Scheme of a Derivation of Collapse from Quantum Dynamics

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Abstract

Two kinds of results regarding local properties of entanglement are given in this work and applied to the problem of collapse:
1. Strong cluster properties of local entanglement in a macroscopic system result directly from the Schrödinger equation when there is interaction with another system. These properties, which cannot be expressed by observables, are nevertheless associated with well-defined local probabilities, though different from standard quantum probabilities. They evolve with a finite velocity under nonlinear wave equations, until complete entanglement between the two systems.
2. When these local properties are extended to the interactions of a macroscopic system with its environment, and especially to fluctuations in these interactions, they induce a substantial level of incoherence in the quantum state of the system.

These properties of clustering and of incoherence can combine together during a quantum measurement, to generate an explicit mechanism of random collapse in which randomness derives from incoherence and the associated outcomes of collapse are governed by Born's probability rule.

Although these results are partly conjectural, they seem suggestive enough for proposing their trend as a renovated strategy for approaching the collapse problem.

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The problem of wave function collapse —asking why reality is unique in a quantum world— came long ago to the foreground of physics and still remains a nagging question: "Can one really understand quantum mechanics?" A series of articles by Schrödinger [1] and one by Einstein, Podolsky and Rosen [2], both published in 1935, left in this regard a long-lasting impression of incompleteness in quantum theory [3,4].

Many tentative answers were proposed. Some essays tried to modify in depth the interpretation of quantum mechanics [5-7], or revised its foundations [8, 9]. Attempts were made also to complete its physics by extraneous phenomena [10], or questioned its exactness [11, 12]. Other conceptions of understanding were also proposed along philosophical lines (13-15].

The approach of the present work is somewhat more conventional and reconsiders anew, with the help of new results, the possibility that *quantum mechanics could predict collapse*.

It may be convenient to say how this work is written, to make it clearer. It arose from a finding of two key results: A first one, regarding local properties of entanglement [16], was the existence of a specific probability for these local properties, which is not reducible to standard quantum probabilities (squares of amplitudes [17]). This line of research was found fecund in suggestive consequences.

A second result was the discovery among these consequences of a generation of incoherence in the state of a macroscopic system, from fluctuations in its interaction with environment.

When joined together, these two results were found to bring out a well-defined and rather simple mechanism of collapse, relying only on quantum dynamics and local entanglement, and which is the main topic of the present paper.

A complete exposition of this proposal on the basis of quantum principles would require however some reconsideration of the standard Von Neumann-Dirac interpretation of quantum mechanics [18, 19], to include an introduction of the principle of cluster decomposition [20]. This is because this principle, advocated as necessary for a consistent axiomatic construction of quantum field theory [20], is closely linked with the local properties of entanglement with which one will have to deal. No attempt of such a revision will be here however, for the sake of brevity, and one will only rely on a set of converging remarks, from which a proposed interpretation of collapse will come out.

The main points of this proposal stand as follow: Collapse would result from an accumulation of many elementary effects, in which for instance two atoms in a macroscopic measuring system enter (or a few elementary constituents of such a system). Every elementary effect would occur when one atom, locally entangled with a measured system, happens to interact with another atom, which is not locally entangled with the measured system and is moreover in an incoherent state (resulting from incoherence in a fluctuating action of environment). The conjunction of these conditions regarding local entanglement and incoherence is sufficient to entail small *transfers of quantum probabilities* between different measurement channels. This effect is impossible in an isolated system because of coherence of its evolution [1, 21], and it will be called here accordingly an elementary *misstep* in coherence, to stress its evasion from Schrödinger's conclusion in the case of an isolated system [1, 21].

This proposal will be developed in two successive parts and several sections. Part I is devoted to an introduction and to a general survey, which aims at making the proposals as

clear as could be. Section 1 in this part deals with preliminaries regarding the approach to be used. Section 2 describes the main aspects of local entanglement and Section 3 draws from them the existence and the main properties of an incoherence resulting from fluctuations in environment. Section 4 deals first with the notion of missteps and draws then from their accumulation an explicit mechanism of collapse.

A second part of the paper deals with more technical aspects of the proposals and the results in Part I, Section 5 completing some points on local entanglement, Section 6 on incoherence and Section 7 on collapse. The conclusions in Section 8 consider how these proposals compare with some basic points in the interpretation of quantum mechanics.

PART I: A SURVEY

1. Preliminaries

These preliminaries are a mixing of experimental, theoretical and conceptual considerations, which are supposed to provide a convenient frame for research on the topic of collapse. They proceed by successive steps as follow.

1. The basic rules of quantum mechanics, regarding wave functions, observables and dynamic laws under the Schrödinger equation, will be considered as universal and expressed in the Von Neumann-Dirac formalism [18, 19].

2. Intrinsic randomness of quantum processes will be granted, the associated quantum probabilities being expressed by squares of quantum amplitudes according to Born's probability rule.

3. Macroscopic dynamics, which uses the Lagrange-Hamilton formalism for collective observables at macroscopic scales, is considered a consequence of Schrödinger's dynamics at these scales [22]. The conservation of uniqueness of this macroscopic world at macroscopic scales is thus insured, together with its macroscopic behavior, but the origin and emergence of this uniqueness raise more difficult questions, which belong to the topics of next sections.

4. Careful experiments, especially in quantum optics, have shown decisively that collapse does never occur at a microscopic level [23]. It is always seen on the contrary in measurement at macroscopic scales, so that measurement theory is only concerned with macroscopic measuring devices.

5. Local properties of entanglement will take a central place in the present approach. They were discovered by Lieb and Robinson in the framework of spin lattices [16], but never seem to have come at the center of attention in research regarding collapse. The present author, who rediscovered them in that framework, called them "intricacy" in ignorance of their previous existence and of their previous name [17]. This specific name will be still used here for the sake of clarity, because local entanglement will often be opposed to global (or algebraic) entanglement[24], when problems regarding measurements will be considered. Clearly distinct names for these different features will be considered suitable in that regard.

6. The existence of intricacy illustrates a quantum principle of "cluster decomposition", which is not often mentioned among the principles of quantum theory (at least in the literature on interpretation). Steven Weinberg stressed the necessity of this principle a consistent axiomatic construction of quantum field theory, rather than starting as

usual from an extension of the model of quantum electrodynamics, in spite of its direct pattern on classical electrodynamics ([20], I, Chapter 4).

Wichmann and Crichton gave the first explicit expression for this principle [25], which "says in effect, that distant experiments yield uncorrelated results" [20]. The idea had appeared however previously in statistical physics, either as clustering behaviors in Green functions, in partition functions or in resolvents [26-28]. It occurred also in studies of multiple scattering and particularly Faddeev's equations [29]. The cluster decomposition principle has therefore many faces, including derivations of Feynman paths in field theory and Faddeev-Popov ghosts in gauge theories ([20], II).

Many forms of "clusters" express a decrease of algebraic connection with distance and, in that sense, weakening in the constraints arising from the framework of Hilbert spaces and observables. No complete and rigorous mathematical construction is known for them however (as far as the present author is aware) but one will see here another example of how they come out, in the case of local entanglement with its application to collapse.

7. Another significant point is concerned with the status of probabilities in quantum mechanics. Von Neumann considered that a quantum probability must refer to a projection operator expressing a value of an observable [18]. In a plainer language, every probability is the square of some quantum amplitude.

Although this definition is quite proper, one will consider here nevertheless that it does not encompass necessarily every kind of probability that can enter in the interpretation of quantum mechanics.

This standpoint is not new and an earlier example occurred in consistent histories [30-32]. They theory involved a wider notion of probability, specific to histories of a system, and they drew benefit from this in logical matters, especially by discarding paradoxes that obscured interpretation for a long time [33]. Other probabilities will be found relevant here, involving intricacy, measuring incoherence or expressing collapse. The equality of these last probabilities with quantum probabilities will be then found to be not a coincidence in nature, but in value, and this equality will be one of the major results of this work.

8. A nontrivial question, which will also be of interest here, is concerned with the notion of quantum system versus the notion of environment, and especially the question: Can one consider the environment of a quantum system as being itself a quantum system?

The main point is of course that a quantum system S is associated with a well-defined set of observables (a C*-algebra), which defines all its properties. It is also associated with a state (most often expressed by a density matrix ρ_S), which yields quantum probabilities for these properties. The fulfillment of these two conditions is far from obvious in the case of the environment around a system S.

There is a vast literature on this question where, briefly said, a party holds for the existence of a wave function Ψ_U of the universe [7] and the opposition rejects this existence as meaningless. One will take here a middle way where Ψ_U will not be given a grand meaning, but only used to define the state ρ_S of a system S as a partial trace over the product $|\Psi_U\rangle\langle\Psi_U|$. One will avoid however considering this use of Ψ_U as making it a source of quantum correlations between local objects and the universe, with ultimate branching effects extending from a local apparatus to the whole universe. The use of Ψ_U will therefore mean only that the same quantum laws hold for a real system in which one is interested, as they do at every place in its environment.

9. As another preliminary, one will recall Pearle's theorem, which is central in CSL theories [11, 12]. One will not assume however, as in these theories, that linear Schrödinger

equations are only approximate [34], and one will take this linearity as exact. The assumptions allowing the validity of Pearle's theorem will need then to be shown consequences of intricacy and incoherence, and that will be done.

These assumptions refer to a microscopic system, denoted by *A,* which will be measured and is in an initial state

$$|A\rangle = \sum_j c_j |j\rangle \qquad (1.1)$$

where distinct eigenvectors $|j\rangle$ of a measured observable are superposed. The theorem assumes moreover that random fluctuations δp_j for the quantum probabilities p_j can occur during some short time interval δt. These fluctuations are supposed Brownian, with correlations proportional to δt so that

$$< \delta p_j \delta p_k > = A_{jk} \ \delta t \ , \qquad (1.2)$$

where the coefficients A_{jk} are supposed to depend only on the probabilities $\{p_j\}$ and the time *t.*

Some more properties will be mentioned in Section 4 and found valid in the present approach. They imply the following theorem, which is a wide extension of a much simpler "gambler's ruin theorem" going back to Huygens and stands as follows:

Theorem (Pearle): There is necessarily a unique outcome of every individual random process obeying the conditions (1.2), in which the probability p_j of a unique state $|j\rangle$ of the measured system *A* becomes equal to 1, while all other probabilities for other channels vanish. The Brownian probability for getting a specific outcome *j* coincides moreover with the initial quantum probability $p_j(0) = |c_j|^2$, in agreement with Born's probability rule [35].

Pearle's theorem was among the main guides of the present work. The problem of finding a physical origin for the existence of fluctuations δp_j in the quantum quantities $p_j(t)$ became then essential and came at center of this research.

10. A last preliminary point is concerned with the place of approximations in the present approach. As a matter of fact, everything in quantum mechanics –and more generally in physics– relies on approximations. Quantum mechanics itself, for instance, has been checked to a high but finite level (of order 10^{-13}) and its agreement with general relativity remains still a question.

The problem of collapse stands in some sense even deeper, since its concern is the uniqueness of real facts, which nothing less than the condition for the existence of physics as a science. The proposals thay will be made here regarding its origin will also involve approximations and this is unavoidable. These approximations will be acknowledged and made apparent, even if not stressed at every step of the discussion. The main quality, which one will ask from them will only be robustness in their conclusions for practical (*i.e.*, realistic) purposes.

To conclude these preliminaries, one may recognize that some of them could have remained unstated, but they were thought nonetheless worth mentioning because the problem of collapse is one where different authors can start from many different prolegomena. The

assumptions that were to be supposed and used in a newfangled approach were therefore thought to need clean statements and this is what one tried to do.

One will now proceed from there on to a survey in which intricacy, then incoherence, and finally a proposed mechanism of collapse will be successively outlined.

2. Intricacy (local entanglement)

Local entanglement (which is called here "intricacy" as explained in Point 5 in the preliminaries) is based on some mathematical properties of the Schrödinger equation, which will be explained in detail in Section 5. It implies several consequences regarding physical applications, which are the main topic of the present introduction.

