Spin Polarized versus Chiral Condensate in Quark Matter at Finite Temperature and Density

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arXiv:1601.01403v1 [hep-ph] 7 Jan 2016

It is shown that the spin polarized condensate appears in quark matter at high baryon density and low temperature due to the tensor-type four-point interaction in the Nambu-Jona-Lasinio-type model as a low energy effective theory of quantum chromodynamics. It is indicated within this low energy effective model that the chiral symmetry is broken again by the spin polarized condensate as increasing the quark number density, while the chiral symmetry restoration occurs in which the chiral condensate disappears at a certain density.

§1. Introduction

One of recent interests in the physics of the strong interaction, namely, in the physics governed by the quantum chromodynamics (QCD), may be to clarify the structure of the phase diagram on the plane with respect to the baryon chemical potential and temperature.[1)] In the region of finite temperature and zero baryon chemical potential, the lattice QCD simulation is workable and the reliable calculation based on the first principle is performed until now. However, in the region of the low temperature and the finite baryon chemical potential, the possibility for various phases has been indicated such as the color superconducting phase,[2),3),4)] quarkyionic phase,[5)] inhomogeneous chiral condensed phase[6),7),8)] and so on.

On the other hand, it has been shown in our previous papers that the spin polarized phase may appear and be realized against the color superconducting phase in both the cases of two-[9)] or three-flavor[10)] in the region with finite quark chemical potential at zero temperature. It is further interesting to investigate possible phases in the region with high density and low temperature from a viewpoint of physics of compact stars, especially, the structure of inner core of the compact stars. It has also been shown in our recent work[11)] that there is a possibility of the existence of the strong magnetic field on the surface of compact stars if there exists the quark spin polarized phase, which leads to the spontaneous magnetization of quark matter due to anomalous magnetic moment of quark, while only symmetric quark matter has been considered.

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In this paper succeeding to Refs.12) and 9), a possibility of the quark spin polarized phase is investigated in the region of finite quark chemical potential and finite temperature by using the Nambu-Lona-Lasinio (NJL) moldel[13),14),15)] with the tensor-type four-point interaction between quarks,[16)] apart from the pseudovector-type four-point interaction.[17),18)] The chiral condensate and the quark spin polarization, namely, the tensor condensate, are considered simultaneously apart from the color superconductor.[19)]

This paper is organized as follows: In the next section, the recapitulation of the NJL model with the tensor-type four-point interaction between quarks is given and in this model the chiral condensate and quark spin polarized condensate are considered simultaneously. In §3, the thermodynamic potential at zero temperature is introduced under the mean field approximation and in §4, the analytic calculation for the thermodynamic potential is presented. In §5, the thermodynamic potential at finite temperature and density is given and derived. Another derivation of the thermodynamic potential is given in Appendix A. Also, the effective potential is evaluated in Appendix B. In §6, the numerical results are given through the calculation of the thermodynamic potential under the various temperature and quark chemical potential. The results are summarized in the phase diagram on the plane with respect to the quark chemical potential and temperature, in which the possible phases, the position of phase boundary and the order of the phase transition are shown, apart from the color superconducting phase. The last section is devoted to a summary and concluding remarks.

§2. NJL model with a tensor-type four-point interaction

Let us consider the NJL-model Lagrangian density with a tensor-type four-point interaction. The Lagrangian density with $su(2)$-flavor symmetry can be expressed as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm S} + \mathcal{L}_{\rm T}, \tag{2.1}$$

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu\partial_\mu - m_0)\psi, \tag{2.2}$$

$$\mathcal{L}_{\rm S} = G_{\rm S}\left\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\vec{\tau}\psi)^2\right\}, \tag{2.3}$$

$$\mathcal{L}_{\rm T} = -\frac{G_{\rm T}}{4}\left\{(\bar{\psi}\gamma^\mu\gamma^\nu\vec{\tau}\psi)\cdot(\bar{\psi}\gamma_\mu\gamma_\nu\vec{\tau}\psi) + (\bar{\psi}i\gamma^5\gamma^\mu\gamma^\nu\psi)(\bar{\psi}i\gamma^5\gamma_\mu\gamma_\nu\psi)\right\}, \tag{2.4}$$

where m_0 is a current quark mass for up and down quark and $\vec{\tau}$ is the Pauli matrices for isospin. It is known that these current quark masses are different slightly for each flavor, but we have used the same value approximately. The first two terms, $\mathcal{L}_0+\mathcal{L}_{\rm S}$, is the original NJL-model Lagrangian density. In this discussion $\mathcal{L}_{\rm T}$ is added into the model. Then, the spin matrix appears from $\mathcal{L}_{\rm T}$ when $\mu = 1$, $\nu = 2$ or $\mu = 2$, $\nu = 1$ as follows:

$$\Sigma_3 = -i\gamma^1\gamma^2 = \begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix}.$$

