Chaotic Inflationary Models in $f(R)$ Gravity

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Abstract

In this paper, we discuss inflationary scenario via scalar field and fluid cosmology for anisotropic homogeneous universe model in $f(R)$ gravity. We consider an equation of state which corresponds to quasi-de Sitter expansion and investigate the effect of anisotropy parameter for different values of deviation parameter. We evaluate potential models like linear, quadratic and quartic which correspond to chaotic inflation. We construct the observational parameters for power-law model of this gravity and discuss the graphical behavior of spectral index and tensor-scalar ratio which indicates consistency of these parameters with Planck 2015 data.

Keywords: Inflation; Slow-roll approximation; $f(R)$ gravity.
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1 Introduction

One of the crucial advancement on the landscape of modern cosmology is the detection of cosmic acceleration of the universe as well as mysteries behind its origin. The most conclusive evidence for the present accelerated epoch appears in the measurements of supernovae type Ia supported by some

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renowned observations such as cosmic microwave background (CMB), large scale structure and weak lensing. The existence of this epoch is due to some hidden source with surprising characteristics referred as dark energy (DE). Astrophysical observations resolve the enigma about the birth of the universe by introducing a standard model known as big-bang model [1]. According to this model, the universe was facing decelerated expansion characterized by matter or radiation dominated phase but this era has some long standing issues like horizon, monopole and flatness. These critical issues lead to an epoch of rapid acceleration named as "inflation". It is defined as an era of few Planck lengths which experiences a rapid exponential expansion due to some gravitational effects [2].

The idea of accelerated epoch was presented by Guth [3] and Sato [4] who proposed that rapid expansion appeared due to the existence of false vacuum. They also claimed that the universe was filled with bubbles at the end of inflation. This idea experienced some shortcomings like it corresponds to de Sitter expansion and the universe becomes inhomogeneous at the end of inflation. Such issues compelled to propose another version of inflation, referred as new inflation or chaotic inflation [5] in which a scalar field behaves like a source of accelerated expansion. The magnitude of this scalar field is assumed to be negatively large but the field starts rolling down slowly towards the origin of potential. At this stage, the potential approaches to its minimum position leading to the end of inflation which initiates the reheating phase [6]. An alternate approach to deal with inflationary scenario is the fluid cosmology. It is the simplest technique which is even supported by imperfect fluids that describe radiations and matter different from standard one [7].

The FRW model describes isotropic and homogeneous nature of the universe, it ignores all structure of the universe along with observed anisotropy in CMB temperature. Bianchi type cosmological models are the simplest anisotropic models to analyze anisotropy effect in the early universe on behalf of present day observations. Akarsu and Kilinc [8] investigated Bianchi type I (BI) model which describes de Sitter expansion via anisotropic equation of state (EoS) parameter. Sharif and Saleem [9] studied locally rotationally symmetric (LRS) BI model to analyze warm inflation through vector fields and found consistency of this anisotropic model with experimental data. The same authors [10] studied the effects of bulk viscous pressure in warm inflation and checked the consistency of cosmological parameters with recent WMAP7 and Planck results.

The accelerated expansion of the universe and its evidence motivate researchers to propose gravitational theories which can extend general relativity to deal with puzzling nature of DE. The $f(R)$ theory is one of such modifications where the Ricci scalar (R) is replaced by an arbitrary function $f(R)$. Mukhanov [11] analzed cosmic inflation with a deviating EoS parameter and formulated consistent range of observational parameters. Bamba et al. [12] investigated slow-roll and observational parameters of inflationary models through reconstruction methods in $f(R)$ gravity. They studied different $f(R)$ models and concluded that power-law model gives the best fit values compatible with BICEP2 and Planck observations. Myrzakulov and his collaborators [13] discussed scalar field, fluid cosmology and $f(R)$ gravity to reconstruct feasible inflationary models. Bamba and Odintsov [14] explored inflationary universe for a viscous fluid model and formulated observational parameters. The same authors [15] discussed inflationary scenario in $f(R)$ gravity, loop quantum cosmology as well as trace anomaly and concluded that for all these inflationary models, tensor-scalar ratio and spectral index of density perturbations are viable for Planck data. Artymowski and Lalak [16] studied modified Starobinsky inflationary model in Einstein as well as Jordan frames and found compatible results for both BICEP2 and Planck observations. Huang [17] analyzed polynomial inflationary $f(R)$ model to study tensor-scalar ratio as well as spectral index compatible with Planck constraints.

The chaotic inflationary model has many attractive features as it describes an inflationary epoch when large quantum fluctuations are present at the Planck time and discusses superheavy particle production, preheating as well as primordial gravitational waves [18]. Myrzakul et al. [19] investigated chaotic inflation for flat FRW model and studied massive as well as massless self-interacting scalar field in the background of higher derivative gravity theories. They found that inflation is viable for massive scalar field but it appears to be unrealistic for quartic potential. Gao et al. [20] explored fractional chaotic inflationary model in the context of supergravity and discussed observational quantities for various fractional exponents. The chaotic inflation has also been studied on brane to discuss chaotic inflation along with supergravity [21].

In this paper, we study inflationary power-law model of $f(R)$ gravity using scalar field and fluid cosmology for anisotropic homogeneous universe. The format of this paper is as follows. Section **2** deals with some basic features of inflationary dynamics and construct inflationary parameters. In sections **3** and **4**, we analyze these two approaches for different values of deviation parameter and discuss the effect of anisotropy parameter graphically. Finally, we conclude the results in the last section.

2 Some Basic Features of Inflation

We consider LRS BI universe model as

$$ds^2 = -\tilde{N}^2(t)dt^2 + a^2(t)dx^2 + b^2(t)(dy^2 + dz^2), \tag{1}$$

where $\tilde{N}$ represents lapse function and scale factor a determines expansion of the universe along x-direction whereas b measures the same expansion in y and z-directions. We consider a linear form, $a = b^m,\ m \neq 0, 1$ which is derived from the constant ratio of shear and expansion scalars [22]. Using this relationship, the above model reduces to the following form

$$ds^2 = -\tilde{N}^2(t)dt^2 + b^{2m}(t)dx^2 + b^2(t)(dy^2 + dz^2). \tag{2}$$

The action of $f(R)$ gravity is given by [23]

$$\mathcal{A} = \int d^4x\sqrt{-g}\left(\frac{f(R)}{2\kappa^2} + \mathcal{L}_m\right), \tag{3}$$

where f is an arbitrary function of R and $\mathcal{L}_m$ is the matter Lagrangian. For perfect fluid, the corresponding field equations become

$$\rho_{eff} = \frac{1}{2\kappa^2}\left(f - Rf_R + \frac{18(2m+1)}{(m+2)^2}H^2 f_R + 6H\dot{f}_R\right), \tag{4}$$

$$\begin{aligned} p_{eff} &= -\frac{1}{2\kappa^2}\left(f - Rf_R + (18H^2 + 12\dot{H})\frac{(2m+1)}{(m+2)^2}f_R + \frac{12(2m+1)}{(m+2)^2}H\dot{f}_R\right. \\ &+ \left. 2\ddot{f}_R\right), \end{aligned} \tag{5}$$

where $R = 12H^2 + 6\dot{H},\ H(t) = \left(\frac{m+2}{3}\right)\frac{\dot{b}}{b}$, $\rho_{eff},\ p_{eff}$ represent Hubble parameter, effective energy density and pressure, respectively. The time derivative of effective energy density leads to

$$\dot{\rho}_{eff} = \frac{1}{2\kappa^2}\left(\frac{36(2m+1)}{(m+2)^2}H\dot{H}f_R + \frac{432(2m+1)}{(m+2)^2}H^3\dot{H}f_{RR} - 288H^3\dot{H}f_{RR}\right), \tag{6}$$