Their derivation begins like an exercise about Schrödinger's equation: One considers a particle, denoted by *A*. A gas of atoms, enclosed in a box, constitutes another quantum system *B*, which is macroscopic. The atoms in *B* interact together pairwise through a potential *V* and they can also interact with the particle *A* through another potential *U*. The particle *A* arrives initially from outside and enters the box at time 0. One can then easily write down the Schrödinger equation governing the composite *AB* system.

One can also play a game, with the intention of showing more easily what is going on. For that purpose, the particle *A* is supposed to carry a red color, whereas all the atoms in *B* are colored white before time 0. The rules of the game specify that, when the red particle *A* interacts with a white atom *a,* this atom becomes red. When such a red atom *a* interacts later with another atom *b*, which is still white, *b* becomes red whereas *a* remains red. The red color is acquired moreover once and for all, with no return, so that when two red atoms interact, they remain red, and two white atoms remain white when they interact. Similarly, when the particle *A* (which is supposed always red) interacts with a red atom, this atom remains red. The rules of the game are thus complete and the transmission of redness appears as an irreversible process, to which the name of "contagion" seems appropriate.

This game is detailed in Section 5, formally and explicitly, as an exercise in which the basic rules stand only on the Schrödinger equation. The white color of one atom is replaced there by an index 0 and the red color by an index 1. One says that an atomic state with index 1 is *intricate* with the particle *A*, whereas a state with index 0 is said non-intricate.

This derivation might look purely formal and arbitrary, but one can also see it as a refinement in the study of Schrödinger's equation, with interesting new aspects coming out as follow: Every atom in a macroscopic system receives an index 1 when it has been influenced earlier by the microscopic system *A*, or an index 0 when it has always been non-influenced. Some significant features of intricacy, which appear then, are its existence as a formal consequence of quantum dynamics and –perhaps more interestingly– intricacy appears as an irreversible property that is not expressible by means of any quantum observable: Intricacy stands thus as a set of properties belonging to the mathematics of quantum mechanics, but not to its standard interpretation.

This lack of relation between intricacy and observables is typical of cluster properties [20], as mentioned in the previous Point 3. In a case when for instance intricacy grows mostly through scattering (between atoms, molecules, or quasi-particles like phonons), a continuous growth of connectedness occurs in the *S*-matrix elements showing evolution of the microscopic measured system *A* together with the elementary constituents (atoms) of the macroscopic measuring system *B*. Like in scattering theory [3], or in many-body theory [26-28], this behavior is primarily a property of connectedness (*i.e.,* a topological and not an algebraic property), particularly in Feynman diagrams where it was firstly introduced [17].

One will also show in Section 5, using Weinberg's axiomatic construction of quantum fields [20], that intricacy (local entanglement) applies also in the framework of quantum field

theory, with intricate and non- intricate fields, indicating thereby a wide domain of significance for the notion of intricacy.

New kinds of probabilities

The field formalism for intricate fields implies also an existence of positive local measures for intricacy, $f_1(x)$, and for non-intricacy, $f_0(x)$, at every point x in the macroscopic system B. These measures are shown to exist in Section 5 for realistic states of an isolated system and also for a system in contact with environment.

One cannot be sure that the sum of the two measures $f_1(x)$ and $f_0(x)$ is always equal to 1, particularly in the case of an isolated system in a pure state. But the relation

$$f_1(x) + f_0(x) = 1 \qquad (2.1)$$

holds certainly true for realistic systems, isolated or not. This is considered a consequence, as shown in Section 5, of a high complexity in the realistic states, which are necessarily highly mixed. There are of course some practical conditions for the validity of these properties, such as dealing with temperatures not too close to zero, where states tend to become pure, but one will not discuss these conditions in detail here and one will consider simply that speaking of intricacy probabilities is legitimate for the cases that will be specified here.

Another point regarding interpretation is worth mentioning however: The properties of intricacy cannot be expressed by observables and they need therefore an enlargement in interpretation.

When one is dealing with a quantum measurement, one must concentrate on a case where the initial state of the microscopic system A, with which the macroscopic system B interacts, is a superposition, as shown in Equation (1. 1).

One will find in Section 5 that there exist in that case a well-defined local "probability of intricacy" $f_j(x)$ for every channels j in (1.1). Every such channel carries then two kinds of probabilities: global standard probabilities p_j, which are associated with algebraic entanglement, and a set $\{f_j(x, t)\}$ of local time-dependent probabilities of intricacy with the various channels j. There is no global probability for non-intricacy —since intricacy is not a global property— but there is a local probability for non-intricacy, which turns out to be given by

$$f_0(x, t) = 1 - \Sigma_j p_j f_j(x,t). \qquad (2.2)$$

Waves of local entanglement (*intricacy*)

Another question is concerned with the evolution of a local probability of intricacy $f_j(x,t)$. This is in principle a consequence of the Schrödinger equation. It will be sufficient for the sake of illustration to consider first only the case when a particle A crosses the box containing the gas along a linear track. There are then only two local probabilities, $f_1(x, t)$ for intricacy with this unique channel and $f_0(x, t)$ for non-intricacy, the two of them satisfying Equation (2.1).

Simple considerations, which are derived in more detail in Section 5, yield predictions for the evolution of the local probability of intricacy $f_1(x, t)$. They rely on standard approximations, which apply to transport processes —like heat diffusion or electric conduction for instance— and they consider that, in spite of the complexity of quantum

processes, a process depending only on collisions between atoms can be described approximately as a diffusion process [37]. The conservation of intricacy for an atom under its collisions with other atoms, for instance, could be described by a diffusion equation

$$\partial f_1/\partial t_{\text{ diffusion}} = D\Delta f_1, \qquad (2.3)$$

where D is a diffusion coefficient.

A change in the state of intricacy occurs on the other hand when an atom, which is non-intricate with probability f_0, collides with an intricate atom: The first atom becomes then intricate by contagion whereas the second one keeps its intricacy. The probability for this second atom to be intricate is $f_1(x, t)$ and the probability for a collision to occur during a short time interval δt is $\delta t/\tau$, where τ is the mean free time between successive collisions for an atom. The variation of $f_1(x, t)$ owing to contagion is therefore

$$\partial f_1/\partial t_{\text{ contagion}} = f_1 f_0/ \tau. \qquad (2.4)$$

In spite of the simplicity of these considerations, one may consider them as sufficiently significant —though approximate—for representing at least the gross features of the most interesting effects. One may therefore write, after taking Equation (2.1) into account, the nonlinear diffusion equation

$$\partial f_1/\partial t = D\Delta f_1 + f_1(1 - f_1)/\tau \qquad (2.5)$$

as a fair though approximate representation for the evolution of the local probability of intricacy $f_1(x, t)$, in spite od its approximate character.

Other similar simple considerations, for a one-dimensional position variable x, indicate that Equation (2.5) is not satisfied by a function $f_1(x, t)$, which would be positive everywhere like does the diffusion equation (2.3). Nonlinearity requires existence of a finite space region, bounded by a moving two-dimensional surface S, which contains the track of Particle A and in which $f_1(x, t)$ is positive and represents a probability distribution for intricate atoms. Farther beyond S, the atoms are still non-intricate and $f_1(x, t)$ is still equal to zero. This behavior is not surprising as a matter of fact and one knows that sharp moving wave fronts are often consequences of nonlinear wave equations [38]of which (2.5) is an example.

This front must be essentially cylindrical in the present case, and centered on the track of the incoming particle A. The probability of intricacy $f_1(x, t)$ is still zero (indicating non-intricacy) beyond the front where the influence of A is not yet felt. Behind the front, it is shown in Figure 1 from a numerical calculation in one space dimension.

This shape of the intricacy wave must be essentially the same in one dimension (in the ideal case of an excitation along a plane), in dimension 2 (the present cylindrical case) and in dimension 3 (for a point source). The probability $f_1(x)$ of intricacy must always be close to 1, behind the front, and this front itself may be expected to extend over a distance of order the mean free path of atoms, as shown in Figure 1.

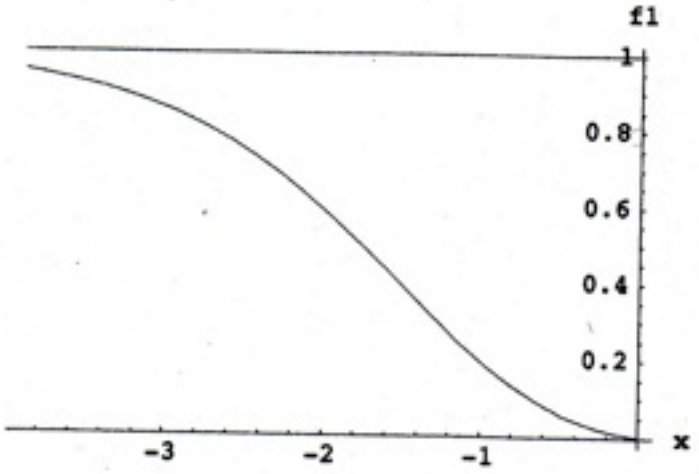

Figure 1: A graph of a wave of local entanglement $f_1(x)$ near its front (located here at $x = 0$), in one dimension or in the case of a cylindrical wave in three dimensions. The scale of abscissas is one mean free path λ and the diffusion coefficient is taken as $D = \lambda^2/6\tau$.

The question regarding the velocity of expansion of this intricacy wave is interesting, but not yet completely solved. The calculation leading to Figure 1 was made with use of the relation $D = \lambda^2/6\tau$ for the diffusion coefficient, which involves the mean free path λ and is valid for a random walk in three dimensions. The wave is then supposed to move with the velocity $v' = v/3^{1/2}$, corresponding to the projection of the random walk of atoms on one direction of motion. One may notice that this velocity coincides then with the velocity of sound c_s in a dilute gas. This interpretation looks reasonable but remains subject to questions.

There are certainly other cases. A look at a possible transmission of intricacy by an electric current, under interactions of conducting electrons with impurities, suggests that in that case the associated velocity would be the Fermi velocity in the conductor. Similarly, in a Geiger counter, a rapidly moving charged particle A can excite nearby atoms and ionize other ones. Photons from the decay of excited atoms would then transport away intricacy at the velocity of light (thought with retardation effects owing to the finite lifetime of the excited states). Electrons, produced together with ions, would also transport intricacy in their own way. The process of intricacy appears therefore remarkably rich and strongly mingled with many other irreversible effects.

3. Fluctuations in Environment and Incoherence

Intricacy has properties unfamiliar in quantum mechanics, as one found in the previous section with its irreversibility and lack of relation with observables. Another of its significant aspects is a link with incoherence, to which one turns now.

The notion of incoherence is reputedly nontrivial in quantum theory. Classically, it has to do with random phases. But, in spite of their consequence, a phase has a delicate status in quantum theory, since it is not associated directly with an observable: It belongs to the imaginary part of the logarithm of a wave function and a logarithm operation is not linear. One must use tricks when dealing with incoherence in a quantum framework. For instance, incoherence of a diffuse light is classically simple but, in a quantum framework, one must often go back to phases of matrix elements and show, for instance, that interferences from two rays in the blue light of sky vanish because of randomness in relevant matrix elements recording distinct scattering events for the two rays [39].

One intends to show now that *fluctuations* in the action of *environment* on a macroscopic system can produce incoherence in the quantum state of that system, but this statement must be made clear. Which notion of incoherence is meant in that case, for instance? Two points will be essential and were emphasized in that statement, namely that only an environment of a *macroscopic* quantum *system* can bring out the kind of incoherence that one wants to discuss. Moreover, the action of environment itself is not responsible for this incoherence, but only fluctuations in that action. As for the nature of the relevant incoherence, one will have to wait for some study before making it clear.

This study will proceed in several steps: Step 1 will consider the behavior of intricacy when an elementary constituent of environment (one external molecule for instance) interacts with a macroscopic system. Step 2 will split this action of environment into an average and fluctuations, with a refined splitting of fluctuations into positive and negative ones, corresponding respectively with excess above average in the action of environment or with a shortage below average. Step 3 will show how intricacy waves carrying fluctuations can be responsible for a transport of random phases, with a randomness originating in fluctuations.

