\epsilon_0 \propto \alpha_g$). Secondly, when $0 < q < q_c \simeq 0.953T$, we have an opposite relation between ϵ_p and ϵ_h, that is, $\epsilon_p < \epsilon_h$. Thirdly, the fermion and antiplasmino have the same energy not only at $q = 0$, but also at $q = q_c$, $\epsilon_p(0.95T) = \epsilon_h(0.95T) = 4.04T$.

In a QCD/QED system, the fermion energy increases monotonically and the collective plasmino mode exhibits a minimum at $q \neq 0$ when momentum q increases. In our system, however, the fermion mode (but not plasmino mode) exhibits as sunken, that is, it has a minimum at $q_1 \simeq 0.26T$ and $\epsilon_p(q_1) \equiv m_p \simeq 3.966T$. Furthermore, the behavior of plasmino mode is significantly different from other systems. In the interesting region, that is, $\epsilon_h > q$, it has a maximum at $q_2 \simeq 0.44T$ and $\epsilon_h \equiv m_h \simeq 4.069T$. At $q > q_2$, $\epsilon_h(q)$ is not a monotonically increasing function but a monotonically decreasing one. This phenomenon may be nominated as plasmino anormal dispersion. To understand the anormal dispersion, we note that, roughly speaking, when the momentum increases, on one hand, the average energy of the trapped particle should generally increase as well, however, on the other hand, the trapping should decrease as the momentum increases. When q is not very large, the trapping decreasing is smaller than the momentum increasing and the plasmino energy is an increasing function. However, and in contrast, when $q > q_2$, the trapping decreasing is larger than the momentum increasing and therefore the plasmino energy is a decreasing function (The point can be seen in Fig. 4). Since the plasmino is related to the electromagnetic properties of the system, studying the plasmino anormal dispersion and its effect is interesting.

We also list decay widths of fermions and plasminos due to Landau damping in Fig. 4. From the figure one finds that obviously $\gamma_h \ll \epsilon_h$ when $q < 3T$ and $\gamma_p \ll \epsilon_p$ when $q < 6T$. Therefore, both fermion and plasmino are well-defined modes. It is interesting that, approximately, the fermion energies can be approximately depicted by $\sqrt{m_p^2 + (q-q_1)^2}$ at moderate q. In other words, the fermion behaves as a "relativity particle" with effective mass m_p. The plasmino energy at moderate q can approximately be depicted by $m_h - 0.086(q-q_2)^2/T + 0.0054(q-q_2)^4/T^3$. Since

$\epsilon_0 \simeq m_p \simeq m_h$, one can nominate ϵ_0 as thermal mass.

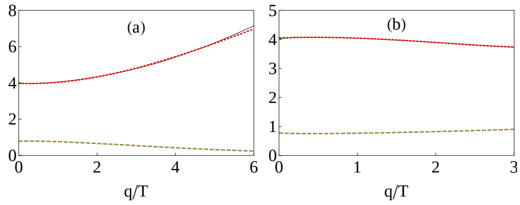

Figure 4: Dispersion relations and decay widths of fermion (a) and plasmino (b). Solid curves are energies and dashed ones are decay widths. Dotted curves are $\sqrt{m_p^2 + (q-q_1)^2}$ (a) and $m_h - 0.086(q-q_2)^2/T + 0.0054(q-q_2)^4/T^3$ (b) respectively .

In summary, at nonzero temperature, we have found that in the long-wavelength region of an intrinsic 2D massless Dirac system there are not only normal collective fermion modes, but also collective plasmino modes, the chiral of which are opposite to the fermion modes. Both energies are on the order of $\epsilon_0 \simeq 0.15 N_c^{1/2} \alpha_g^{3/4} (1 - 0.13/(N_c \alpha_g))^{1/4} T \simeq 4T$. Since in the interesting region $\gamma_p \ll \epsilon_p$ and $\gamma_h \ll \epsilon_h$, both fermion and plasmino are well defined modes. However, there are sharp differences between the discussed system and the QCD/QED system. Firstly, ϵ_0 is proportional to $\alpha_g^{3/4} T$ but not the normal one of $\alpha_g T$. Secondly, at $0 < q < q_c$, we have relation $\epsilon_h > \epsilon_p$ but not the normal one $\epsilon_h < \epsilon_p$ which is valid in QCD/QED systems. Thirdly, the mode which has a minimum at $q \neq 0$ is not plasmino but fermion; on the contrary, the plasmino has a maximum at $q_2 \neq 0$. Although the fermion energy increases monotonically with increasing momentum at $q > q_1$, the plasmino energy decreases monotonically with increasing momentum at $q > q_2$. In this Letter we nominated the interesting phenomenon as anormal dispersion. We believe that our predictions can be tested in a 2D massless Dirac system, specifically, a graphene system, at finite temperature. Note that the material conductivity is related to fermion degree. Our discussions may help to understand the confliction of graphene dc conductivity between experiments and theoretical calculations [14] at $T \to 0$.

The plasmino mode was first predicted in a QCD system. However, the existence of plasmino in QCD is still under debated. We predict that in a 2D massless condensed system, specifically, a graphene system, one can observe the plasmino mode, which is on the order of 0.1eV at room temperature. The prediction can be detected, for instance, by Infrared spectroscopic techniques on graphene. It is hoped that the results of this study will be helpful in designing new type of light-emitting devices.

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