The effective ingredients appear due to the presence of matter contents or scalar field. A linear relationship of these effective quantities leads to a significant parameter, i.e., EoS parameter ($\omega_{eff} = \frac{p_{eff}}{\rho_{eff}}$) which is used to characterize different phases of the universe. This divides DE phase in eras like quintessence for $-1 < \omega_{eff} \leq -1/3$ whereas $\omega_{eff} < -1$ and $\omega_{eff} = -1$ correspond to phantom era and cosmological constant (describes de Sitter expansion), respectively. The non-vanishing accelerated expansion of the universe is represented by these values of ω_{eff}. For vanishing rapid acceleration, there must be a small deviation such as $\omega_{eff} \simeq -1$ instead of $\omega_{eff} = -1$. This deviation leads to quasi-de Sitter expansion and yields a sufficient duration of inflation which elegantly enters into the deceleration phase when deviating EoS parameter approaches to the order of unity [11].

To observe quasi-de Sitter inflationary epoch, we consider EoS parameter of the following form

$$\omega_{eff} \simeq -1 + \frac{\nu}{(1+N)^{\mu}}, \quad \mu,\ \nu > 0, \tag{7}$$

where ν is of order unity and N denotes the e-folds until the end of inflation. The corresponding conservation law ($\dot{\rho}_{eff} + 3H\rho_{eff}(1+\omega_{eff}) = 0$) gives

$$-\frac{d\rho_{eff}}{dN} + \frac{3\rho_{eff}\nu}{(N+1)^{\mu}} = 0. \tag{8}$$

Here, $\frac{d}{dt} = -H(t)\frac{d}{dN}$ and hence we obtain the following solutions

$$\rho_{eff} \simeq \gamma(N+1)^{3\nu}, \quad \mu = 1, \tag{9}$$

$$\rho_{eff} \simeq \gamma \exp\left(\frac{-3\nu}{(\mu-1)(N+1)^{\mu-1}}\right), \quad \mu \neq 1, \tag{10}$$

where γ is the integration constant and for $N = 0$, $\rho_{eff} \simeq \gamma$ when $\mu = 1$ whereas $\rho_{eff} \simeq \gamma \exp[-3\nu/(\mu-1)]$ when $\mu \neq 1$ at the end of inflation. The Hubble flow parameters are given by

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\dot{\epsilon_1}}{H\epsilon_1}, \tag{11}$$

where $\dot{H}$ is negative and ϵ_1, ϵ_2 are positive quantities. During inflation, ϵ_1 and ϵ_2 must be very small such as $\epsilon_1 << 1$ and $\epsilon_2 << 1$. The inflating universe vanishes when $\epsilon_1,\ \epsilon_2$ take the value of unity [24]. To measure the extent of inflation, we have

$$\mathcal{N} \equiv N|_{t=t_i} = \left(\frac{m+2}{3}\right)\int_{t_i}^{t_f}\frac{\dot{b}(t)}{b(t)}dt, \tag{12}$$

where t_f and t_i represent cosmological time at ending and beginning of inflation, respectively. The approximate extent of inflation is found to be 70 but according to fluctuation spectrum of CMB, this limit of the e-folds becomes more smaller, i.e., $40 < \mathcal{N} < 60$. For anisotropic universe, the amplitude of scalar and tensor power spectrum ($\Delta^2_{\mathcal{R}},\ \Delta^2_{\mathcal{T}}$), scalar spectral index (n_s) and tensor-scalar ratio (r) are defined [25] as

$$\begin{aligned}\Delta^2_{\mathcal{R}} &= \frac{\kappa^2H^2}{8\pi^2\epsilon_1}, \quad \Delta^2_{\mathcal{T}} = \frac{2\kappa^2H^2}{\pi^2}, \quad n_s = 1 - \frac{d\ln\Delta^2_{\mathcal{R}}}{dN},\\ r &= \frac{\Delta^2_{\mathcal{T}}}{\Delta^2_{\mathcal{R}}}, \quad H = \frac{(m+2)}{3}\left(\frac{\dot{b}}{b}\right), \quad N = \mathcal{N}.\end{aligned} \tag{13}$$

For general power-law model $f(R) = f_0R^n,\ n \neq 0, 1$, where $f_0,\ n$ are positive constants [26], the field equations are reduced to

$$\begin{aligned}\rho_{eff} &= \frac{1}{2\kappa^2}\left(\frac{18(2m+1)}{(m+2)^2}H^2nf_0R^{n-1} + 6n(n-1)f_0HR^{n-2}\dot{R}\right.\\ &+ \left.(1-n)f_0R^n\right),\end{aligned} \tag{14}$$

$$\begin{aligned}p_{eff} &= -\frac{1}{2\kappa^2}\left((1-n)f_0R^n + (18H^2+12\dot{H})\frac{(2m+1)}{(m+2)^2}nf_0R^{n-1} + 2(n-1)\right.\\ &\times \left. n\ f_0R^{n-2}\{\frac{6(2m+1)}{(m+2)^2}H\dot{R} + \ddot{R}\} + 2n(n-1)(n-2)f_0R^{n-3}\dot{R}^2\right).\end{aligned} \tag{15}$$

The value of $H(t)$ and its derivative can be found using slow-roll approximation in Eqs.(6) and (14) as

$$H^2 = \left(\frac{4\kappa^2(m+2)^2\rho_{eff}}{12^nf_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{n}}, \tag{16}$$

$$\dot{H} = \left(\frac{2\kappa^2(m+2)^2\dot{\rho}_{eff}H}{12^nf_0n\{2(1-n)(m+2)^2+3n(2m+1)\}H^{2n}}\right). \tag{17}$$

Using these values in Eq.(11), we obtain

$$\epsilon_1 = \frac{3(1+\omega_{eff})}{2n}, \quad \epsilon_2 = -\frac{d}{dN}[\ln(1+\omega_{eff})]. \tag{18}$$

These parameters can be written in terms of e-folds as

$$\epsilon_1 \simeq \frac{3\nu}{2n(N+1)^{\mu}}, \quad \epsilon_2 \simeq \frac{\mu}{(N+1)}. \tag{19}$$

For $\mu < 1$, ϵ_1 is a dominant parameter whereas for $\mu > 1$, ϵ_2 dominates. When $\mu = 1$, both parameters play a key role to discuss inflation at the perturbational level.