A model

One will deal with a case where the system in which one is interested is a Geiger counter, which one denotes by *B*. It contains an atomic gas (for instance argon) and one wants to concentrate attention on what happens in this gas, which is enclosed in a solid box. The environment surrounding the system is supposed to be an ordinary atmosphere under standard conditions of pressure and temperature. One also assumes for simplicity that the system and its environment are in thermal equilibrium. No other system enters in the discussion and one is therefore considering for instance the state of the system B *before* a measurement.

As mentioned in Point 8 in Section 1, the quantum state ρ_B of the system can be defined as a partial trace over a formal state $|\Psi_U\rangle\langle\Psi_U|$ of the universe, or a part of it. The evolution of the system *B* cannot be separated completely from its interaction with environment, and this is the question one wants to discuss. One considers anyway B as a well-defined quantum system, because its constitutive particles are themselves well defined and this system can be therefore associated with a definite C*-algebra of observables.

The theoretical status of environment is another matter. In principle, one can define it also as a quantum system at any sharp time *t*, by considering it for instance as consisting of all the molecules that are present at that time inside some sphere *S* enclosing *B*. A state $\rho_E(t)$ of this sharply defined environment would also be given by a partial trace over $|\Psi_U\rangle\langle\Psi_U|$. This construction cannot be extended to a finite time interval however, because the relevant algebra of observables changes incessantly with the motion of molecules in the atmosphere, which cross incessantly the ideal boundary *S* from inside and from outside. One should hold therefore the prospect of identifying this environment with a quantum system ill defined and questionable.

This identification can make sense nevertheless when one restricts attention to a unique external molecule *M* colliding with the solid boundary of *B*, and one will now concentrate attention on that case.

Step 1: *Interaction with a unique molecule*

According to Section 2, an intricacy wave is produced within the macroscopic system *B* by a collision of a unique external molecule and this intricacy wave, which has a finite velocity, spends a finite time before crossing the whole system and bringing it into complete

entanglement with the outgoing state of the molecule. The previous discussion of intricacy shows that this time is of order

$$\Delta t = L/c_s, \tag{3.1}$$

where L denotes a typical linear size of the box and c_s the velocity of the wave, which coincides with the velocity of sound in the gas.

One might try to give a more detailed description and say that, when a molecule M hits the solid envelope of the box, some phonons are produced in the envelope and carry intricacy with M into it through phonon-phonon collisions. When the intricacy wave reaches the inner boundary of the solid envelope, it brings out fluctuations in surface phonons (surface vibrations) with which the atoms in the gas interact. The process of transmission of intricacy is therefore rather complex but its essentials, which consist in the existence and in the behavior of a resulting intricacy wave, are nevertheless rather simple, as far as their macroscopic features are concerned.

Step 2: *Average and fluctuations in the action of environment*

The main point from the previous step that one will exploit now is the evidence for a rather long duration Δt in (3.1) for transient intricacy effects. When one takes account of the incessant flux of external molecules, this delay means that at any time t, the number of intricacy waves, which are present in the system B and moving in it, is equal to the total number of molecules having hit the box during the time interval $[t - \Delta t, t]$.

This number is very large and its magnitude leads one to distinguish an average effect and fluctuations in the action of environment. Along with the previously defined "real" state $\rho_B(t)$ of the system B, one introduces therefore an average state $<\rho_B>$, which is supposed to describe only the macroscopic features of the system under the average action of environment. In the present simple case, one may take this average state as expressed by

$$<\rho_B> = \exp[-(H_B + H_{pB})/k_BT - \lambda I). \tag{3.2}$$

where H_B is the Hamiltonian of the B system and H_{pB} stands for the average action of atmospheric pressure on external atoms of the solid box. One will not need to write down an exact expression of H_{pB} for present purposes and, regarding the parameter λ, it only insures a unit trace for $<\rho_B>$.

This procedure allows many variants and is widely used in statistical physics, where $<\rho_B>$ can also account for every available information regarding a system, or provide its macroscopic description [40]. Alternatively, one could also introduce (3.2) as an average of $\rho_B(t)$ over a finite time interval [41]

In practice, one will use here essentially $<\rho_B>$ as a reference allowing to separate the effect of fluctuations in the environment as associated with the difference

$$\Delta\rho_B(t) = \rho_B(t) - <\rho_B> . \tag{3.3}$$

The matrix $\Delta\rho_B(t)$, which is strongly time-dependent, is the quantity in which one is mostly interested.

Its trace vanishes, since ρ_B and $<\rho_B>$ have both unit traces. The really meaningful data in $\Delta\rho_B(t)$ can be identified accordingly with the positive and negative parts of (3.3), under a splitting

$$\Delta\rho_B(t) = \rho_{B+}(t) - \rho_{B-}(t)\,, \tag{3.4}$$

where the matrix ρ_{B+} involves the eigenvectors of $\Delta\rho_B$ with positive eigenvalues, and $-\rho_{B-}$ involves the eigenvectors with negative eigenvalues. One will accordingly rely much on the expression

$$\rho_B(t) = <\rho_B> + \rho_{B+}(t) - \rho_{B-}(t)\,, \tag{3.5}$$

where ρ_{B+} and ρ_{B-} are positive matrices.

Step 3: *Fluctuations and incoherence*

The decomposition (3.5) of $\rho_B(t)$ involving an average and a sum of fluctuations with opposite signs, separates explicitly the positive and negative effects of fluctuations, to which it attributes respectively positive and negative probabilities. The meaning of negative probabilities can be considered as obvious in that case and needs no special comment.

The average number of collisions during a short time interval δt is given by

$$dN = n_e\, v_e\, S\, \delta t, \tag{3.6}$$

where n_e denotes the average number of atmospheric molecules per unit volume, v_e their average velocity and S the external area of the box (A factor $3^{-1/2}$ relating the average velocity of a molecule with its component along a normal to the surface is neglected). Values of n_e around 10^{19}-10^{20} atoms per cubic centimeter, and of order 10^5 cm per second for v_e, can be considered representative.

Intricacy introduces new features in the action of environment, and especially an effect of persistence according to which an individual collision by a molecule keeps influencing the state ρ_B during the rather long time Δt. The main consequence of this persistence is a permanent presence of many waves of intricacy in the macroscopic system *B,* with the large average number

$$N_W = n_e\, v_e\, SL/\, c_s. \tag{3.7}$$

This number is remarkable. One might say at first sight that, since intricacy has no relation with observables, these waves have no physical consequence. But properties regarding coherence versus incoherence, or definite phases versus random ones, are precisely the ones in which one is interested in the present case and one must thus look at them more carefully.

The various collisions, in number N_W, which leave a mark on the state $\rho_B(t)$ at every instant t, were due to collisions of external molecules occurring in various places on the box enclosing B, at various times between t - Δt and t. The corresponding probability law is a Poisson distribution and the corresponding fluctuations in the number of colliding molecules during the time Δt (or the number of associated intricacy waves), is therefore given by

$$N_f = \Delta N_W = N_W{}^{1/2} = (n_e\, v_e\, SL/\, c_s)^{1/2}\,. \tag{3.8}$$

Some of these fluctuating events are associated with local time-dependent excesses in the flux of external molecules and other ones are associated with shortages. The first kind of events,

linked with excesses, generates positive fluctuations in N_W and the second type generates negative ones.

When expressed formally, these positive and negative fluctuations contribute respectively to the matrices $\rho_{B+}(t)$ and $-\rho_{B-}(t)$ in (3.5), and hence they provide a simple interpretation for the associated plus and minus signs. One may also notice that these contributions from fluctuations do not express generally the totality of the matrices $\rho_{B+}(t)$ and $-\rho_{B-}(t)$, which exist also when the system B is isolated. One will have then also to identify in applications which fraction of ρ_{B+} and ρ_{B-} is actually associated with fluctuations in the action of environment.

One arrives thus at a central question regarding incoherence: Even as a matter of principle, there is no way for distinguishing whether an individual molecule hitting the box B contributes either to $<\rho_B>$, ρ_{B+}, or $-\rho_{B-}$. One cannot specify particularly which molecules contribute to a positive or to a negative fluctuation, or where and when a molecule arrived on the box.

As far as fluctuations are concerned, this situation has exactly the same consequences as if a number N_ϕ of molecules, in states belonging to the matrix ρ_{B+} with positive probabilities, had started anywhere a moving intricacy wave in B at an arbitrary time between $t - \Delta t$ and t.

To discuss such phases without long calculations, one may consider an extreme example where the environment consists in a coherent beam of mono-energetic molecules. The phase of the wave function of each molecule, when it collides with the box, is then perfectly defined by the place and the time of arrival of the molecule. Nevertheless, the complete lack of criteria for distinguishing average from fluctuations, and moreover positive from negative fluctuations, implies a random distribution for the phase they carry in ρ_{B+} and in ρ_{B-}. This result is obviously valid for more realistic types of environment and it can be considered to imply that *fluctuations in the action of environment generate incoherence in the quantum state of a macroscopic system.*

A pattern of incoherence

A systematic and careful study of incoherence from environment should be necessary for drawing reliable conclusions about its amount and its detailed effects. As for its existence, the next Section 4 will show that its action could be responsible for the existence of collapse and, although this is not a proof for its existence, it stands as an incentive for its study. This study is not yet available however and Section 6, in which some of the necessary elements are sketched, will only be for that reason indicative or uncertain in some of its parts.

Some of these elements, with which one will be concerned, are the following ones:

- One can consider as valuable the relation of incoherence with the density matrix ρ_B, according to which only the matrices ρ_{B+} and ρ_{B-} can carry incoherence, with plus and minus signs introducing a random competition between them.

- The probability W for incoherence among the states of individual atoms has an upper bound

$$W \leq 4/3\pi, \tag{3.9}$$

which is shown in the Appendix but should be considered with caution in applications.

- There are sensible reasons for considering valuable the properties of intricacy waves carrying incoherence and their associated fronts, as well as a uniform distribution for them in the system *B*.

- Regarding the random phases, which are associated with these waves: A significant point is that different phases with different origins can add up, for instance in an incoming wave function before scattering. The point one wants to mention and which will be used in the discussion of collapse is that a sum of random phases is equivalent in its physical effects with a unique random phase with uniform distribution in the interval $[0, 2\pi]$.

- A subtler question with much significance is concerned with the transfer of randomness from the fluctuating states of external colliding molecules to a random character of phases in the states of individual atoms in the apparatus *B*. It still needs more investigation.

To sum up and in spite of difficulties and remaining uncertainties in the pattern of incoherence resulting from these considerations, one will pursue the study of collapse in the next section on the assumption of its existence. Some relevant considerations can be summarized by a simple assumption according to which: In a macroscopic measuring apparatus *B* where the dynamics is dominated by collisions between atoms or other elements, every atomic state in any place can be supposed to have nonzero "probabilities" $\pm W$ for being in a state with a random phase belonging either to the matrix ρ_{B+} or to ρ_{B-}.

This assumption will be considered as the most problematic one in the present work and the one contributing most to make it conjectural.

4. Sketch of a collapse mechanism

One can now come back to collapse and make an explicit proposal for its origin and mechanism. Two distinct effects enter then and were already mentioned. The first one is the existence of intricacy between the measured system *A* and the macroscopic measuring system *B*. The second effect consists of some incoherence in the state of this measuring system, which is also a consequence of intricacy but with fluctuations in its environment.

Collapse appears thus a quantum process, under a peculiar randomness owing to intricacy. This effect does not act directly on the quantum states of the *AB*-system or its observables, but *collapse acts on the quantum probabilities of various channels and nothing else*.

One will find for instance that a unique quantum probability for a unique measurement channel reaches finally the extreme value 1, which always marks certainty and adds reality at a macroscopic level. One will find that, nevertheless, the inner evolution of the system in every channel continued to progress till the end with all its specific phenomena and the evolution of its observables, before reaching at last this level of reality. The quantum probabilities of various channels were meanwhile the only quantities, which underwent variations and showed differences with the ideal case of an isolated system. All other characteristics of the system behaved on the contrary during that time as Schrödinger had found them to do in his historic paper [1].

