

Neutrino-Deuteron Reactions in the $\Delta(1232)$ Region

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Abstract The results from an investigation of the incoherent single pion electroweak production on the deuteron in the $\Delta(1232)$ region are reported. The importance of including the nucleon-nucleon final state interactions in extracting the neutrino-nucleon cross sections from the data obtained from the experiments on the deuteron target is demonstrated.

KEYWORDS: neutrino-deuteron reactions

1. Introduction

It has been well recognized that the neutrino(ν)-nucleus reactions in the about 1-3 GeV region are dominated by the excitation of the $\Delta(1232)$ resonance. To determine the neutrino properties from the data of ν -nucleus reactions, it is necessary to develop theoretical models for calculating the nuclear effects in this resonance region. Our approach in this direction consists of two steps: (1) develop a model which can describe the cross sections of electroweak single pion production on proton (p) and neutron (n), (2) use the ν -nucleon reaction amplitudes generated from the constructed model to calculate the nuclear effects by using the Δ -hole model which had been well-developed in the studies of pion-nucleus and photon-nucleus reactions. The objective of this report is to review the progress we have made [1–5] in the step 1. The results from the step 2 had been published in Ref. [6], but will not be covered in this contribution.

Since there exists no neutron target, the available data on ν -neutron reactions were extracted from analyzing the data obtained from the experiments on the deuteron target, as done in analyzing the data from Argonne National Laboratory (ANL), Brookhaven National Laboratory (BNL) and European Organization for Nuclear Research (BEBC-CERN) [7–14]. The essential assumption of these analyses is that in the region near the peak of the quasi-free nucleon knock out process, one of the nucleons in the deuteron does not participate in the reaction mechanism and can be treated as a spectator in evaluating the cross sections on the deuteron target. For analyzing future experiments, it is essential to examine the extent to which this spectator approximation procedure is valid. In this contribution, we report on our results [5] from investigating this problem.

In section 2, we briefly describe our model for the nucleon. The results on the deuteron target are presented in section 3. A summary is given in section 4.

2. Dynamical model for the electroweak pion production on the nucleon

A model for investigating the pion production from ν -nucleus reactions must first describe successfully all of the following processes on the proton (p) and neutron (n):

$$\pi + p(n) \rightarrow \pi + N, \quad (1)$$

$$\gamma + p(n) \rightarrow \pi + N, \quad (2)$$

$$e + p(n) \rightarrow \pi + N, \quad (3)$$

$$\nu + p(n) \rightarrow l + \pi + N, \quad (4)$$

$$\bar{\nu} + p(n) \rightarrow l + \pi + N, \quad (5)$$

where l denotes the outgoing leptons, and the final $\pi + N$ can be of any πN state permitted by the electroweak conservation laws within the Standard Model. In Refs. [1–3], we have developed such a model by constructing a Hamiltonian of the following form

$$H = H_0 + H_1 + H_{em} + H_{cc} \quad (6)$$

with

$$H_1 = [v_{\pi N, \pi N} + \Gamma_{\pi N, \Delta}] + [h.c.], \quad (7)$$

$$H_{em} = [v_{\pi N, \gamma N} + \Gamma_{\gamma N, \Delta}] + [h.c.], \quad (8)$$

$$H_{cc} = [v_{\pi N, W^{\pm} N} + \Gamma_{W^{\pm} N, \Delta}] + [h.c.], \quad (9)$$

where H_0 is the free Hamiltonian for all of the particles in Eqs.(1)-(5), H_1 , H_{em} and H_{cc} describes the πN scattering Eq.(1), the electromagnetic processes Eqs.(2)-(3) induced by the photon (γ), and the weak charged current (cc) processes Eqs.(4)-(5) induced by the W^{\pm} bosons, respectively. In the above equations, $[h.c.]$ denote the hermitian conjugates of the interaction terms within the square brackets. The two-body interactions $v_{\pi N, \pi N}$, $v_{\pi N, \gamma N}$, and $v_{\pi N, W^{\pm} N}$ are determined by the tree-diagrams of the well known phenomenological Lagrangian with N , Δ , π , ρ , and ω fields and the associated electroweak currents. The vertex interactions $\Gamma_{\pi N, \Delta}$, $\Gamma_{\gamma N, \Delta}$, and $\Gamma_{W^{\pm} N, \Delta}$ are determined by fitting the data of the processes Eq.(1), Eqs.(2)-(3), and Eqs.(4)-(5), respectively.