The effective energy density fluctuations are measured by the amplitude of scalar power spectrum. The scalar power spectrum is given by

$$\Delta_{\mathcal{R}}^2 = \frac{\kappa^2(m+2)^2H^2}{4\pi^2\epsilon_1\{6(2m+1)f_R + 72(2m+1)H^2f_{RR} - 48(m+2)^2H^2f_{RR}\}}. \tag{20}$$

Using $f(R)$ power-law model and Eq.(7) with (9) and (10), we obtain scalar power spectrum and spectral index as

$$\begin{aligned}
\Delta_{\mathcal{R}}^2 &\simeq \gamma^{\frac{2}{n}-1}\left(\frac{4\kappa^2(m+2)^2}{12^nf_0\{2(1-n)(m+2)^2+3n(2m+1)\}H^{2n}}\right)^{\frac{2}{n}} \\
&\times \frac{(\mathcal{N}+1)^{1-3\nu+\frac{6\nu}{n}}}{4\pi^2\nu}, \quad \mu = 1, &(21)\\
\Delta_{\mathcal{R}}^2 &\simeq \gamma^{\frac{2}{n}-1}\left(\frac{4\kappa^2(m+2)^2}{12^nf_0\{2(1-n)(m+2)^2+3n(2m+1)\}H^{2n}}\right)^{\frac{2}{n}} \\
&\times \frac{(\mathcal{N}+1)^{\mu}}{4\pi^2\nu}\exp\left[\left(\frac{-3\nu}{(\mu-1)(\mathcal{N}+1)^{\mu-1}}\right)\left(\frac{2}{n}-1\right)\right], \\
\mu &\neq 1, &(22)\\
1-n_s &\simeq \frac{1-3\nu+\frac{6\nu}{n}}{\mathcal{N}+1}, \quad \mu = 1, &(23)\\
1-n_s &\simeq \frac{\mu(\mathcal{N}+1)^{\mu-1}+3\nu(\frac{2}{n}-1)}{(\mathcal{N}+1)^{\mu}}, \quad \mu \neq 1. &(24)
\end{aligned}$$

The tensor-scalar ratio for the EoS parameter (7) is

$$r \simeq 8\kappa^2\nu\gamma^{1-\frac{1}{n}}\left(\frac{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{n}} \tag{25}$$

$$\times\ (\mathcal{N}+1)^{-1+3\nu-\frac{3\nu}{n}}, \quad \mu=1, \tag{26}$$

$$r \simeq 8\kappa^2\nu\gamma^{1-\frac{1}{n}}(\mathcal{N}+1)^{-\mu}\left(\frac{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{n}}$$

$$\times\ \exp\left[\left(\frac{-3\nu}{(\mu-1)(\mathcal{N}+1)^{\mu-1}}\right)\left(1-\frac{1}{n}\right)\right], \quad \mu\neq 1. \tag{27}$$

We can investigate reconstruction of different models for $\mu=1,\ \mu\neq 1$. Since n_s is smaller than unity in both cases and $\epsilon_1,\ \epsilon_2$ are positive, thus an elegant exit from inflation is possible in this case. Moreover, recent observations form Planck 2015 [27] predict the values of spectral index and tensor-scalar ratio as $n_s=0.9666\pm 0.0062$ (68%CL) and $r<0.10$ (95%CL).

3 Inflationary Model for $\mu=1$

In this section, we reconstruct inflationary model corresponding to spectral index (23). The corresponding Hubble flow functions and EoS parameter take the form

$$\epsilon_1 \simeq \frac{3\nu}{2n(N+1)}, \quad \epsilon_2 \simeq \frac{1}{(N+1)}, \tag{28}$$

$$\omega_{eff} = -1+\nu\left(\frac{\gamma}{\rho_{eff}}\right)^{\frac{1}{3\nu}}. \tag{29}$$

At the ending phase of inflation, γ represents effective energy density for $\rho_{eff}=\gamma$. The tensor-scalar ratio turns out to be

$$r \simeq 8\kappa^2\nu\gamma^{1-\frac{1}{n}}\left(\frac{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{n}}$$

$$\times\ \left(\frac{1-3\nu+\frac{6\nu}{n}}{1-n_s}\right)^{-1+3\nu-\frac{3\nu}{n}}. \tag{30}$$

Now, we investigate viability of inflationary scenario in the context of scalar field and fluid cosmology.

3.1 Inflation via Scalar Field

Inflation can also be analyzed by introducing a minimally coupled scalar field (ϕ) subject to a potential $V(\phi)$. In this case, Lagrangian takes the form

$$\mathcal{L}_\phi = -\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi). \tag{31}$$

The sum of kinetic energy $\left(\frac{\dot{\phi}^2}{2}\right)$ and potential energy ($V(\phi)$) defines effective energy density ρ_{eff} while their difference gives effective pressure p_{eff}. Thus the EoS parameter is

$$\omega_{eff} = \frac{p_{eff}}{\rho_{eff}} = \frac{\frac{\dot{\phi}^2}{2} - V(\phi)}{\frac{\dot{\phi}^2}{2} + V(\phi)}. \tag{32}$$

The energy conservation law implies that

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{33}$$

where prime denotes derivative with respect to ϕ. This equation is known as Klein-Gordon equation also referred as scalar wave equation.

Chaotic inflation is used to discuss early inflating universe in which chaotic conditions originate some fluctuation patches. This occurs when inflaton field $\phi \gg M_{Pl}$ as well as negatively very large at the beginning and ends if $\phi \sim M_{Pl}$. In this case, inflaton moves towards the origin of potential and starts oscillating. Due to this behavior of inflaton field, the corresponding models are also referred as large field models. Chaotic inflation can also be described by quasi-de Sitter solution for $H = H_{dS}$ (H_{dS} being constant represents the quasi-de Sitter solution) when $\omega_{eff} \simeq -1$ and slow-roll approximation is valid. The slow-roll approximation is carried out when inflaton and matter or radiation interactions are useless as well as kinetic energy is much smaller than the potential energy [3].

The slow-roll approximation technique is used to analyze inflation via slow-roll parameters ($\epsilon,\ \eta$) given by

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = -\frac{\dot{H}}{H^2} - \frac{\ddot{H}}{2H\dot{H}} \equiv 2\epsilon - \frac{\dot{\epsilon}}{2\epsilon H}. \tag{34}$$

In terms of Hubble flow functions, these parameters can be expressed as

$$\epsilon = \epsilon_1, \quad \eta = 2\epsilon_1 - \frac{\epsilon_2}{2}, \tag{35}$$

which are valid for

$$\left|\frac{\dot{H}}{H^2}\right| \ll 1, \quad \left|\frac{\ddot{H}}{2H\dot{H}}\right| \ll 1. \tag{36}$$

In inflationary era, strong energy condition is violated which leads to

$$\dot{\phi}^2 \ll V(\phi), \quad \ddot{\phi} \ll 3H\dot{\phi}. \tag{37}$$

Under the slow-roll approximation, this yields

$$3H\dot{\phi} \simeq -V'(\phi), \quad 3H\ddot{\phi} \simeq -V''(\phi)\dot{\phi}. \tag{38}$$