This final emergence of reality through collapse will be found a random effect, with a peculiar from of randomness: not intrinsic quantum randomness, but something subtler, or less familiar. The underlying randomness in collapse arises from a competition between positive and negative fluctuations in the action of environment (or local time-varying excesses versus shortages in this action). Collapse appears nonetheless to be a quantum effect, *i.e.* a consequence of the Schrödinger equation, though with new aspects: It affects fine details in

the *history* of wave functions, but not the wave functions themselves or the associated values of observables. One might express this behavior by saying that the mechanism of collapse refines on the consequences of Schrödinger's dynamics, through more attention for the history of the state of system. Conversely, and remarkably, one will see that whether this action on the state belongs to the Schrödinger representation of quantum dynamics, it cannot apparently be express in the framework of Heisenberg's representation. One sees again at work the characteristic behavior of intricacy of a refinement in the history of wave functions together with a lack of relation with observables, but now completed by a drastic action on quantum probabilities in circumstances allowing collapse.

Missteps in entanglement and incoherence

One considers now a possible basic process, which would underlay collapse. To approach it, one notices first that if the quantum probability of some channel can reach randomly the value 1, this effect cannot be total and sudden like a quantum jump, if it results from a dynamical process at macroscopic scale. It must probably proceed through small variations in the probabilities of various channels. If so, a key point must be to understand how exchanges can occur between the quantum probabilities of different measuring channels.

One proposes that this process consists in an accumulation of many small individual events, all of which occur at a microscopic level and involve only two –or a few– individual atoms (or particles, molecules, quasi-particles…). To give a name to such an elementary process, one will call it a *misstep* in coherence, to stress its departure from coherence when compared with the unwanted consequences of coherence in Schrödinger's basic paper [1].

To define a misstep properly, one will consider the example of a collision between two atoms, *a* and *b*, under the two following conditions: (*i*) The initial state $|a\rangle$ of Atom *a* is intricate with a definite state $|j\rangle$ of the measured system *A*. (*ii*) The initial state $|b\rangle$ of Atom *b* is incoherent and also non-intricate with the system *A*. (*iii*) The collision between the two atoms *a* and *b* is incoherent.

These conditions refer to previous statements or remarks: Condition (*i*), for instance, implies that since the initial state of Atom *a* is intricate with the state $|j\rangle$ of *A*, it is also algebraically entangled with that state. The fact that intricacy (local entanglement) implies global (standard) entanglement is essential in this respect.

When saying in Condition (*ii*) that the state of Atom *b* is incoherent, one means that this state $|b\rangle$ carries a random phase, owing to an incoherent action of fluctuations in the action of environment. One will denote this random phase by ϕ_b. Condition (*iii*) is however necessary and is valid in two cases: The state $|a\rangle\rangle$ of the first atom involves no random phase, or it involves also a random phase ϕ_a and the difference ϕ_a - ϕ_b is also random. The case of two incoherent states of the two atoms with the same random phase must be excluded because their collision is then coherent.

A few assumptions and restrictions will make the discussion of a misstep shorter. The first one restricts the discussion to a case when there are only two channels $|1\rangle$, $|2\rangle$, in the initial state (1.1) of the measured system. One may think for instance that the state $|1\rangle$ represents a charged particle, along a trajectory crossing a Geiger counter *B*, whereas the state $|2\rangle$ is associated with a trajectory that is not crossing *B*.

A convenient restriction is associated with the origin of incoherence in fluctuations in action of environment. Positive fluctuations are associated with excesses in the flux of

external molecules and negative fluctuations with shortages. The positive ones are mathematically associated with positive eigenvalues in the spectrum of the matrix ρ_{B+}, negative ones with negative eigenvalues of $-\rho_{B+}$. One may speak then of positive and negative probabilities in an unambiguous way. Since Condition *(ii)* requires the state of *b* to be incoherent, this state must therefore belong either to ρ_{B+} or to $-\rho_{B-}$ and not to $<\rho_B>$. Since the states of Atom *a* and Atom *b* interact, they must both belong to the same matrix $\pm\rho_{\pm}$. A restriction in the discussion will be to consider only explicitly the case of ρ_{B+}. The state $|j\rangle$ in Condition (*i*) will be taken as $|1\rangle$ and the state $|A\rangle$ still in the form (1.1) when the collision occurs between the two atoms.

It will be useful to compare the present case, where incoherence is present, with the case of a coherent *AB*-system, considered by Schrödinger [1]. Condition (*i*) assumes an initial state $|a\rangle$ of Atom *a* intricate with the state $|1\rangle$ of the measured system *A* and, this intricacy implies entanglement between the two state vectors. In Schrödinger's framework where intricacy did not enter, Condition (*i*) can mean only that $|a\rangle$ is entangled with $|1\rangle$. Condition (*ii*) requires on the other hand in the present case that the initial state $|b\rangle$ of Atom *b* should be both incoherent and non-intricate with *A*. Its non-intricacy means that the initial state of the subsystem *A*-*b* has the form

$$|A,b\rangle = (c_1|1\rangle + c_2|2\rangle)\otimes|b\rangle \qquad (4.1)$$

Bur according to Condition (*i*), the initial state of *a* participating in the collision is entangled with $|1\rangle$, so that in the Hilbert space describing the measured particle and these two atoms, one is dealing with the state

$$|A,a,b\rangle = c_1|1\rangle\otimes|a\rangle\otimes|b\rangle. \qquad (4.2)$$

The collision between the two atoms (*a, b*) is governed by a matrix element

$$\langle ab,out|T(|a\rangle\otimes|b\rangle) \qquad (4.3)$$

of a collision matrix *T*, related to the *S*-matrix by $S = I + iT$. But the existence of entanglement between Atom *a* and the measured system *A* requires consideration of the Hilbert space for the three systems. The (*a, b*) collision is governed $c_2|2\rangle\otimes|b\rangle$ only by a matrix element

$$\langle 1|\otimes\langle ab,out|T|a\rangle\otimes|b\rangle\otimes|1\rangle c_1, \qquad (4.4)$$

whereas the term $c_2|2\rangle\otimes|b\rangle$ in (4.1) does not participate in the (*a, b*) collision. Finally, this is only a heavy way for showing after Schrödinger that no transition occurs between different channels as an effect of a collision between two atoms, except that two conditions have been however introduced, mentioning that all interactions are coherent and no effect of intricacy is considered.

Pursuing the comparison, one considers then Condition (*iii*), according to which the collision between the two atoms (*a, b*) is incoherent. One may remember in this regard that Condition (*ii*) supposed also that the state $|b\rangle$ of Atom *b* is incoherent, and therefore carries a

random phase ϕ_b. Condition (*iii*) requires therefore that the initial state $|a\rangle$ either carries no random phase or carries one such phase ϕ_a, distinct from ϕ_b. In both cases, the matrix element (4.4) expressing a collision between the two atoms vanishes under averaging on random phases, when conditions (*i-iii*) are satisfied. This behavior is drastically different from the one in the coherent case with no intricacy, where the matrix element (4.4) does not vanish in Channel $|1\rangle$ where it remains concentrated.

This unusual behavior, which is specific to the occurrence of random phases and a consideration of intricacy, warrants a digression on fundamental matters. It does not refer to the usual interpretation of incoherence in quantum theory, which makes no distinction between quantum systems and non-systems. When one expresses for instance that the states of two diffused photons in the blue light of sky are incoherent, the standard usual method consist in referring to the previous histories of these photons, which involve independent scattering events on uncorrelated atmospheric molecules. The explanation relies then only on observables, whereas a direct reference to phases would only remain a pedagogical analogy with classical waves, since phases are not observables [40].

The present standpoint is different: One saw in Section 3 that the random phases in which one is interested originate from fluctuations in the action of environment after removal of the average effect and a distinction between positive (or active) and negative (or missing) contributions in ρ_{B+} and $-\rho_{B-}$, respectively. This selection cannot be associated neither with an explicit mathematical or physical process, which would be expressed in terms of observables. The only way for making it sensible is to introduce a character of *non-system* for the environment and one is therefore dealing with a *new interpretation.*

To conclude on practical matters, one can say that the quantity

$$\left|\langle a,b,out|T|a\rangle\otimes|b\rangle\right|^2, \qquad (4.5)$$

which controls the probability for the atoms (*a, b*) to scatter does not vanish. This collision is then a typical example of a contagion of intricacy under which the outgoing state of Atom *b* comes the intricate with state $|1\rangle$ of *A* and therefore entangled with it. Its incoming state, however, was *non-intricate* and therefore *entangled* with *both* states $|1\rangle$ and $|2\rangle$. The initial probability, which this atom had to be entangled with $|2\rangle$ has therefore been transferred to a probability of entanglement (and intricacy) with $|1\rangle$.

To avoid confusion, one must stress that *the quantity* (4.5) *does not represent then a quantum probability in the Born-Dirac-Von Neumann sense, but a contribution to a change in probabilities of intricacy*.

One may thus assert as a conclusion that a misstep in coherence in the collision of two atoms, under the previously stated conditions, can be responsible for elementary transfers of quantum probabilities between different measurement channels.

Fluctuations in probabilities

How many missteps occur everywhere in a measuring system, and what fluctuations do they produce in the quantum probabilities p_j? One turns next to this question.

Somewhat surprisingly, one will find that this question is rather easy and can be discussed immediately in the case of arbitrarily many different channels, including also the one when the system B consists itself in several separate and arbitrarily distant pieces.

One thus considers a measuring device, which can be made of various parts but in which all the detecting parts are assimilated to gases. This is still a strong restriction, but it yields a simple paradigm, but which one may expect to extend to less simple cases in a later stage.

If one denotes by x a point anywhere in this collection of separate subsystems, one can define everywhere local probabilities $f_j(x, t)$ for intricacy of the whole system with all channels j, as well as a local probability $f_0(x, t)$ for non-intricacy satisfying Equation (2.2).

Many missteps in coherence occur during a short time interval δt. They are random events with well-defined probabilities and one wants to calculate these probabilities. One saw that an individual misstep occurs when an atom *a,* intricate with a channel j, collides with an atom b in a state that is both incoherent and non-intricate. Variations in channel probabilities are then produced and they can show either positive or negative signs.

When Atom b undergoes a collision during the time interval δt the corresponding probability is $\delta t/\tau$, where τ is the mean free time. The state of this atom is supposed non-intricate so that the corresponding probability is equal to $f_0(x, t)$, if the collision occurs near some point x anywhere in the whole system. The state of Atom b is also incoherent and this property is essential, because incoherence goes along with external fluctuations, which can be positive or negative.

One considers first the case of positive fluctuations and recall that Atom b has then a probability W for being incoherent. Since the state of Atom a is supposed intricate with a state $|j\rangle$, and the associated probability involves two distinct conditions: (*i*) Atom a belongs to Channel j, which carries the probability p_j. (*ii*) Atom a is moreover intricate with the state $|j\rangle$ and this property has a probability $f_j(x, t)$. The a priori probability for occurrence of this misstep under the supposed conditions is therefore equal to $Wp_jf_j(x, t)\, f_0(x, t)\delta t/\tau$.

One looks then at the associated changes δp_k in the quantum probabilities of various channels. Atom b, which was initially non-intricate, was accordingly entangled with every channel $|k\rangle$ with a probability p_k. After collision, Atom b had become entangled with $|j\rangle$. If however the state $|b\rangle$ of Atom b had been non-intricate and coherent (as in Schrödinger's reference case, only its part $p_j^{1/2}|b\rangle$, entangled with $|j\rangle$, would have participated in the collision. This collision would have then only contributed to the growth of intricacy with Channel $|j\rangle$ with no variation in any quantum probability.

All the probabilities p_k for entanglement of Atom b with various channels $|k\rangle$, with $k \neq j$, participate in the misstep. The probability for Atom b to be entangled with channel $|j\rangle$ switches from p_j to 1 under the (*a, b*)-collision and it increases therefore the associated probability for this entanglement by a contribution 1 - p_j. Simultaneously, the initial entanglement of Atom b with every channel $|k\rangle$ ($k = j$ or $k \neq j$) switches to entanglement with that channel $|j\rangle$ or remain in it. As for the probability for intricacy with a channel k with $k \neq j$, it switches from p_k to 0 and undergoes therefore a variation - p_k.