With the Hamiltonian defined above, the reaction amplitudes on the nucleon with a πN final state can be written [1] as

$$T_{\pi N, \alpha N}(E) = t_{\pi N, \alpha N}(E) + \bar{\Gamma}_{\pi N}(E) \frac{1}{E - m_{\Delta}^0 - \Sigma(E)} \bar{\Gamma}_{\alpha N}^{\dagger}(E), \quad (10)$$

where $m_{\Delta}^0 = 1299$ MeV is the bare mass of the Δ , and $\alpha N = \pi N, \gamma N, W^{\pm} N$. The non-resonant $\pi N \rightarrow \pi N$ amplitude $t_{\pi N, \pi N}(E)$ and the dressed $\Delta \rightarrow \pi N$ vertex $\bar{\Gamma}_{\pi N}(E)$ are defined by

$$t_{\pi N, \pi N}(E) = v_{\pi N, \pi N} \left[1 + \frac{1}{E - H_0 + i\epsilon} t_{\pi N, \pi N}(E) \right], \quad (11)$$

$$\bar{\Gamma}_{\pi N}(E) = \left[1 + t_{\pi N, \pi N}(E) \frac{1}{E - H_0 + i\epsilon} \right] \Gamma_{\pi N, \Delta}, \quad (12)$$

$$\bar{\Gamma}_{\pi N}^{\dagger}(E) = \Gamma_{\pi N, \Delta}^{\dagger} \left[1 + \frac{1}{E - H_0 + i\epsilon} t_{\pi N, \pi N}(E) \right]. \quad (13)$$

Keeping only the first order in electroweak coupling, the amplitudes for the electroweak processes are accordingly defined by

$$t_{\pi N, \beta N}(E) = \left[1 + t_{\pi N, \pi N}(E) \frac{1}{E - H_0 + i\epsilon} \right] v_{\pi N, \beta N}, \quad (14)$$

$$\bar{\Gamma}_{\beta N}^\dagger(E) = \Gamma_{\beta N, \Delta}^\dagger + \bar{\Gamma}_{\pi N}^\dagger(E) \frac{1}{E - H_0 + i\epsilon} v_{\pi N, \beta N}, \quad (15)$$

where $\beta N = \gamma N, W^\pm N$. The Δ self-energy in Eq.(10) is defined by

$$\Sigma(E) = \bar{\Gamma}_{\pi N}^\dagger(E) \frac{1}{E - H_0 + i\epsilon} \Gamma_{\pi N, \Delta}. \quad (16)$$

The parameters of the model have been determined by fitting all of the available data of the processes Eqs.(1)-(5) in the $\Delta(1232)$ energy region. The overall good fits to the data have been given in Refs. [1-3] as well as in many experimental papers on the electromagnetic processes Eqs.(2)-(3). Here we only give two examples. In Fig.1 we show that the predicted structure functions of $p(e, e'\pi^0)p$ are in good agreement with the data. Another example is shown in Fig.2 for the fits (solid curves) to the available total cross section data of $\nu + N \rightarrow l + \pi + N$. We note here that the calculated total cross sections for the neutron target are not in good agreement with the data. Before we improve our model, it is however necessary to examine the extent to which the spectator approximation used in extracting these data from the data on the deuteron target is valid, as discussed in section 1. We address this important question in the next section.

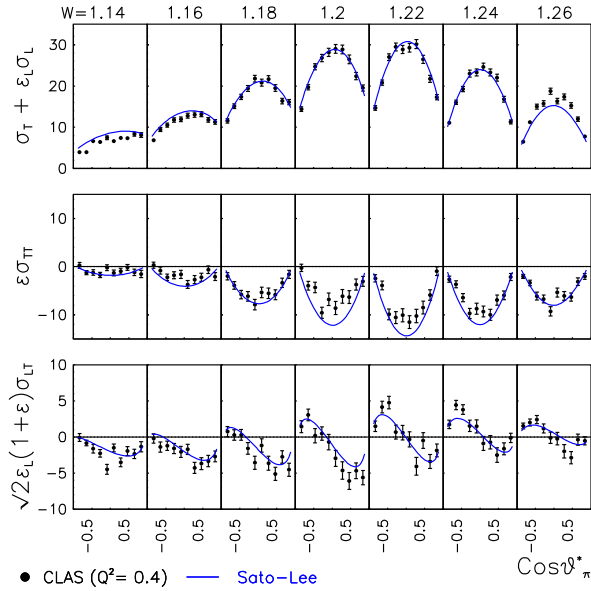


Fig. 1. Predictions of Refs. [1, 2] are compared with the structure function data of $p(e, e'\pi^0)p$. The data are from Ref. [15]