In order to formulate inflationary model for Eq.(29), we obtain a relationship between kinetic energy and potential of the field using Eq.(32) as

$$\dot{\phi} \simeq \frac{\sqrt{\nu}\gamma^{\frac{1}{6\nu}}}{V(\phi)^{\frac{1-3\nu}{6\nu}}}. \tag{39}$$

For the potential, we take the first equation of (38) which yields

$$\begin{aligned} V(\phi) &= \left[3\sqrt{\nu}\gamma^{\frac{1}{6\nu}}\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}\right]^{\frac{6n\nu}{n+3n\nu-3\nu}} \\ &\times \left(\frac{n+3n\nu-3\nu}{6n\nu}\right)^{\frac{6n\nu}{n+3n\nu-3\nu}}(-\phi)^{\frac{6n\nu}{n+3n\nu-3\nu}}. \end{aligned} \tag{40}$$

For $\nu = \frac{1}{6}$, the EoS parameter becomes

$$\omega_{eff} = -1 + \frac{1}{6}\left(\frac{\gamma}{\rho_{eff}}\right), \tag{41}$$

and the corresponding inflaton takes the form

$$\phi = \phi_i + \sqrt{\frac{\gamma}{6}}(t - t_i),$$

where ϕ_i is the integration constant. This corresponds to the negatively large field at initial phase of inflation. For such scalar field, the potential and Hubble function become

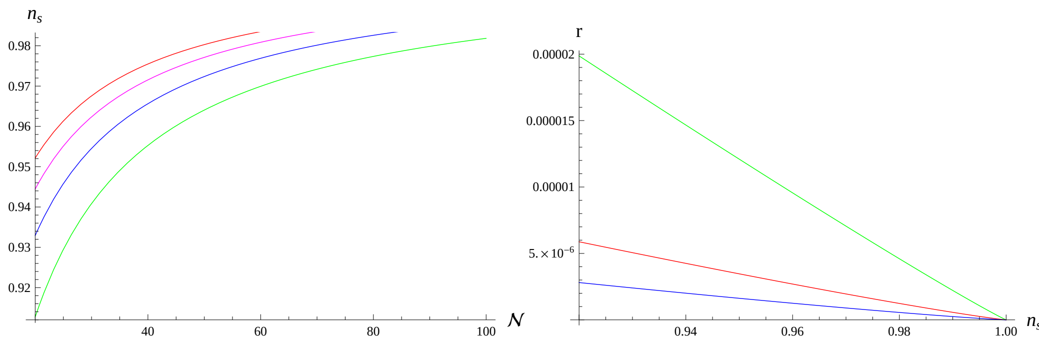

Figure 1: n_s versus $\mathcal{N}$ (left) for $n = 0.75$ (green), $n = 1.1$ (blue), $n = 1.5$ (magenta) and $n = 1.98$ (red) whereas r versus n_s (right) for $n = 0.9,\ m = 0.3$ (green), $n = 0.8,\ m = 0.5$ (blue) and $n = 0.75,\ m = 0.8$ (red).

$$
\begin{aligned}
V(\phi) &= \frac{\chi_1}{(\frac{2n}{3n-1})}(-\phi)^{\frac{2n}{3n-1}},\\
H(t) &= \left[\left(\frac{\chi_1(3n-1)}{2n}\right)\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)\right]^{\frac{1}{2n}}\\
&\times (-\phi)^{\frac{1}{3n-1}},
\end{aligned}
$$

where χ_1 is constant given as

$$
\begin{aligned}
\chi_1 &= \left[\left\{\sqrt{\frac{3}{2}}\gamma\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}\right\}^{2n}\right.\\
&\times \left.\left(\frac{3n-1}{2n}\right)^{1-n}\right]^{\frac{1}{3n-1}}.
\end{aligned}
$$

Notice that $V(\phi)$ is linear for $n = 1$. The slow-roll parameters, spectral index and tensor-scalar ratio are

$$
\epsilon = \frac{1}{4n(N+1)}, \quad \eta = 2(1-n)\epsilon, \quad n_s = 1 - \frac{n+2}{2n(\mathcal{N}+1)}, \tag{42}
$$

$$
\begin{aligned}
r &= \frac{4\kappa^2}{3}\gamma^{1-\frac{1}{n}}\left(\frac{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{n}}\\
&\times \left(\frac{n+2}{2n(1-n_s)}\right)^{-\frac{1+n}{2n}}.
\end{aligned} \tag{43}
$$

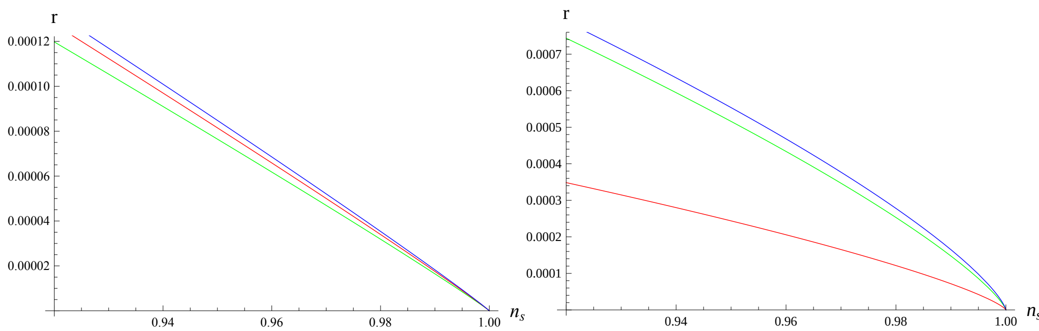

Figure 2: r versus n_s (left)$n = 1.1,\ m = 0.3$ (green), $n = 1.1,\ m = 0.5$ (blue) and $n = 1.1,\ m = 0.8$ (red) while r versus n_s (right) for $n = 1.8,\ m = 0.3$ (green), $n = 1.92,\ m = 0.5$ (blue) and $n = 1.98,\ m = 0.8$ (red).

In Figure **1**, the left plot indicates that the e-folds starts decreasing as n increases whereas the right panel shows that tensor-scalar ratio is compatible for all considered values of n and m. The consistent behavior of r is shown in both plots of Figure **2**.

When $\nu = \frac{1}{3}$, Eq.(29) becomes

$$\omega_{eff} = -1 + \frac{1}{3}\left(\frac{\gamma}{\rho_{eff}}\right). \tag{44}$$