A last step is concerned with the effect of all these missteps on quantum probabilities, when a first atom a is intricate with a channel $|j\rangle$. One can specify this effect by looking at some Atom b and counting the average number of atoms with which it can undergo collision

during the time interval δt: The distance along which this atom b can travel freely before collision is a random variable y with a probability distribution $\exp(- y/l)dy$. The number of atoms with which this Atom b can undergo its first collision is thus given by

$$N_a = \int n_a \exp(-|x|/\lambda) d^3x = 8\pi n_a \lambda^3. \quad (4.6)$$

Conversely, the probability for each one of these atoms to be the one suffering the collision with b is $1/N_a$.

One disposes then of all the data for computing the standard deviations for the fluctuations δp_j during the time δt, with their correlation coefficients. Some of these fluctuations belong to the positive matrix ρ_{B+} and as many to $- \rho_{B-}$. A straightforward calculation yields then for overall effects:

$$<\delta p_j> = 0, \quad (4.7)$$

$$<(\delta p_j)^2> = W p_j (1 - p_j) C_j \delta t/\tau, \quad (4.8a)$$

$$<\delta p_j \delta p_k> = - W p_j p_k (C_j + C_j) \delta t/\tau, \quad \text{for } j \neq k. \quad (4.8b)$$

The time parameter τ is again the mean free time, spent by an atom between two successive collisions. The coefficients C_j are given by

$$C_j = \int n_a f_j(x) f_0(x) dx / N_a . \quad (4.9)$$

Equations (4.7-9) provide the main result of the present work. The part that is played by intricacy in the process appears in them through the local probabilities $f_j(x)$ and $f_0(x)$ measuring local intricacy and non-intricacy. Incoherence, which originates in fluctuations in the action of environment, is represented by the probability W for an atomic state to be incoherent. The factor $1/N_a$ represents the probability for a specific atom to be the one that comes to interact with a specific atom of Type b, among all those that could do so.

When the measuring system involves several separate detectors, separate domains of integrations occur for the positions x, with local values for the parameters n_a, N_a.

The Brownian mechanism of collapse

The nature of collapse becomes clear when this final effect is conceived as an accumulation of many missteps in coherence: The randomness of fluctuations δp_j, together with the linear behavior in δt of the correlations in Equations (4.7-9), are characteristic of a Brownian processes. This result is not surprising and was anticipated years ago by Philip Pearle as a possibility [11]. The main surprise –if there is one– is that this Brownian behavior results here from quantum mechanics, and not its violation [35].

Pearle's theorem, which was mentioned as Point 9 in Section 1, requires more conditions for predicting collapse: The random motion of various channel probabilities $\{p_1, p_2, \ldots\}$ must allow every one of them to have a possibility of vanishing during the process, and have then no chance for getting back thereafter to nonzero values. These conditions are satisfied by Equations (4.) for the correlation coefficients, because they imply that a "probability current", defined by

$$J_j = \partial{<}(\delta p_j)^2{>}/\partial p_j \qquad (4.10)$$

does not vanish when p_j vanishes. This current remains positive moreover on the boundary of the domain of in probability space where p_j vanishes, and this is the condition for no return, these boundary properties insuring a one-way disappearance of a channel when its quantum probability happened to vanish.

Pearle's theorem asserts finally that the ultimate outcome of the whole process is necessarily that one of the quantities p_j will come out equal to 1 at the end of the process. All the probabilities for other channels will have then disappeared when their associated quantum probabilities vanished. Moreover, because of non-vanishing of the current (4.10) on the boundary, the moving point with coordinates $\{p_j(t)\}$ in probability space encounters the boundary after a finite time and the time scale of collapse is also finite.

A last though essential prediction of Pearle's theorem is that the "Brownian" probability for some channel j to be the last one reaching probability 1 is given by

$$\text{Probability for } p_j(t) \to 1) = p_j(0) = \left|c_j\right|^2, \qquad (4.11)$$

which is Born's probability rule.

The main conclusion of the present work is therefore this proposal for "wave function collapse", with an explicit mechanism and a derivation of the Born probability rule. These results are considered direct consequences of the Schrödinger equation in its description of quantum evolution, extended to the history of this evolution and local entanglement. One recalls also that the range of quantum observables cannot share this extension, so that no observation can ever be expected to check this mechanism: even if explained, collapse is expected to remain beyond actual watching.

Some more comments are added also in Section 7.

PART II: COMPLEMENTS AND CONCLUSIONS

5. Complements on Intricacy

Lieb and Robinson discovered local properties of entanglement in the framework of spin lattices [16]. They evolve with a finite "Lieb-Robinson velocity", which has been experimentally observed [42] and there are many publications on this subject. Their possible relation with collapse, however, was only mentioned recently [17].

A minor question arises regarding the name that one uses for these properties in the present work: will : Lieb and Robinson called them "local entanglement", as one did here also when introducing them. There is however an inconvenience in that name, when used in measurement theory: Local entanglement appears there often opposite to standard (algebraic) entanglement, when one says that local entanglement implies entanglement whereas the converse is not true. The two notions are not equivalent in that framework and one will therefore keep using here the word "intricacy" rather than "local entanglement".

Intricacy, conceived as influence

Intricacy is an essential element of the problem of collapse, in the present approach. It does so as mainly a source of a new kind of probabilities and as a physical effect, though

which incoherence is generated by fluctuations in action of environment. One introduced intricacy in Section 2, but a more detailed description is necessary for completeness and is the main topic of the present section.

As done in Section 2, one will again introduce intricacy on an example where an energetic alpha particle A crosses an argon gas along some track in a Geiger counter B. Local effects of the charged particle can be then described by quantum theory, according to the Bethe-Heitler theory for slowing down of charged particles. One thus gets various quantum probabilities for excitation or ionization of atoms, which are observable effects of the incoming particle A. One can also add these probabilities to get a local quantity $F_1(x)$ measuring the probability for an atom, near a point x in the system B, to have been affected by action of the particle A through a change in its state. This probability $F_1(x)$ is larger near a track than away from it.

Quantities like $F_1(x)$, which measure the *action* of the detected particle on the medium inside the system, are familiar in the literature on measuring devices. But one is mainly interested here in another quantity, denoted by $f_1(x, t)$, which is supposed to express the *influence* of the particle A on the atoms in B, near a point x in the medium and at a time t. A simple example of such an influence is given by Van der Waals scattering of the charged particle on atoms near its track. The rapidly varying electric field, which is produced at the place of an atom of by the charged particle passing by can produce a small change in the momentum of this atom, and this is another influence, which marks the presence and the action of the incoming charged particle, weaker than its previously considered actions on inner states but nevertheless significant as an influence.

One will therefore consider more generally that an influence of the charged particle on an atom extends to any change that can occur in the state of that atom, not only in its inner state but also its state of motion, so that an atomic state can be said to have come under the influence of the particle A if it differs from what it would have been if this particle had not arrived. This notion, as one pointed out in Section 2, does not refer directly to the state of the composite AB system at time t but to its *history* since the time when A began to interact with B. In accordance with that interpretation, one will not consider only direct influences of the particle A on atoms through a direct interaction, but also every secondary influence resulting from a later action of directly influenced atoms on new ones, and so on for an influence of already influenced atoms on not yet influenced ones.

Dynamics of intricacy

A simple model can help making this idea of influence clearer: The macroscopic system B is supposed to consist of a gas of identical atoms, interacting together through a two-body potential V (which one will also sometimes denote by $V_{aa'}$ or $V(x_a , x_{a'})$ when dealing with specific atoms a and a'). In place of a charged particle A acting through its electric field, one will use a model in which every atom a can interact with the particle A through a two-body potential, denoted either by U, U_{Aa} or $U(X_A , x_a)$ when some atom a is specified or when one deals with the positions of Particle A and of that atom a.

With these conventions, intricacy can be associated with an index 1, which indicates that some state of some atom under consideration received an earlier influence from A, either directly or indirectly. Another index 0 marks on the contrary that no such influence has (yet) occurred.

According to quantum dynamics, a wave function ψ of the AB system evolves under the Schrödinger equation

$$i\hbar\partial\psi / \partial t = H\psi \, . \qquad (5.1)$$

A first construction is then to use this equation for an account of the influence of Particle A on the set of all atoms. A simple procedure for that purpose consists in considering that every atom carries an index of intricacy r, which can take either the value 1 or 0. One may notice for illustration that this procedure looks much like one that is sometimes used in lectures on quantum mechanics, when a spin 1/2 is introduced for atoms when spin had not yet be mentioned in earlier lectures. A procedure for introducing spin in the formalism consists then in introducing new indices together with associated matrices. This is essentially what one is doing here now for introducing the idea of influence/intricacy.

One must use then three 2×2 matrices acting on intricacy indices, namely:

$$P_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, S = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \tag{5.2}$$

When thinking of an analogy with spin 1/2, one can say that P_1 is a projection matrix, which is associated with intricacy and with the index 1, whereas P_0 is associated with non-intricacy and index 0. These two matrices look then much like the ones projecting on spin-1/2 states on states with spin up ($s_z = +1/2$) or states with spin down ($s_z = -1/2$). The matrix S in (5.2) picks up in the present case the intricacy index 0 and brings it to 1 (like the spin-1/2 matrix $s_{+=}$ ($s_x + is_y$)/2 does when it acts on spin indices).

One will not need other matrices and especially not the matrix $S^\dagger$, which is the adjoint of S. If it were used, its action would erase an intricacy index 1 and replace it by the non-intricacy index 0, and the dismissal of $S^\dagger$ is characteristic of the ideas of influence or intricacy: Influence can either begin under a direct interaction of an atom with the particle A or grow through a chain of transmission under successive interactions of atoms. It is thus seen as a quality that an atom can acquire and which can be transmitted to other atoms, but which can never be cancelled. When intricacy is understood in that way, its growth is irreversibly transmitted from atom to atom, much like through a process of a contagion. One might also say at a deeper level that this approach, which relies explicitly on the history of the system, makes clear that intricacy is a form of cluster property [20].

One can express formally this idea by replacing the potential U_{Aa}, for interaction of the particle A with an atom a, by the 2×2 matrix

$$\boldsymbol{U}_{Aa} = U_{Aa} (S_a + P_{1a}), \tag{5.3}$$

which brings non-intricacy to intricacy and conserves it when it had been previously acquired. Similarly, the two-body potential $V_{aa'}$ for interaction between two atoms a and a' can be replaced by a 4 ×4 intricacy matrix:

$$\boldsymbol{V}_{aa'} = V_{aa'} (P_{0a} \otimes P_{0a'} + P_{1a} \otimes P_{1a'} + P_{1a} \otimes S_{a'} + S_a \otimes P_{1a'}). \tag{5.4}$$

The first term in the parenthesis is concerned with two initially non-intricate atoms and keeps them non-intricate. The second term conserves similarly previously acquired intricacies in the two atoms. The last two terms represent contagion, which happens systematically when one intricate atom interacts with a non-intricate one and brings it to intricacy.

When these operations are introduced for all the two-body interactions in the composite system AB, the Hamiltonian H in the Schrödinger equation (5.1) becomes an operator H' involving now intricacy, in a way much similar to what happens when the quantum dynamics of spinless atoms is extended to spin-1/2 atoms. Kinetic energy terms -

$\nabla_a^2/2m$, which represent the free motion of atoms, do not change intricacy and can be therefore considered as carrying simply a 2×2 unit matrix I_a.

This construction can be then extended to wave functions and yields then a dynamics of intricacy. Like in the analogy with spin, one does not use in that case any more a unique wave function ψ, but a set $\{\psi_s\}$ of many wave functions ψ_s, indexed by a string s of N intricacy indices $(i_1, i_2, \ldots, i_N)$ taking either the values 0 or 1 (the last index N denoting then the number of atoms in the system). If one denotes more briefly this set $\{\psi_s\}$ by ψ', its evolution is given formally by an equation $N\times N$ matrix equation similar to Schrödinger's equation and written as

$$i\hbar\partial\psi'/\partial t = H'\psi', \tag{5.5}$$

where all these changes have been performed.