3. Electroweak reactions on the deuteron

In our investigations [4, 5], the amplitudes of the incoherent electroweak pion production on the deuteron consist of an impulse (imp) term, a nucleon-nucleon (NN) final state interaction term, and a pion-nucleon (πN) final state interaction term:

$$T_{\pi NN, \beta d} = \langle \pi [N_1 N_2]_A | J^{(Imp)}(E) + J^{(NN)}(E) + J^{(\pi N)}(E) | \beta \Phi_d \rangle, \quad (17)$$

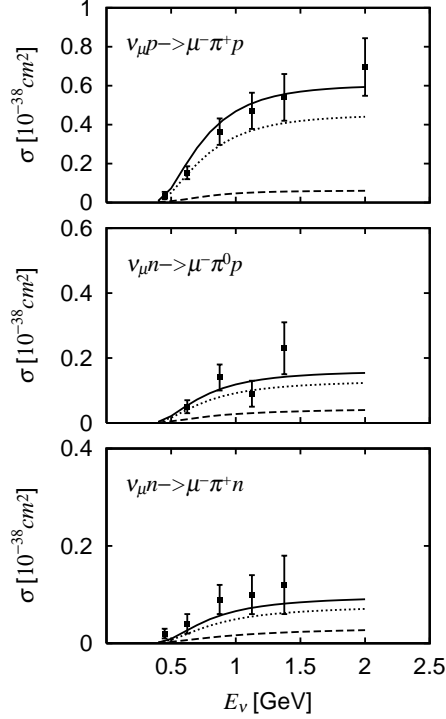


Fig. 2. The total cross sections of ν -nucleon reactions. The solid curves are from the full calculations of Ref. [3]. The dotted curves are from the calculations including only the contributions from the Δ resonant amplitudes. The dashed curves are from the calculations including only the contributions from the non-resonant amplitudes. The data are from Ref. [16]

where $\beta = \gamma, W^\pm$, Φ_d is the deuteron bound state wavefunction, and $\langle \pi[N_1 N_2]_A |$ is a plane-wave πNN state with an anti-symmetrized NN component. Each term in Eq.(17) can be calculated from the single-nucleon matrix elements $T_{\pi N, \beta N}$ defined in section 2, and the NN scattering amplitude $T_{NN, NN}(E)$ which can be generated from various realistic NN potentials. Schematically, they can be written as

$$J^{(Imp)}(E) = [T_{\pi N_1, \beta N_1}(E_{\pi N_1})] + [1 \rightarrow 2], \quad (18)$$

$$J^{(NN)}(E) = [T_{N_1, N_2, N_1, N_2}(E_{N_1 N_2}) \frac{|\pi N_1 N_2 \rangle \langle \pi N_1 N_2|}{E - H_0 + i\epsilon} T_{\pi N_1, \beta N_1}(E_{\beta N_1})] + [1 \leftrightarrow 2], \quad (19)$$

$$J^{(\pi N)}(E) = [T_{\pi, N_2, \pi, N_2}(E_{\pi N_2}) \frac{|\pi N_1 N_2 \rangle \langle \pi N_1 N_2|}{E - H_0 + i\epsilon} T_{\pi N_1, \beta N_1}(E_{\beta N_1})] + [1 \leftrightarrow 2], \quad (20)$$

where E_{ab} is the energy associated with particles a and b , as specified in Ref. [5]. In Fig.3, we illustrate each term in the calculations of the amplitudes of the $\nu(p_l) + d(p_d) \rightarrow l'(p_{l'}) + \pi(k) + N_1(p_1) + N(p_2)$ process.

3.1 Test of the model in the study of $\gamma + d \rightarrow N + N + \pi$ reactions

We first test our calculation procedures by comparing the calculated cross sections of $\gamma + d \rightarrow N + N + \pi$ reactions with the data. Our results for the total cross sections of $\gamma + d \rightarrow \pi^0 + n + p$ are shown in the left-side of Fig.4. When only the impulse term $J^{(Imp)}$ is included, we obtain the dashed curve. It is greatly reduced to dot-dashed curve when the np final state interaction term $J^{(NN)}$ is added in the calculation. When the πN final state interaction term $J^{(\pi N)}$ is also included in our full calculation, we obtain the solid curve. Clearly, the np re-scattering effects are very large while the πN re-scattering give negligible