At the beginning of inflation, this effective parameter leads to the following from of inflaton field, potential and Hubble function as

$$\begin{aligned} \phi &= \phi_i + \sqrt{\frac{\gamma}{3}}(t - t_i), \quad V(\phi) = \chi_2(1 - \frac{1}{2n})(-\phi)^{\frac{1}{1-\frac{1}{2n}}}, \\ H(t) &= \left[\chi_2\left(1 - \frac{1}{2n}\right)\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}\right)\right]^{\frac{1}{2n}} \\ &\times \quad (-\phi)^{\frac{1}{2n(1-\frac{1}{2n})}}, \end{aligned}$$

where

$$\chi_2 = \left[\left(1 - \frac{1}{2n}\right)^{\frac{1}{2n}}(6\gamma)^{\frac{1}{2}}\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}\right)^{\frac{1}{2n}}\right]^{\frac{1}{1-\frac{1}{2n}}},$$

and $V(\phi)$ called a generalized quadratic potential of chaotic inflation. The

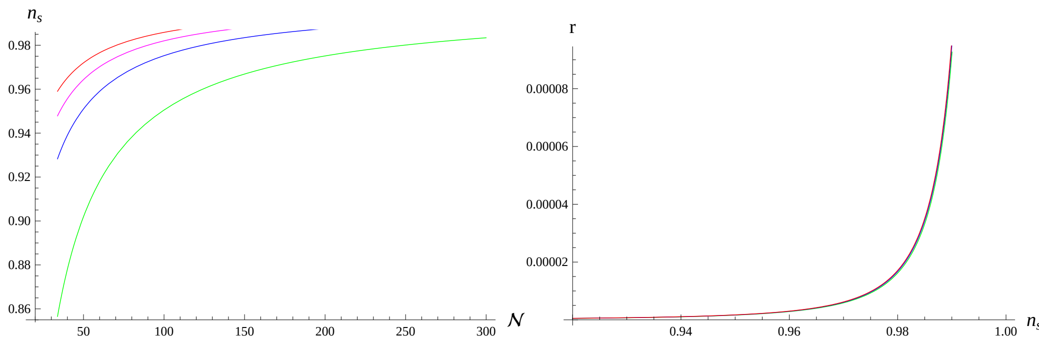

Figure 3: n_s versus $\mathcal{N}$ (left) for $n = 0.4$ (green), $n = 0.8$ (blue), $n = 1.1$ (magenta) and $n = 1.4$ (red) and r versus n_s (right) for $n = 0.4,\ m = 0.3$ (green), $n = 0.4,\ m = 0.5$ (blue) and $n = 0.4,\ m = 0.8$ (red).

corresponding slow-roll parameters, spectral index and tensor-scalar ratio are

$$\epsilon = \frac{1}{2n(N+1)}, \quad \eta = (2-n)\epsilon, \quad n_s = 1 - \frac{2}{n(\mathcal{N}+1)}, \tag{45}$$

$$r = \frac{8\kappa^2}{3}\gamma^{1-\frac{1}{n}}\left(\frac{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{n}} \times \left(\frac{2}{n(1-n_s)}\right)^{-\frac{1}{n}}. \tag{46}$$

The left graph in Figure **3** describes that for quadratic potential, the e-folds are getting smaller as n gets larger. The right plot shows that r is consistent with Planck constraint whether we increase or decrease the value of anisotropy parameter whereas Figure **4** indicates the same results for r.

For $\nu = \frac{2}{3}$, we have

$$\omega_{eff} = -1 + \frac{2}{3}\left(\frac{\gamma}{\rho_{eff}}\right). \tag{47}$$

The scalar field and $H(t)$ can be expressed as

$$\phi = \phi_i + \sqrt{\frac{2\gamma}{3}}(t-t_i), \quad V(\phi) = \frac{\chi_3}{\frac{4n}{3n-2}}(-\phi)^{\frac{4n}{3n-2}},$$

$$H(t) = \left[\frac{\chi_3(3n-2)}{4n}\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)\right]^{\frac{1}{2n}} \times (-\phi)^{\frac{2}{3n-2}}.$$

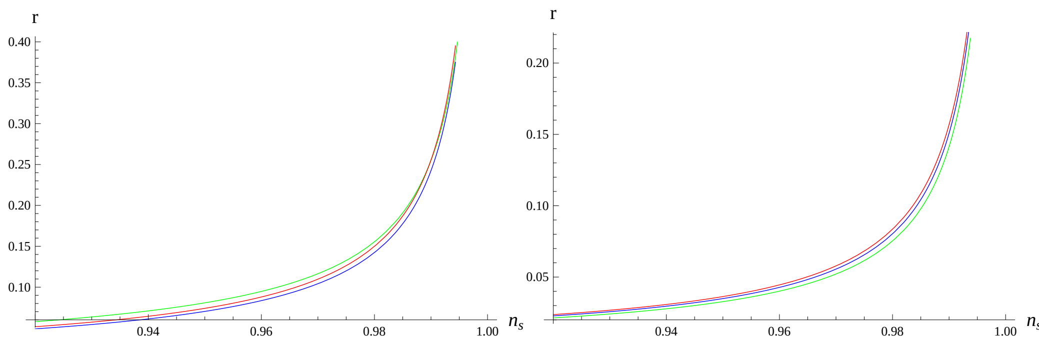

Figure 4: r versus n_s (left)$n = 1.1,\ m = 0.3$ (green), $n = 1.1,\ m = 0.5$ (blue) and $n = 1.1,\ m = 0.8$ (red) while r versus n_s (right) for $n = 1.4,\ m = 0.3$ (green), $n = 1.3,\ m = 0.5$ (blue) and $n = 1.3,\ m = 0.8$ (red).

The potential of the scalar field corresponds to quartic potential model of chaotic inflation for $n = 1$ with coupling constant given as

$$\chi_3 = \left[\sqrt{6}\left(\frac{3n-2}{4n}\right)^{n+2}\gamma^{\frac{1}{4}}\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}\right]^{\frac{4n}{3n-2}}.$$

In this case, the slow-roll and observational parameters become

$$\epsilon = \frac{1}{n(N+1)}, \quad \eta = \frac{(4-n)\epsilon}{2}, \quad n_s = 1 - \frac{4-n}{n(\mathcal{N}+1)}, \tag{48}$$

$$r = \frac{16\kappa^2}{3}\gamma^{1-\frac{1}{n}}\left(\frac{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{n}} \times \left(\frac{4-n}{n(1-n_s)}\right)^{-1+\frac{2}{n}}. \tag{49}$$

In Figure **5**, the left panel represents that there exists an inverse relation between n and $\mathcal{N}$, i.e., e-folds decreases when n increases and vice-versa. The best fit value of the e-folds is obtained for $n = 1.5$ with quartic potential. The right plot indicates that we obtain a consistent range for $m = 0.3,\ 0.5$ and 0.8 whereas n remains the same, i.e., $n = 1.1$. Figures **6** and **7** yield a compatible range of tensor-scalar ratio for different values of m and n.

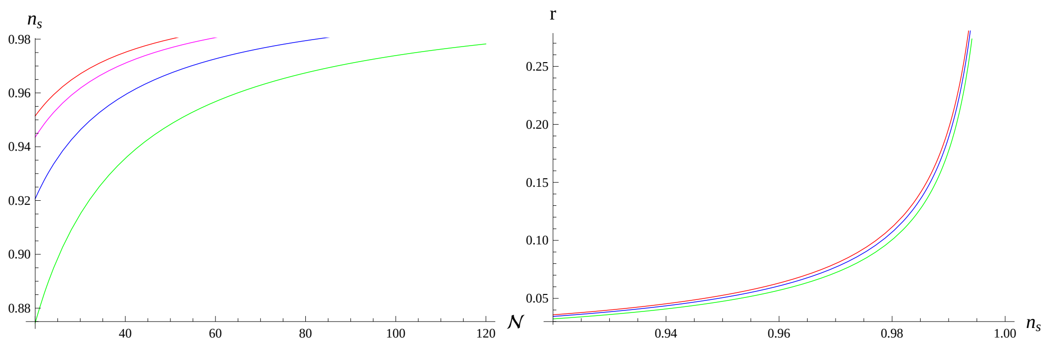

Figure 5: n_s versus $\mathcal{N}$ (left) for $n = 1.1$ (green), $n = 1.5$ (blue), $n = 1.83$ (magenta) and $n = 1.98$ (red) and r versus n_s (right) for $n = 1.1,\ m = 0.3$ (green), $n = 1.1,\ m = 0.5$ (blue) and $n = 1.1,\ m = 0.8$ (red).