Equation (5.5) is very similar to an ordinary Schrödinger equation, except that the operator H' is now a $2^N\times 2^N$ matrix with differential operators as elements. A very significant difference is however that the evolution operator H' *is not self-adjoint*. The absence of matrices $S_a^\dagger$, which would be adjoints of the matrices S_a in (5.4) and could bring back intricacy to non-intricacy, is responsible for this essential feature of H'.

Equation (5.5) makes sense nevertheless and it has solutions, like the basic Schrödinger equation (5.1). One can say in that sense that (5.5) draws only new consequences of (5.1) by exhibiting how the Schrödinger equation can be used, after only some rewriting, to describe directly intricacy in an explicit way.

The relation between the two evolution equations (5.1) and (5.5) goes much farther and the standard Schrödinger equation (5.1) can also be derived from Equation (5.5) giving the evolution of intricacy. This inverted derivation can be obtained by introducing the sum

$$\psi'' = \Sigma_s \psi_s. \tag{5.6}$$

One can easily show, from the explicit expressions (5.3-4) for interactions, that this function ψ'' satisfies the basic Schrödinger equation (5.1). When one considers furthermore initial conditions, the equality $\psi'' = \psi$ is obviously valid at time 0, before occurrence of any interaction between the systems A and B and when all atoms were still non-intricate. The wave function ψ_{s0} was then the only one in the sum (5.6) (if the index s_0 denotes the string of intricacy indices (0000…0)). One had thus $\psi_{s0}(0) = \psi''(0) = \psi(0)$ at time $t = 0$ and, since the two functions $\psi(t)$ and $\psi''(t)$ obey the same Schrödinger equation (5.1), the equality $\psi(t) = \psi''(t)$ is valid at all later times. Equation (5.5), which describes the evolution of intricacy, is therefore exactly equivalent to the standard Schrödinger equation (5.1).

The theory of intricacy appears thus simply as a corollary of quantum dynamics, which it does not modify in any way but to which it affords an opportunity for extending its interpretation.

Note: One may add as a further comment, useful in applications, how intricacy describes a collision between the particle A and an atom a (this description would be the same for a collision between two atoms). The contagion effect can be then analyzed by means of the Lipmann-Schwinger equation for two-particle scattering [36], which yields a simple result: The collision involves only a switch in the potential $U_{Aa} \rightarrow \boldsymbol{U}_{Aa}$ as in (5.3) and brings out simply a replacement $T_{Aa} \rightarrow \boldsymbol{T}_{Aa} = T_{Aa}(S_a + P_{1a})$ for the T-matrix of collision theory (which is related to the S-matrix by $S = I + iT$). One uses here frequently this result for

macroscopic systems in which the dynamics is dominated by two-particle collisions ("multiple scattering").

Symmetry properties of intricate wave functions

Some symmetry properties in the components ψ_s of the wave function ψ result from the Bose-Einstein or the Fermi-Dirac symmetry between atoms. One will consider only the Bose-Einstein case. A simple consideration of Equation (5.5) shows then that every function ψ_s is symmetric under a permutation of two atoms (a, a') carrying both the index 1 in the string s, or under a permutation of two atoms carrying the same index 0. The symmetry between indices a and a' in the last two terms of the interaction (5.4) implies moreover that the final state of a collision bringing one of two colliding atoms to intricacy is symmetric between the atoms: This behavior is practically the same as in the argument by which Dirac [19] showed impossibility of deciding which is which when two atoms have collided.

Irreversibility of intricacy

A basic feature of intricacy is its irreversible character. The example of a charged particle showed that the number of intricate atoms in a detector must grow with time, so that Equation (5.5) implies also that all the atoms in B become intricate after a finite time, total entanglement being then reached once and for all.

This evolution is *irreversible* and its behavior is implied, from a mathematical standpoint, by the non-selfadjoint character of the evolution operator H' in (5.5). One noticed already this point when mentioning that the first term in Equation (5.2), together with the last two terms (5.3) in H', involved the non-selfadjoint 2×2 matrix S creating intricacy, with no compensating terms which would have reestablished self-adjointness in H'. The evolution of intricacy under Equation (5.5) is therefore absolutely irreversible, in view of the general relation between self-adjointness of the Hamiltonian and time reversibility of the Schrödinger equation [20].

As already mentioned in Section 2, this property of irreversibility for intricacy goes along with its association with the history of the two interacting systems and not with their instantaneous wave functions.

Field formalism for intricacy

A quantum field version of intricacy can also be shown. Its existence allows one to expect a wide range of validity for this notion, although the corresponding domain has not yet been thoroughly circumscribed [17].

Intricacy can thus be extended to quantum field theory. One will only sketch here the example of non-relativistic undistinguishable atoms obeying Bose-Einstein symmetry. Before introducing intricacy, a convenient description consists in using quantum fields, $\phi^\dagger(x)$ and $\phi(x)$ for creation and annihilation and a quantum state $|\psi\rangle$ of the gas in the counter B in the form [43]

$$|\psi\rangle = \int dx_1\, dx_2 \ldots dx_N\, \psi(x_1, x_2, \ldots x_N)\ \phi^\dagger(x_1)\ \phi^\dagger(x_2) \ldots \phi^\dagger(x_N)\ |0\rangle . \qquad (5.7)$$

The function ψ in the right-hand side denotes here the standard non-relativistic wave function of the system, a variable x_j denoting the position of an atom. The reference state $|0\rangle\rangle$ is the

vacuum state of quantum field theory, which satisfies the properties $\phi(x)\left|0\right\rangle = 0$. the local fields ϕ and $\phi^\dagger$ satisfy the commutation relations

$$[\phi(x), \phi^\dagger(x')] = \delta(x - x'). \tag{5.8}$$

One can also introduce intricate quantum fields $\phi_r(x)$ with intricacy indices $r = 0$ or 1. They satisfy the commutation relations

$$[\phi_r(x), \phi_{r'}(x')] = 0,$$

$$[\phi_r(x), \phi_{r'}^\dagger(x')] = \delta_{rr'} P_r\, \delta(x - x'), \tag{5.9}$$

where P_r is one of the matrices P in (5.2). Another field $\alpha(x)$ is then used to describe the incoming A-particle.

One can write down the previous evolution operator H' showing the propagation of intricacy as a field operator

$$H = H_{A0} + H_{B0} + V_{AB} + V_B. \tag{5.10}$$

The term H_{A0} represents there the kinetic energy of the A-particle, H_{B0} the kinetic energy of atoms, whether intricate or not. The term V_{AB} describes the interaction of the particle A with some atom so that the outgoing state of this atom comes out intricate whether it was initially intricate or not. The last term V_B represents two-body interactions between atoms with account of the contagious behavior of intricacy.

In the simple case when all the particles and atoms are non-relativistic with no spin, these operators are given according to previous notations by

$$H_{A0} = \int dy\, \alpha'^\dagger(y)(-\nabla^2/(2m_A))\alpha(y), \tag{5.11a}$$

$$H_{B0} = \int dx\, \{\phi_1^\dagger(x)(-\nabla^2/(2m_a))\, \phi_1(x) + \phi_0^\dagger(x)(-\nabla^2/(2m_a)\, \phi_0(x)\}, \tag{5.11b}$$

$$V_{AB} = \int dxdy\, \alpha'^\dagger(y)\, \phi_1^\dagger(x) U(x, y)(\phi_1(x) + \phi_0(x))\, \alpha(y), \tag{5.11c}$$

$$V_B = (1/2)\int dxdx'\, \{\phi_0^\dagger(x)\, \phi_0^\dagger(x') V(x', x)\, \phi_0(x)\, \phi_0(x') + \phi_1^\dagger(x)\, \phi_1^\dagger(x') V(x', x)\, \phi_1(x)\, \phi_1(x')$$

$$+ \phi_1^\dagger(x)\, \phi_1^\dagger(x') V(x', x)\, \phi_1(x)\, \phi_0(x') + \phi_1^\dagger(x)\, \phi_1^\dagger(x') V(x', x) \phi_0(x)\, \phi_1(x')\}. \tag{5.11d}$$

Equation (5.10) can be easily extended to many type of particles, whether relativistic or non-relativistic, and also to many types of interaction. Although one does not consider this approach as universal, one may deem it nonetheless to be representative for many kinds of interactions involving a microscopic system and a macroscopic one. One will suppose anyway that this is sufficient for expecting a wide range of validity for the notion of intricacy, at juncture between quantum physics and macroscopic behavior.

One must also mention to avoid misgivings that the fields $\phi_r(x)$ and $\phi_r^\dagger(x)$ are not operators in a definite Hilbert space (so that $\phi_r(x) + \phi_r^\dagger(x)$ is not actually an observable). A rigorous mathematical formulation would probably make them act as operators in a sheaf of

Hilbert spaces, in which every Hilbert space would represent quantum states of atoms with definite intricacy. But this extension towards the theory of sheaves has not yet been attempted carefully, so that the present approach remains still at a rough mathematical level.

Probabilities of intricacy

One introduced earlier two local probabilities, $F_1(x, t)$ and $F_0(x, t)$, for direct influence of the incoming particle A on the inner states of atoms. One wants now to define more properly local probabilities, $f_1(x, t)$ and $f_0(x, t)$ describing respectively some measures for intricacy and non-intricacy, at some time t in a neighborhood of some point x in the gas.

The field formalism provides easily this notion. One writes down for that purpose the initial state $|\psi\rangle$ of the macroscopic system before interaction as one did in Equation (5.7), with non-intricate fields $\phi_0^\dagger(x)$ replacing now standard fields $\phi^\dagger(x)$. One lets then the operator H' in (5.10-11) act on that state and generate an intricacy representation for a state $|\psi_{AB}(t)\rangle$ of the composite A-B system when its two components (A, B) interact. Because a macroscopic system of actual interest is never in a pure state, one will use more generally an intricate density matrix $\rho'_{AB}(t)$, which evolves according to

$$i\partial\rho'_{AB}/\partial t = [H', \rho'_{AB}]. \tag{5.12}$$

One introduces also two operators for the total numbers of intricate and of non-intricate atoms, as

$$N_r = \int dx\ \phi_r^\dagger(x)\ \phi_r(x), \tag{5.13}$$

with $r = 1$ or 0. Eventually, one also uses local operators for the number of intricate or non-intricate atoms $N_{r\beta}$ in some space cell β in the gas, as the integral (5.13) with space points x belonging to that cell. Local probabilities for intricacy and non-intricacy, $f_1(x)$ and $f_0(x)$, where x now denotes the center of a small cell β, are then defined as the ratios

$$f_r(x) = Tr(N_{r\beta}\ \rho'_{AB})/N_\beta\,, \tag{5.14}$$

where N_β is the average number of atoms in the cell β.

Finally, one mentions again that Equation (2.4) describing contagion is still far from a rigorous proof. One will not elaborate however on that point, which is partly due to the fact that the quantum statistical theory of irreversible processes stands still itself on partially incomplete foundations [44]. Equation (2.4) is nevertheless used in the present work with no more argument than the satisfactory though often approximate success of similar considerations in many domains, for at least practical purposes.

6. A few complements on incoherence

One pointed out in Section 3 a special form of incoherence, which results in a macroscopic system from fluctuations in the action of environment. This effect was then attributed to the impossibility of considering an environment as a well-defined quantum system (except when it is a subsystem of a wider well-defined system).

One will add here only a few remarks regarding a relation between incoherence and a dominance of multiple scattering, which has also some relation with cluster properties. The model of an atmospheric molecule *M* colliding with a box in which an atomic gas *B* is enclosed will still be used.

The model of multiple scattering can be used when two-particle scattering events dominate the dynamics of a system. It is almost always an approximation. An example is shown in Figure 2, where the collision of the molecule *M* is shown occurring on one atom at some time 0 and later interactions between the atoms are described by Feynman graphs, the figure showing one of these graphs.