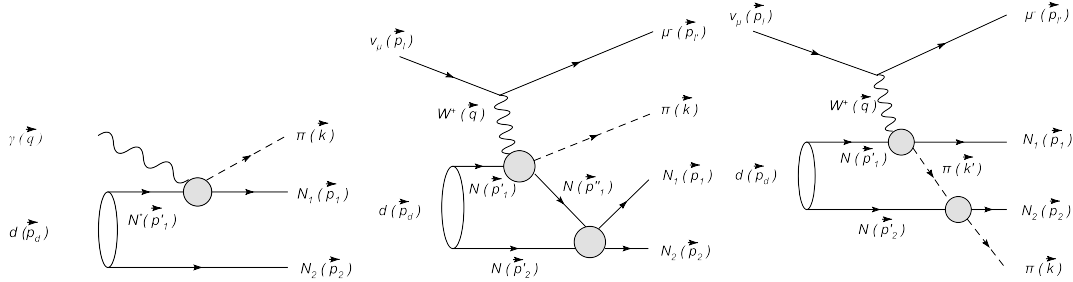


Fig. 3. The impulse (left), NN final state interaction (center), and πN final state interaction (left) mechanisms of $\nu(p_l) + d(p_d) \rightarrow l'(p_l') + \pi(k) + N_1(p_1) + N(p_2)$

contributions.

Similar comparisons for the total cross sections of the $\gamma + d \rightarrow \pi^- + p + p$ process are shown in the right side of Fig.4. Here we see that both the pp and πN final state interactions are weak. Comparing the two results shown in Fig.4, we see the large differences between np and pp final state interactions.

This results shown in Fig.4 are consistent with the results of Refs. [17–21], and can be understood qualitatively from the properties of the initial deuteron wave function and the final NN wave functions. We first observe that the final πNN interactions are mainly due to the s-wave NN states in the considered energy region. For $\pi^0 np$ final state, the dominant final np state is ${}^3S_1 + {}^3D_1$ which has the same quantum number as the initial deuteron state. Since the radial wave functions of the deuteron and the scattering state in this partial wave must be orthogonal to each other, one expects that the loop integrations over these two wave functions are strongly suppressed compared with those from the impulse approximation calculations. On the other hand, there is no such orthogonality relation for the ${}^1S_0 pp$ in the $\pi^- pp$. Consequently the final state interaction effect in the $\gamma + d \rightarrow \pi^0 + n + p$ is much stronger than that in the $\gamma + d \rightarrow \pi^- + p + p$.

We see in Fig.4 that our full calculations (solid curves) are in reasonable agreement with the data in both the shapes and magnitudes, while some improvements are still needed in the future. Thus our approach based on Eqs.(17)-(20) is valid for predicting the $\nu + d \rightarrow \mu + \pi + N + N$ cross sections, as given in the next subsection.

3.2 Predictions of $\nu + d \rightarrow \pi + N + N$ cross sections

In section 1, we describe a spectator approximation which was used in the previous analyses [7–13] to extract the neutrino-induced single pion production cross sections on the proton and neutron from the data on the deuteron target. Here we use our model to examine the extent to which this approximation is valid.

To be specific, we consider the case that the spectator nucleon is at rest. If there is no final state interactions, the $\nu + d \rightarrow l^- + \pi^+ + n + p$ cross section is only from the pion production on the other nucleon which is also at rest in the deuteron rest frame. Then the cross sections measured at the kinematics that the final proton (neutron) at rest $\vec{p}_p = 0$ ($\vec{p}_n = 0$) are simply the cross sections of $\nu_\mu + n \rightarrow \mu^- + \pi^+ + n$ ($\nu_\mu + p \rightarrow \mu^- + \pi^+ + p$). These are the dashed curves in Fig. 5. When the NN final-state interaction terms are included, we obtain the dotted curves in Fig. 5. The solid curves are obtained when the πN final state interaction is also included in the calculations. Clearly, the NN re-scattering can significantly change the cross sections while the πN re-scattering effects are weak. It is also important to note that the NN re-scattering effects on the cross sections for $\vec{p}_p = 0$ are rather different that