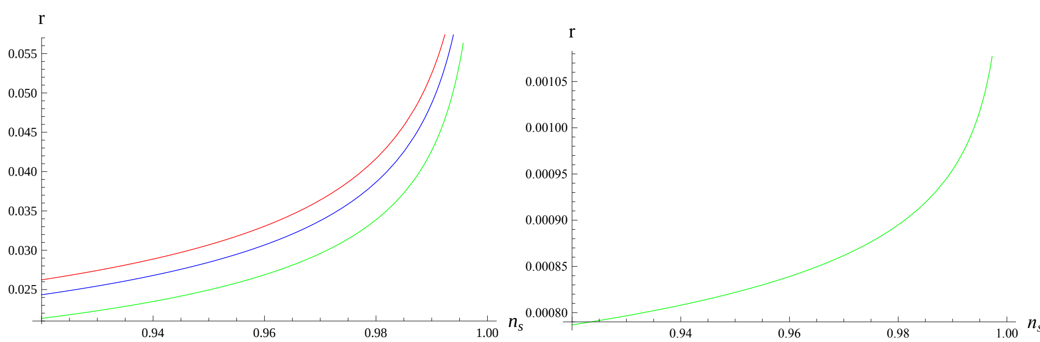

Figure 6: r versus n_s (left) for $n = 1.5$ (green), $n = 1.5$ (blue), $n = 1.5$ (magenta) whereas r versus n_s (right) for $n = 1.83,\ m = 0.3$ (green).

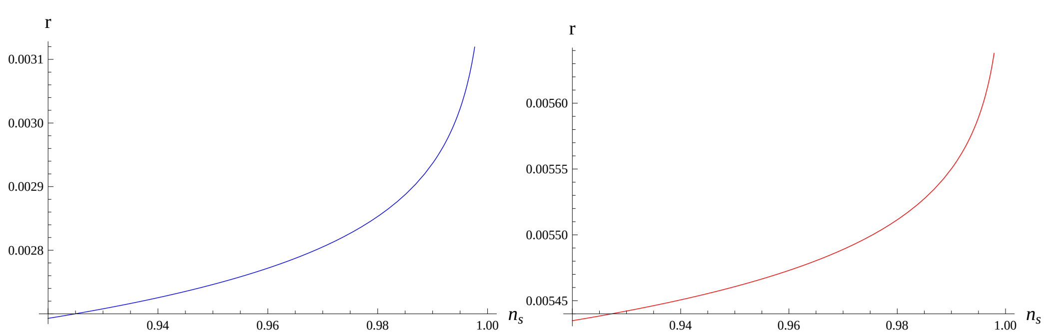

Figure 7: r versus n_s (left) for $n = 1.92,\ m = 0.5$ (blue) while r versus n_s (right) for $n = 1.98,\ m = 0.8$ (red).

3.2 Inflation via Fluid Cosmology

This is a well-known approach to any cosmological phenomenon which deals with perfect as well as imperfect fluid corresponding to ordinary radiation or matter in the universe. A straight forward description of rapid and uniform accelerated expansion of the universe is given by exotic matter which is governed by EoS different from radiation or ordinary matter. To discuss a graceful exit of rapid acceleration into a deceleration phase, quasi-de Sitter expansion in which EoS parameter depends on energy density. We consider the EoS parameter as

$$\omega(\tilde{\rho}) = -1 + \nu\left(\frac{\gamma}{\tilde{\rho}}\right)^{\frac{1}{3\nu}}, \tag{50}$$

where $\tilde{\rho}$ and $\tilde{p}$ represent energy density and pressure of inhomogeneous fluid. The energy density from conservation law and Hubble parameter for $\nu = \frac{1}{6}$ are

$$\begin{aligned}\tilde{\rho} &= \left[\mu_0 - \gamma^2\left(\frac{4n-1}{12n}\right)\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}\right. \\ &\times \left.(t-t_i)\right]^{\frac{2n}{4n-1}}, \qquad (51)\\ H &= \left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}\left[\mu_0 - \gamma^2\left(\frac{4n-1}{12n}\right)\right. \\ &\times \left.\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}(t-t_i)\right]^{\frac{1}{4n-1}}, \qquad (52)\end{aligned}$$

where μ_0 is an integration constant of the quasi-de Sitter expansion. Inflation occurs when t approaches to t_i for which ϵ_1 and ϵ_2 become

$$\epsilon_1 = \frac{1}{4n}\left(\frac{\gamma}{\tilde{\rho}}\right)^2, \quad \epsilon_2 = \left(\frac{\gamma}{\tilde{\rho}}\right)^2.$$

These parameters recover the expressions of slow-roll parameters, spectral index and tensor-scalar ratio when $\nu = \frac{1}{6}$ for the scalar field. Equations (9) and (50) lead to a relationship between number of e-folds and energy density of inhomogeneous fluid as

$$N + 1 = \left(\frac{\tilde{\rho}}{\gamma}\right)^2.$$

This provides a condition for the ending of inflation, i.e., ($\tilde{\rho} = \gamma$) and scale factor takes the form

$$a(t) = a_f \exp\left[1 - \left(\frac{\tilde{\rho}}{\gamma}\right)^2\right],$$

where a_f denotes the scale factor at the end of inflation. For $\nu = \frac{1}{3}$, the energy density and Hubble parameter become

$$\begin{aligned}
\tilde{\rho} &= \left[\mu_0 - \gamma\left(\frac{2n-1}{2n}\right)\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}\right)^{\frac{1}{2n}}\right. \\
&\times \left.(t - t_i)\right]^{\frac{2n}{2n-1}}, \qquad (53) \\
H &= \left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}\right)^{\frac{1}{2n}}\left[\mu_0 - \gamma\left(\frac{2n-1}{2n}\right)\right. \\
&\times \left.\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}\right)^{\frac{1}{2n}}(t - t_i)\right]^{\frac{1}{2n-1}}. \qquad (54)
\end{aligned}$$

Using Eqs.(53) and (54), we obtain Hubble flow functions at $t = t_i$ as

$$\epsilon_1 = \frac{1}{2n}\left(\frac{\gamma}{\tilde{\rho}}\right), \quad \epsilon_2 = \left(\frac{\gamma}{\tilde{\rho}}\right).$$