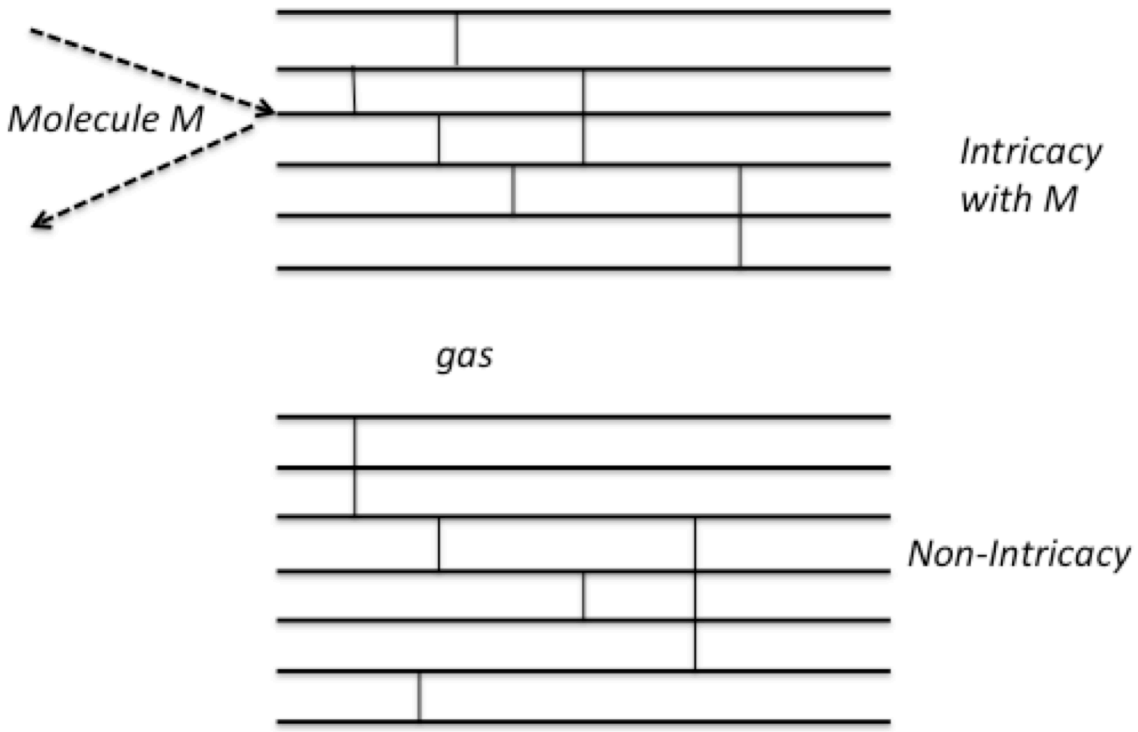

Figure 2: A Feynman graph for intricacy with dominant scattering

One saw earlier how the molecule collides in reality with the solid envelope of the gas, and also how intricacy is initially transported by phonons, but this aspect is left aside here. Scattering dominance is expressed in the graph by vertical lines relating two propagators for atoms (shown by horizontal lines). The approximation consists in considering that these vertical lines do not represent matrix elements of the potential V between atoms, but matrix elements of the two-body scattering matrix *T.* The graph itself is supposed to contribute to the evolution of the system during a time interval [0, t].

The main consequence of this approximation is a separation of atoms between a subset S_i of them, which became intricate with M, and a non-intricate set S_{ni}. These sets are of course different for different graphs, but one may consider the sum of all possible graphs as an approximation (under scattering dominance) of the growth of intricacy between the molecule M and the gas B.

The most relevant feature of such a graph is concerned with the phase of the incoming wave function of the molecule, which is shared by the set S_i. This phase cannot ne present in the states of atoms in the set S_{ni} and this distinction in the behavior of phases is specific to multiple scattering. It is only approximate when multiple scattering is only dominant, but this approximation is enough in the cases that were considered in Section 4 on collapse. In the present graph, this difference has been exhibited by drawing systematically the propagation lines for intricate atoms on top of the graph and for non-intricate atoms in the part below.

One may recall in this regard the standard meaning of phases in the conventional interpretation of quantum theory. The primary datum regarding the state of M, is then its meaning as a *ray,* with no definite phase. An arbitrary choice of a phase, say φ_M, allows then both a meaning and a use for a definite representative state vector $|M\rangle$. A selection of a pure state vector $|B\rangle$ for the system B requires also the choice an arbitrary phase φ_B. These two systems M and B are initially separate and their associated phases are considered independent, until time 0 when they happen to interact. One considers then usually that they become altogether a composite system M-B with associated phase $\varphi_{MB} = \varphi_M + \varphi_B$ [21]. The initial phase φ_B and the final one φ_{MB} are then global (affecting the whole wave functions) and also arbitrary, but not random: The difference is that an arbitrary phase has a fixed value once and for all, for instance φ_M, whereas a random phase can take any value in the interval $[0, 2\pi]$. The difference is that one must integrate relevant matrix elements over this interval when one deals with probabilities involving random phases.

When one considers in that sense a matrix element for an operator between two states of atoms, one with propagator in the upper half of Figure 2 and the other one in the lower half, this matrix element vanishes under integration on φ_M. This is the usual expression of incoherence and it means that atomic states in the two halves of the graph are incoherent. One may also express this behavior by saying that, in the approximation of dominance of multiple scattering, the properties of intricacy and of non-intricacy are mutually incoherent.

The transition from pure to mixed states, which brings out local properties for intricacy $f_1(x, t)$ and non-intricacy $f_0(x, t)$, allows then also to consider these local

probabilities as respective measures for incoherence and coherence, regarding random phases in the environment.

This comment applies of course only to primarily incoherent molecules with states belonging to positive or negative fluctuations in the action of environment. It affords a meaning to the random phases that were used in Section 3 and the discussion of missteps in coherence in Section 4. It also shows that the proposed theory of collapse is presently restricted to cases where multiple scattering can be held dominant, at least in some parts of a measuring device. It looks enough however that, later on, *one* external system relaying the measurement satisfies the condition of dominance of multiple scattering for collapse to happen finally. Anyway, the proposed theory is still in infancy and its possible variants are so many that one will not push forward farther here these questions.

Light as environment

One may also mention as a last comment what happens with the part that light plays in the environment. This point may be worth mentioning because the case of incoherent light is the most familiar regarding coherence versus incoherence and it turns out to be exceptional when incoherence from environment is concerned.

There is generally no significant intricacy when light is concerned. When rather than considering molecules, hitting a solid boundary, one considers photons, which *cross* for instance an atomic gas, the wave functions of photons behave in a strongly coherent way. It is a well-known fact that the scattered waves on various atoms interfere constructively to regenerate together a collective wave, which moves at a velocity smaller than c because of time delay in scattering events [36]. The fact that the basic velocity c of basic photon waves does not depend on their frequency is essential in this result.

7. Complements on collapse

The description of a collapse process in Section 4 was general though also sketchy. It left open many questions but one intends to mention only two of them now, which deal with estimates of orders of magnitude. A few rough considerations on the general case will be also mentioned to point out some technical difficulties and also apparently wide opportunities.

A simple case

One considers an academic case in which the system A to be measured is again an energetic charged particle and the measuring system B is the Geiger counter that was kept used as paradigm. The initial state of A consists simply in a superposition of two tracks with associated states, the first one $|1\rangle$ crossing the detector and the second one $|2\rangle$ not crossing it.

One is mainly interested in putting some numbers into Equations (4.4-6) expressing the collapse effect, to get an estimate for a time scale T of collapse, given by

$$T \approx \tau / WC_1 \qquad \text{with } C_1 = \int n_a f_1(x) f_0(x) dx / N_\beta \ , \tag{7.1}$$

with previous notations.

The detection of Particle A goes through several steps beginning with a growth of ionization in the detector, ending with the production of a spark. The spark occurs only when

the size of the region containing free electrons becomes comparable with the dimension L of the apparatus. The associated intricacy wave coincides then essentially with the whole region of ionization, which is a cylinder with axis on the track. The radius of this cylinder is then of order O(L) and the width of the wave front is of order O(λ), the mean free path of atoms. One thus gets estimates

$$N_\beta = n_a \lambda^3 \times \mathrm{O}(1),\ C_1 = n_a L^2 \lambda \times \mathrm{O}(1),\ T = (\tau/W).(L/\lambda)^2 \times \mathrm{O}(1) \qquad (7.2)$$

If the mean free time τ of atoms is for instance of order 10^{-10} s and the value of the probability of incoherence W is not extremely much below its upper bound $4/3\pi$, and with ratio L/λ significantly larger than 1, one may expect very small values for the time scale of collapse in this model.

The general case

One can say little however about orders of magnitude in general and, if one assumes the present approach to make sense, many questions arise.

An example is provided by Schrödinger's famous example of a decaying radioactive source. Nothing forbids a priori a use of the estimates (7.2) for the time of collapse in that case, but there is a problem: Why does one observe a unique track in a Geiger counter? In principle, one should consider the possibility of many tracks starting in every direction and one would describe them as so many competing channels. But there are many possible ones and their number could appreciably lengthen the time scale of collapse.

This is not only a quantitative matter but also a matter of principle and one must envision it as such. It seems difficult to guess the right answer would be, but a somewhat far-fetched possibility is worth mentioning.

This answer, if one may call it so, goes back to Von Neumann's chain of measuring devices measuring previous measuring devices [18]. He used it to show that a quantum superposition remains stable along such a chain, but the answer coming out here is quite different: There is on the contrary a contagion of intricacy in the growing set of measuring devices with the measured system, and collapse becomes more and more rapid when more of them become involved.

The main point is that, in the present theory, every physical property of interest must be expressed by collective observables (expressing a local amount of ionization for instance), evolves independently from the values that are taken at some time by the quantum probabilities $\{p_j\}$ of various channels, until all these channel probabilities vanish, except for one reaching the value 1. One might say in this regard that collapse is invisible till the end all the observables continue to evolve as if there were tendency towards collapse(except for the projection operators expressing not yet null probabilities, which are observables in a mathematical sense but not something observable by physical means).

To describe it differently, one might say that as long as some channel has not yet become closed through waning of its associated probability, all its inner physical processes continue to evolve and to act in it as if it had a chance to be the final winner of probability 1 together with the property of uniqueness and the status of reality.

Von Neumann's chain enters this game in a way similar to another proposal, which was promoted earlier by Zurek as Quantum Darwinism [45]: Every external system coming under influence of the measuring device B and able in some sense to "observe" it, adds its contribution to the parameters C_j in (4.6) for the rate of collapse, and thereby accelerates its process. The effect grows exponentially with the number of systems in "mutual agreement" during the collapse process.

Another proposal, aiming also at a derivation of a unique result in a quantum measurement, has been proposed by Allahverdyan, Balian and Nieuwenhuyzen to derive from quantum dynamics [46]. It has too much difference however with the present proposal for allowing a sufficiently careful discussion here.

8. Conclusions

One proposed here an explanation for "wave function collapse", the phenomenon under which a unique datum comes out randomly from the measurement of a quantum quantity by a macroscopic device. According to this proposal, collapse would be a quantum effect, resulting directly from the Schrödinger equation, perturbed by an environment.

This is of course a risky statement, especially because it was already discarded long ago [1], at least in the ideal case of no environment. To draw conclusions about it, showing why and how it could have a chance of holding a part of truth, one may begin by summarizing more explicitly the proposal and then point out strong and weak points in its present developments.

Summary of the proposal

The proposed arguments went as follow: When a composite system consists in a microscopic one, to be measured, and a macroscopic measuring one, the Schrödinger equation for its evolution has remarkable properties of local entanglement [16, 17] (which were named here "intricacy"). These properties were not envisioned for a long time as possible agents in collapse although they seem worth much attention in this regard.

These properties of intricacy, which are concerned with a growing degree of influence from the microscopic measured system on regions of the measuring one, have remarkable characters: They cannot be expressed by quantum observables nor can they be extracted mathematically from the wave function at a sharp time: They belong to the history of the wave function. As so, they are irreversible and their consideration exceeds the bounds of the standard interpretations of quantum mechanics [18, 19].

A second constitutive element in the present proposal consists in a supposed existence of a very special form of incoherence in the quantum state of a macroscopic quantum system. This incoherence would result from fluctuations in the action of the environment on that system.