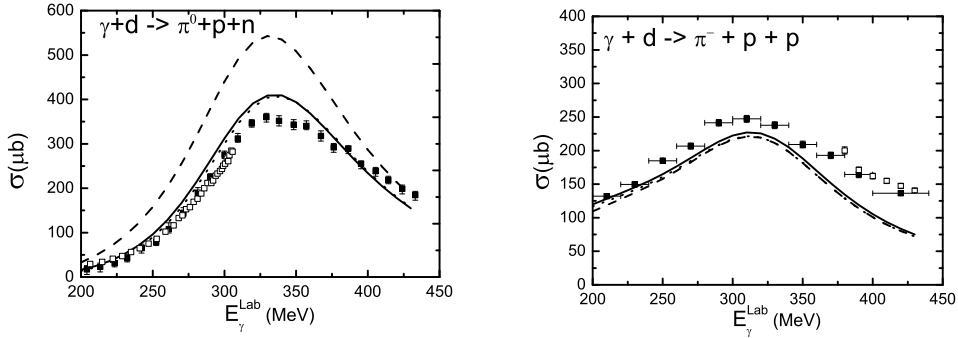


Fig. 4. The total cross sections of $\gamma + d \rightarrow \pi^0 + n + p$. The dashed, dotted, and solid curves are from the calculations include only the impulse term, the impulse + (NN final state interaction), and the Impulse + (NN final state interaction) + (πN final state interaction), respectively. Data are from Refs. [22–24].

for $\vec{p}_n = 0$.

The results shown in Fig. 5 strongly suggest that the spectator assumption used in the previous analyses [7–13] is not valid for the π^+ process $\nu + d \rightarrow \mu^- + \pi^+ + n + p$. This result is due to the large np re-scattering effects, as explained in the subsection 3.1.

We have also examined the results for $p_s = 0$ for the π^0 process $\nu + d \rightarrow \mu^- + \pi^0 + p + p$. Here we find that the spectator assumption is a good approximation for extracting the cross section on the nucleons from the deuteron target. This is of course due to the weak pp final state interactions, as can be seen in the right side of Fig.4.

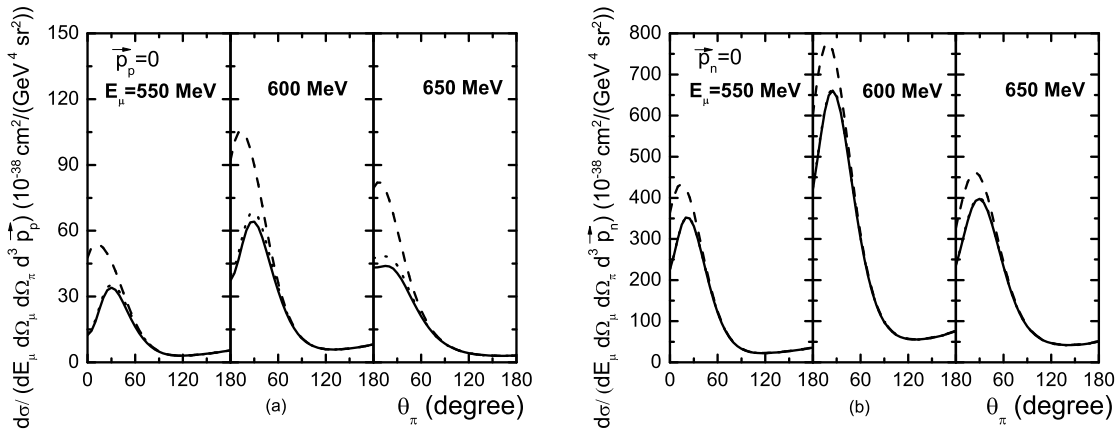


Fig. 5. The differential cross sections $d\sigma/dE_{\mu^-} d\Omega_{\mu^-} d^3\vec{p}_n$ of $\nu_{\mu} + d \rightarrow \mu^- + \pi^+ + p + n$ as function of pion scattering angle in the laboratory system. The left (right) figure is for the proton ($\vec{p}_p = 0$ (neutron $\vec{p}_n = 0$)) spectator kinematics. The outgoing muon energy is $E_{\mu^-} = 550, 600,$ and 650 MeV. The dashed, dotted, and solid curves are from calculations including only the Impulse term, Impulse + (NN final state interaction), and Impulse + (NN final state interaction) + (πN final state interaction), respectively. The dotted and solid curves are almost indistinguishable since the πN final state interaction effects are very small.

4. Summary

We have studied the incoherent electroweak pion production reactions on the deuteron target. It is found that the predicted $\gamma + d \rightarrow \pi^0 + n + p$, $\pi^- + p + p$ cross sections agree well with the available data. The cross sections of $\nu + d \rightarrow l^- + \pi^+ + n + p$, $l^- + \pi^0 + p + p$ have been predicted. It is demonstrated that the NN final state interactions have large effects on the processes with np final states. Our results indicate the need of re-analyzing the available data on the deuteron target to extract more accurately the cross sections on the neutron and proton targets.

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