The resulting slow-roll parameters, spectral index and tensor-scalar ratio turn out to be the same as for $\nu = \frac{1}{3}$ in the scalar field. The scale factor becomes

$$a(t) = a_f \exp\left[1 - \left(\frac{\tilde{\rho}}{\gamma}\right)\right],$$

where $N + 1 = \frac{\tilde{\rho}}{\gamma}$. When $\nu = \frac{2}{3}$, the energy density, Hubble parameter and its flow functions for inhomogeneous fluid are

$$\begin{aligned}
\tilde{\rho} &= \left[\mu_0-\gamma^{\frac{1}{2}}\left(\frac{n-1}{n}\right)\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}\right.\\
&\times \left.(t-t_i)\right]^{\frac{2n}{n-1}}, \qquad (55)\\
H &= \left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}\left[\mu_0-\gamma^{\frac{1}{2}}\left(\frac{n-1}{n}\right)\right.\\
&\times \left.\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}(t-t_i)\right]^{\frac{1}{n-1}}, \qquad (56)\\
\epsilon_1 &= \frac{1}{n}\left(\frac{\gamma}{\tilde{\rho}}\right), \quad \epsilon_2=\left(\frac{\gamma}{\tilde{\rho}}\right)^{\frac{1}{2}}.
\end{aligned}$$

The parameters ϵ_1 and ϵ_2 recover the slow-roll and observational parameters formulated for $\nu=\frac{2}{3}$ with scalar field.

Finally, we take EoS for $\nu=\frac{1}{3}$ and use the field equations (4) and (5) which yield

$$2\ddot{f}_R-6H\dot{f}_R\left[1-2\left(\frac{2m+1}{(m+2)^2}\right)\right]+12\dot{H}\left(\frac{2m+1}{(m+2)^2}\right)=-\frac{2\kappa^2\gamma}{3}. \qquad (57)$$

The Hubble parameter and its derivative take the form

$$H=\sqrt{\frac{\kappa^2\gamma(m+2)^2(N+1)}{9(2m+1)}}, \quad \dot{H}=\frac{\kappa^2\gamma(m+2)^2}{18(2m+1)},$$

which can also expressed in terms of e-folds at the end of inflation as

$$\frac{dH}{dN}=\frac{1}{2}\sqrt{\frac{\kappa^2\gamma(m+2)^2}{9(2m+1)(N+1)}}.$$

Thus, we can reconstruct $f(R)$ model by inserting the above derivative in Eq.(57) leading to

$$\frac{a(N+1)}{3}\ddot{f}_R+a\dot{f}_R\left[\frac{1}{6}+(N+1)\left(1-\frac{2}{a}\right)\right]-f_R=-1,$$

where $a=\frac{(m+2)^2}{2m+1}$. Integrating the above equation, we obtain

$$\begin{aligned} f_R(R) &= \sqrt{N+1}\exp\left(-\frac{3(a-2)N}{a}\right)\left[c_1 U\left(-\frac{1-a}{a-2},\frac{3}{2},\frac{3(a-2)(N+1)}{a}\right)\right. \\ &+ \left. c_2 L^{\frac{1}{2}}_{\frac{1-a}{a-2}}\left(\frac{3(a-2)(N+1)}{a}\right)\right]+1. \end{aligned} \tag{58}$$

Here, c_1 and c_2 are integration constants whereas U represents confluent hypergeometric function and L denotes associated Laguerre polynomial. This equation (58) can also be expressed in terms of R by taking

$$R=\kappa^2\gamma(4N+3),\quad N=\frac{3}{4}\left(\frac{(2m+1)R}{(m+2)^2\kappa^2\gamma}-1\right), \tag{59}$$

which implies that

$$\begin{aligned} f_R(R) &= 1+\frac{1}{2}\sqrt{\frac{3R}{a\kappa^2\gamma}+1}\exp\left(\frac{-9(a-2)}{4a}\left(\frac{R}{a\kappa^2\gamma}-1\right)\right) \\ &\times \left[c_1 U\left(-\frac{1-a}{a-2},\frac{3}{2},\frac{9(a-2)}{4a}\left(\frac{R}{\kappa^2 a\gamma}+1\right)\right)\right. \\ &+ \left. c_2 L^{\frac{1}{2}}_{\frac{1-a}{a-2}}\left(\frac{9(a-2)}{4a}\left(\frac{R}{a\kappa^2\gamma+1}\right)\right)\right]. \end{aligned} \tag{60}$$

Its integration leads to

$$\begin{aligned} f(R) &= R+\frac{1}{2}\int\sqrt{\frac{3R}{a\kappa^2\gamma}+1}\exp\left(\frac{-9(a-2)}{4a}\left(\frac{R}{a\kappa^2\gamma}-1\right)\right) \\ &\times \left[c_1 U\left(-\frac{1-a}{a-2},\frac{3}{2},\frac{9(a-2)}{4a}\left(\frac{R}{\kappa^2 a\gamma}+1\right)\right)\right. \\ &+ \left. c_2 L^{\frac{1}{2}}_{\frac{1-a}{a-2}}\left(\frac{9(a-2)}{4a}\left(\frac{R}{a\kappa^2\gamma+1}\right)\right)\right]+c_3, \end{aligned}$$

where c_3 is an integration constant. This $f(R)$ model corresponds to Starobinsky inflationary model for $n,\ m,\ f_0=1$.

4 Inflationary Model for $\mu\neq 1$

Here, we would like to investigate the existence of the viable inflationary models for $\mu=2$ with spectral index and tensor-scalar ratio given in Eqs.(24)

and (27). In this case, the EoS (7) and Hubble flow functions (19) reduce to

$$\omega_{eff} = -1+\left(\frac{1}{9\nu}\right)\log^2\left[\frac{\rho_{eff}}{p_{eff}}\right], \tag{61}$$

$$\epsilon_1 = \frac{3\nu}{2n(N+1)^2}, \quad \epsilon_2=\frac{2}{(N+1)}, \tag{62}$$

where ϵ_2 is much larger than ϵ_1. The observational parameters $(n_s,\ r)$ become

$$n_s = 1-\frac{2}{(\mathcal{N}+1)}, \tag{63}$$

$$r = 2\kappa^2\nu\gamma^{1-\frac{1}{2n}}(1-n_s)^2\left(\frac{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{n}} \times \exp\left[-\frac{3\nu(1-n_s)(n-1)}{2n}\right], \tag{64}$$

where $(\mathcal{N}+1)^2=\frac{4}{(1-n_s)^2}$. In this case, tensor-scalar ratio is found to be inconsistent with Planck constraint whereas $\mathcal{N}=59$.