The main practical result in the present work is an identification of a microscopic mechanism of "missteps in coherence", which is supposed the basic tool of collapse. It consists in an interaction between two atoms (or other microscopic constituents). It occurs under specific conditions, namely: (*i*) The state that is considered for one of the two atoms is locally entangled (intricate) with some channel of the measured system (some eigenvector of the measured observable). (*ii*) The second atom is non-intricate. (*iii*) The interaction between the two atoms is incoherent (from incoherence in environment). The associated misstep in coherence consists then in the existence of transitions between the quantum probabilities of different measurement channels, with no directly observable consequence.

Collapse appears then simply as an accumulation of a large number of missteps in coherence, through Brownian variations in the quantum probabilities of various measurement channels [11]. The result of the process is a final outcome in a unique channel, which occurs at random under govern of Born's rule for the associated probability. This rule is thus no more

an axiom of quantum theory, but a consequence of the other fundamental laws of quantum mechanics.

An evaluation of the proposal

One may try then to appreciate the value of this proposal. The various elements in its chain of statements and of conjectures have different status. Some of them can be considered as reasonably well established whereas other ones may look questionable. One can therefore briefly review these differences, to try appreciating the value of the theory, its degree of conjecture and the problems that would have to be solved for making it valuable.

I propose to consider the existence of local properties of entanglement ("intricacy") as reasonably well established, including their propagation with a finite velocity [16].

The description of this propagation by local probabilities (for intricacy or non-intricacy) stands on a lesser firm ground however. Its derivation, which relies on a use of theories of irreversible processes, would certainly need more thorough study.

The idea that some incoherence in the state of a system could arise from fluctuations in the action of its environment appears to be new. It came out naturally as a consequence of intricacy within the present framework. But it appears to have deep aspects, which are not yet sufficiently clear, and which could indicate for that reason weak links in the construction. The origin of this incoherence in a competition between positive and negative fluctuations (or in some sense from fluctuations inside fluctuations) seems to be specific to the problem at hand and needs certainly a careful critical study.

The prediction of collapse and of its properties on these grounds seems valuable on the contrary, at least under the proviso that the previous points could be granted.

An argument of elegance seems its main justification. There was something harmonious in the way under which the various elements in this construction tended to call each other to existence, and they seem finally to fit remarkably well together. This is not of course a scientific argument, but it might have some sense with regard to the beautiful framework of quantum theory. Anyway, as one should say, the present theory stands only as a conjecture, which is proposed for more investigation with the hope that something in it can remain or could be an occasion for finding better variants or wider openings.

Relations with problems of consistency

The present suggestions are only meant as conjectures, which a mathematical investigation could confirm, reject or modify. Although such an attempt would be probably difficult, it might open interesting perspectives on several questions regarding consistency in quantum mechanics, of which some aspects can be already noticed. One will mention them only from the standpoint of philosophical questioning.

1. Older questions regarding differences between the approaches by Heisenberg and by Schrödinger to quantum mechanics come back again in the present pattern, although they are now considered as having been brought to harmony [46]. The Schrödinger representation, based on wave functions, would remain fully valid in the presently proposed standpoint, and it would be further extended with its prediction of intricacy. The Heisenberg representation, in which physical quantities (observables) stand primary, would not extend that far and would not encompass collapse with its uniqueness of Reality. The philosophical consequences could be interesting, and perhaps also mathematical remedies (like a recourse to sheaf theory), but they cannot be sketched here.

2. The questions regarding a wave function of the universe would also be affected. One considered convenient this idea of a universal state in the premises of Section 1, as a way for avoiding difficulties with localized states and their mutual relations. This convention did not mean however agreement with Everett's conception of multiple non-communicating branches in the wave function of the universe.

The present approach implies on the contrary a unique wave function for a unique universe, with only local transitory effects (except maybe in some cosmological considerations about the early universe). This stability results mainly from the properties of incoherence, which were mentioned in sections 3 and 6 and enter in the phenomenon of collapse.

The argument goes as follows: At the time when a measurement occurs, the incoherence that is present in the state of the measuring device results from previous interactions of this device with its environment, which occurred during a finite lapse of time (equal to $\Delta t = L/c_s$ in previous notations). No part of the environment at a distance larger than $c\Delta t$ can therefore have any influence on collapse, which is therefore necessarily localized into a finite region. More distant regions can only be influenced by the unique outcome of a collapse event and their states cannot involve quantum uncertainties about that fact, which has become a classical property [22].

3. The question of self-consistency of quantum mechanics is too vast and deep for being discussed in a useful way here. One may nevertheless evoke some of its aspects with reference to the "Bohr-Einstein dialogue", which was made of quotations from these two creators at beginning of the book by Wheeler and Zurek [4]).

One will not try to deal in detail with Einstein's reservations about an intrinsic character of quantum randomness. Only three points may be mentioned in this regard. The first one is that the randomness of collapse is pushed back in the present approach to an indeterminate association of environment with the formal notion of quantum system.

The second point regards the uniqueness of macroscopic reality. The proposed origin of uniqueness in a collapse effect implies in this regard that a unique macroscopic reality can be considered as a consequence of a multitude of collapse effects along the history of the universe and in a multitude of places, every such elementary outcome being unique.

The deterministic behavior of macroscopic physics, which is the concern of the last point, was shown already elsewhere a consequence of quantum dynamics [22].

To conclude on this question regarding the status of randomness, one will propose that randomness (as a reservoir of opportunities) can be attributed an ontological status, whereas macroscopic reality, with its uniqueness and causality, would emerge from the history of the universe, with its own wider laws. This position could of course be a matter of much discussion and elaboration, but this is not the place for developing it further.

Another comment regards Bohr's standpoint. It starts from a conceptual consequence of the proposed mechanism of collapse. It relies on two points: (*i*) No mathematical observables can enter and play a part in this mechanism. (*ii*) Every actual property in quantum physics must be formally expressible by such observables (projection operators) [18]. The mechanism in Section 4 implies therefore that no conceivable experiment could allow an empirical access to this collapse process, namely to observe its unfolding, *as a matter of principle.*

This conclusion, if it came to be justified by a proof, would be much in favor of Bohr's wise and cautious attitude regarding the nature of collapse (a word that he seems to have never used, but a deep thought [4]). On another side, this idea that the root of everything

accessible to actual knowledge would be, by its own nature, inaccessible to perception has something fascinating for the philosophy of knowledge, close in spirit to d'Espagnat's idea of a "veiled reality" [13].

Acknowledgements

I thank much Philippe Blanchard, Robert Dautray, Jürg Frölich and Phillip Pearle for their encouragement or their help during this work, with special gratitude to the memory of Bernard d'Espagnat. Franck Laloë's help in clarifying these ideas was especially precious. I also benefitted from lucid criticisms, particularly from Stephen Adler, Heinz-Dieter Zeh and again Phillip Pearle, about earlier proposals of mine on the origin of incoherence.

Appendix: A bound on the probability of incoherence

This appendix is concerned with the probability W for incoherence. One will rely and its average density matrix $<\rho_B>$ has the simple form (3.2). The results in Section 3 show that a high disorder results from the action of environment (at least when the product $n_e v_e$ in (3.6) is not very small). The eigenvectors of ρ_B at a sharp time are then sums of many incoherent terms with correlation ranges of order a mean free path. One will say that this situation is one of "high disorder", which makes the density matrix ρ_B random, "for all practical purposes". Another feature making this randomness much stronger is the very high degeneracy of the eigenvectors of $<\rho_B>$, which makes them extremely sensitive to even very small perturbations.

One may assume however, in the simple model under consideration, that the energy distribution in (3.2) is not appreciably affected by fluctuations in the environment so that thermal equilibrium is not appreciably disturbed.

As a first step in the study of ρ_B, one uses this stability in the energy distribution by splitting r_B into a direct sum of terms ρ_{Bn}, associated with separate energy intervals $[E_n, E_n + \Delta E]$ with ΔE, not too large. One has then simply

$$\rho_B = \prod_n \rho_{Bn}. \qquad \text{(A.1)}$$

One will denote by N the dimension of this finite matrix. This number is very large in spite of the smallness of ΔE, because of a very high degeneracy in the energy spectrum.

One will denote also by $|\mu\rangle$ the eigenvectors of ρ_n, and by p_μ the corresponding eigenvalues. Similarly, one denotes by $<\rho_n>$ the matrix resulting in the same way from an average $<\rho_{Bn}>$ in $<\rho_B>$and by $|m\rangle$ its eigenvectors (which are also eigenvectors of the Hamiltonian H_B). All the eigenvalues of $<\rho_n>$ have then the same value $p = 1/N$.

High disorder implies a strong random behavior of ρ_n with two main consequences: (*i*) The orientation of the orthogonal basis of vectors $\{|\mu\rangle\}$ with respect to the basis $\{|m\rangle\}$is

random. (*ii*) The distribution of the positive eigenvalues $\{p_\mu\}$ is random with average value p (this value resulting from the unit trace of ρ_n).

One will also make a last assumption, which expresses a complete randomness in the distribution of eigenvalues p_μ, and resulting again from the extreme instability in the quantum states of $<\rho_n>$. This very strong assumption is the following assumption:

Assumption of maximal randomness: The probability distribution for the random values p' of the eigenvalues $\{p_\mu\}$ is given by

$$g(p') = (1/p)\exp(-p'/p). \qquad \text{(A.2)}$$

One will consider that this assumption is justified by a complete lack of determination in the individual eigenvectors $|\mu\rangle$, which implies a minimal value for the corresponding measure of information (algorithmic entropy)

$$-\int g(p') \log[g(p')]\, dp'. \qquad \text{(A.3)}$$

When writing the equation expressing a minimum for (A.3) and taking account of the average value p for the random variable p', one gets (A.2). One may notice also that this result implies the following value for the standard deviation $\Delta p'$:

$$(\Delta p')^2 = p'. \qquad \text{(A.4)}$$

One can then state the main consequence of these preliminaries. which is expressed by the following lemma

Lemma: When written as an $N\times N$ matrix in the basis $\{|m\rangle\}$ of eigenvectors of the Hamiltonian H_B, the matrix $\Omega = \rho - <\rho>$ is a normalized Wigner random matrix [47].

One recalls that a self-adjoint matrix Ω with dimension N is a (normalized) Wigner random matrix when all its matrix elements are independent complex random numbers $\omega_{mm'} = \langle m|\Omega|m'\rangle$, real for $m = m'$. If one denotes by Av the operation of taking average values on functions depending on these numbers, one has the relations

$$Av(\omega_{mm'}) = 0, \qquad Av(|\omega_{mm'}|^2) = 1/N, \qquad \text{(A.5)}$$

where the normalization $1/N$ in the second equation results form normalization. More general Wigner matrices correspond to cases where the right-hand side is a constant though not necessarily equal to $1/N$.

The supposed properties of randomness in the eigenvalues of ρ_n and in the orientation of its eigenvectors imply that the difference $\Delta\rho_n = \rho_n - <\rho_n>$ should be a Wigner random matrix. There is however a constraint, which is due the exactly zero trace with no fluctuation of $\Delta\rho_n$. This condition is not satisfied by a strictly defined Wigner matrix, whose trace fluctuates. The relevant corrections are however of order $1/N$ on every prediction that one will be using and is therefore negligible: One recalls that the number N of states under

consideration is an exponential in the number of atoms in the system and that the normalization $1/N$ results from the property $Tr(\Delta\rho_n^2) = (\Delta p)^2 = p = 1/N$.

If one writes down then the eigenvalues q' of $\Delta\rho_n$ in the form $q' = px$, Wigner's "semicircle theorem" [47] determines the probability distribution $h(x)$ for the values of x, with $-2 \leq x \leq 2$, which is given by

$$h(x)dx = (4 - x^2)^{1/2}\, dx/2\pi. \qquad \text{(A.6)}$$

This distribution implies an average value $4/3\pi$ for the positive values of x and the opposite for the negative values, with From there on, one can go back to from this result on ρ_{Bn} to the matrix ρ_B and conclude, after separating the positive and negative parts in the difference $\rho_B - <\rho_B> = \rho_{B+} - \rho_{B-}$, the simple result

$$W = Tr(\rho_{B+}) = Tr(\rho_{B-}) = 4/3\pi. \qquad \text{(A.7)}$$

The assumptions, which lead to this result, must be taken with caution however and this is why one considers them only as an extreme case and an upper bound.

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