For the inflation via scalar field, the kinetic energy and potential function are formulated using Eqs.(32), (38) and (61) as

$$\dot{\phi} \simeq \frac{1}{3}\sqrt{\frac{V(\phi)}{\nu}}\left(\frac{\gamma}{V(\phi)-1}\right), \tag{65}$$

$$V(\phi) \simeq \left[\left\{c_4-\sqrt{\frac{2}{\nu}}\left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}\phi\right\} \times \frac{(1-3n)\gamma}{2n\,{}_2F_1\left(1,\frac{3}{2}-\frac{1}{2n};\frac{5}{2}-\frac{1}{2n};\frac{V(\phi)}{\gamma}\right)}\right]^{\frac{3n-1}{2n}}, \tag{66}$$

where c_4 is an integration constant and ${}_2F_1$ represents hypergeometric function. The potential function, slow-roll and observational parameters for

$c_1 = 2,\ \nu = \frac{1}{2}$ become

$$
\begin{aligned}
V(\phi) &\simeq \left[\left\{1 - \left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}\right)^{\frac{1}{2n}} \phi\right\}\right. \\
&\times \left.\frac{(1-3n)\gamma}{n_2F_1\left(1, \frac{3}{2} - \frac{1}{2n}; \frac{5}{2} - \frac{1}{2n}; \frac{V(\phi)}{\gamma}\right)}\right]^{\frac{3n-1}{2n}}, \qquad (67) \\
\epsilon &= \frac{3}{4n(N+1)^2}, \quad \eta = -\frac{1}{(N+1)} + \frac{3}{2n(N+1)^2}, \qquad (68) \\
n_s &= 1 - \frac{2}{\mathcal{N}+1}, \qquad (69) \\
r &= \kappa^2\gamma^{1-\frac{1}{2n}}(1-n_s)^2\left(\frac{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{n}} \\
&\times \exp\left[-\frac{3(n-1)(1-n_s)}{4n}\right]. \qquad (70)
\end{aligned}
$$

The potential function corresponds to Starobinsky model for $m,\ n = 1$ [28].

In order to discuss inflation via fluid cosmology, we take inhomogeneous fluid so that EoS takes the form

$$
\omega(\tilde{\rho}) = -1 + \frac{1}{9\nu}\log^2\left(\frac{\tilde{\rho}}{\gamma}\right). \qquad (71)
$$

Inserting Eq.(16) in the conservation law, we obtain

$$
\tilde{\rho} = \gamma\left[1 - \left(\frac{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{2n}} \frac{3\nu}{\gamma^{\frac{1}{2n}}(t_e - t)}\right], \qquad (72)
$$

$$
\begin{aligned}
H &= \left[1 - 3\nu\left(\frac{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{2n}} \frac{1}{\gamma^{\frac{1}{2n}}(t_e - t)}\right]^{\frac{1}{2n}} \\
&\times \left(\frac{4\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2 + 3n(2m+1)\}}\right)^{\frac{1}{2n}} \gamma^{\frac{1}{2n}}, \qquad (73)
\end{aligned}
$$

where t_e denotes time at the end of inflation. In this limit $t << t_e$, the

Hubble parameter become

$$\epsilon_1 = \frac{3\nu}{2n\gamma^{\frac{1}{n}}(t_e-t)^2}\left(\frac{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{n}}, \quad (74)$$

$$\epsilon_2 = \frac{2}{\gamma^{\frac{1}{2n}}(t_e-t)}\left(\frac{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}{4\kappa^2(m+2)^2}\right)^{\frac{1}{2n}}. \quad (75)$$

To describe the duration of inflation, the number of e-folds and scale factor turn out to be

$$N = \left(\frac{4\kappa^2(m+2)^2\gamma}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}(t_e-t)-1,$$

$$a(t) = a_f \exp\left[1-\left(\frac{4\gamma\kappa^2(m+2)^2}{12^n f_0\{2(1-n)(m+2)^2+3n(2m+1)\}}\right)^{\frac{1}{2n}}(t-t_e)\right].$$

The parameters ϵ_1 and ϵ_2 recover the expressions given in Eqs.(62)-(64).

5 Concluding Remarks

This paper is devoted to study inflation via two approaches scalar field and fluid cosmology in $f(R)$ gravity using LRS BI universe model. When the inflaton field starts from a large field value and then rolls down towards the minimum value of potential function, the field value is about to vanish at this point. This is known as chaotic inflation in which inflaton field is greater than M_{Pl} and ends when inflaton field is nearly close to M_{Pl}. Models which correspond to chaotic inflation are known as large field models. To investigate such type of inflation, we have taken EoS with a deviation parameter which describes quasi-de Sitter expansion and leads to an elegant exit from inflation to deceleration phase. We have furnished some basic features of inflation and formulated Hubble flow functions as well as slow-roll parameters in fluid cosmology and scalar field for a power-law model of $f(R)$ gravity.

We have analyzed inflation by taking different values of ν with $\mu=1$ and $\mu\neq 1$. For $\mu=1$, we have found expressions of kinetic and potential energies for scalar field with $\nu=\frac{1}{6}$, $\frac{1}{3}$ and $\frac{2}{3}$ which yield linear, quadratic and quartic potential models, respectively and analyzed the effect of anisotropy parameter. We have calculated slow-roll parameters, spectral index and tensor-scalar ratio for all these values. The results can be summarized as follows.

- For $\nu = \frac{1}{6}$, we have found the range of n for $m = 0.3,\ 0.5,\ 0.8$ as $0.9 \leq n \leq 1.8,\ 0.8 \leq n \leq 1.92,\ 0.75 \leq n \leq 1.98$. This range indicates that when anisotropy is increased, the range for n is also increased. In this range, r shows consistency with Planck observations whereas compatible number of e-folds are $\mathcal{N} = 54,\ \mathcal{N} = 41,\ \mathcal{N} = 34,\ \mathcal{N} = 29$ for $n = 0.7,\ 1.1,\ 1.5,\ 1.98$, respectively. We have found that e-folds are getting smaller when n is larger.

- In case of $\nu = \frac{1}{3}$, the range of n is $0.4 \leq n \leq 1.4,\ 0.4 \leq n \leq 1.3,\ 0.4 \leq n \leq 1.3$ for the above values of anisotropic parameter. For $n = 0.4,\ 0.8,\ 1.1$ and 1.4, the e-folds are found to be $\mathcal{N} = 148,\ \mathcal{N} = 74,\ \mathcal{N} = 53$ and $\mathcal{N} = 29$, respectively. The tensor-scalar ratio is compatible with recent observational data for the above mentioned interval of n.

- When $\nu = \frac{2}{3}$, the range of n becomes $1.1 \leq n \leq 1.83,\ 1.1 \leq n \leq 1.92,\ 1.1 \leq n \leq 1.98$ for the same anisotropy values. The e-folds gives $\mathcal{N} = 78,\ 49,\ 34,\ 30$ for $n = 1.1,\ 1.5,\ 1.83$ and 1.98, respectively. The tensor-scalar ratio turns out to be compatible with Planck constraint.

In fluid cosmology, we have calculated Hubble flow functions $(\epsilon_1,\ \epsilon_2)$ which recover expressions of the spectral index and tensor-scalar ratio for the scalar field. We have also evaluated the value of $f(R)$ which corresponds to Starobinsky inflationary model for $n,\ f_0,\ m = 1$. For $\mu \neq 1$, we have taken $\mu = 2$ and constructed observational parameters for ν and expressions found in fluid cosmology. We again recover these observational parameters as well as Hubble flow functions. We have investigated inflation with scalar field and developed expressions of kinetic and potential functions. In this case, the tensor-scalar ratio is incompatible to Planck constraints whereas $\mathcal{N} = 59$ for all values of n and ν. It is worth mentioning here that all our results can be recovered in isotropic and homogeneous universe for $n,\ m,\ f_0 = 1$ [13].

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