Since we use the mean field approximation, then we get mean field Lagrangian density as

$$\begin{aligned}\mathcal{L}_{\rm MFA} =&\bar{\psi}(i\gamma^\mu\partial_\mu - m_0)\psi + G_{\rm S}\left\{2\langle\bar{\psi}\psi\rangle(\bar{\psi}\psi) - \langle\bar{\psi}\psi\rangle^2\right\} \\ &+ \frac{G_{\rm T}}{2}\left\{2\langle\bar{\psi}\Sigma_3\tau_3\psi\rangle(\bar{\psi}\Sigma_3\tau_3\psi) - \langle\bar{\psi}\Sigma_3\tau_3\psi\rangle^2\right\}, \end{aligned} \tag{2.5}$$

where $\langle\,\rangle$ means vacuum expectation value. Here τ_3 is the third component of the Pauli matrix for isospin. When it operates on ψ for up-quark (down-quark), the matrix changes into 1 (-1) as its eigenvalue. Thus we could safely express $\bar{\psi}\Sigma_3\tau_3\psi$ as follows:

$$\bar{\psi}\Sigma_3\tau_3\psi \to \bar{\psi}\Sigma_3\psi\tau_f,$$

where $\tau_f = 1$ (-1) when $f =$ up-quark (down-quark). Let us define the following quantities:

$$F \equiv -G_{\rm T}\langle\bar{\psi}\Sigma_3\psi\rangle, \qquad M \equiv -2G_{\rm S}\langle\bar{\psi}\psi\rangle, \qquad M_{\rm q} \equiv m_0 + M.$$

F and M are especially important quantities, because if F and/or M are not equal to zero, then spin polarization and/or chiral condensation occur. Here, $M_{\rm q}$ is just a constituent quark mass. Substituting these quantities into Eq.(2.5), we convert $\mathcal{L}_{\rm MFA}$ into

$$\mathcal{L}_{\rm MFA} = \bar{\psi}(i\gamma^\mu\partial_\mu - M_{\rm q})\psi - F(\bar{\psi}\Sigma_3\psi) - \frac{M^2}{4G_{\rm S}} - \frac{F^2}{2G_{\rm T}}. \tag{2.6}$$

Let us switch from Lagrangian formalism to Hamiltonian formalism by Legendre transformation. First we must obtain the canonical momentum π_α:

$$\pi_\alpha = \frac{\partial\mathcal{L}_{\rm MFA}}{\partial\dot{\psi}_\alpha} = i\psi^\dagger_\alpha,$$

where α means an index for spinor and isospin. We, therefore, get the Hamiltonian density:

$$\begin{aligned}\mathcal{H}_{\rm MFA} &= \pi_\alpha\dot{\psi_\alpha} - \mathcal{L}_{\rm MFA} \\ &= \bar{\psi}(-i\vec{\gamma}\cdot\vec{\nabla} + M_{\rm q})\psi + F(\bar{\psi}\Sigma_3\psi) + \frac{M^2}{4G_{\rm S}} + \frac{F^2}{2G_{\rm T}}. \end{aligned} \tag{2.7}$$

Thus, the Hamiltonian is expressed as

$$H_{\rm MFA} = \int d^3x\,\psi^\dagger\gamma^0(-i\vec{\gamma}\cdot\vec{\nabla} + M_{\rm q} + F\Sigma_3)\psi + V\frac{M^2}{4G_{\rm S}} + V\frac{F^2}{2G_{\rm T}},$$

where V is the volume of this system. We transform $\psi(x)$ by the Fourier transformation as follows:

$$\psi(x) = \int\frac{d^3p}{(2\pi)^3}\tilde{\psi}(p)e^{-ip\cdot x}.$$

Substituting it into $H_{\rm MFA}$, we obtain

$$H_{\rm MFA} = \int\frac{d^3p}{(2\pi)^3}\tilde{\psi}^\dagger\gamma^0(\vec{\gamma}\cdot\vec{p} + M_{\rm q} + F\Sigma_3)\tilde{\psi} + V\frac{M^2}{4G_{\rm S}} + V\frac{F^2}{2G_{\rm T}}. \tag{2.8}$$

What we must do is to diagonalize $H_{\rm MFA}$. Not diagonalized terms are

$$\begin{aligned} h_{\rm MFA} &\equiv \gamma^0(\vec{\gamma}\cdot\vec{p} + M_{\rm q} + F\Sigma_3) \\ &= \begin{pmatrix} F\sigma_3 + M_{\rm q} & \vec{p}\cdot\vec{\sigma} \\ \vec{p}\cdot\vec{\sigma} & -F\sigma_3 - M_{\rm q} \end{pmatrix} \\ &= \begin{pmatrix} F+M_{\rm q} & 0 & p_3 & p_1 - ip_2 \\ 0 & -F+M_{\rm q} & p_1+ip_2 & -p_3 \\ p_3 & p_1-ip_2 & -F-M_{\rm q} & 0 \\ p_1+ip_2 & -p_3 & 0 & F-M_{\rm q} \end{pmatrix}. \end{aligned}$$

Since $h_{\rm MFA}$ is a hermitian matrix, it is diagonalized by a unitary matrix. The eigenvalues are obtained as

$$\pm E_{\vec{p}}^{(\eta)} = \pm\sqrt{p_3^2 + \left(\sqrt{p_1^2+p_2^2+M_{\rm q}^2} + \eta F\right)^2}, \tag{2.9}$$

where $\eta = \pm 1$.

§3. Thermodynamic potential at zero temperature

Next, we introduce a quark chemical potential μ and a number density operator $\mathcal{N}$ in order to discuss on finite density system at zero temperature. The thermodynamic potential is defined as follows:

$$\Phi = \mathcal{H}_{\rm MFA} - \mu\mathcal{N}. \tag{3.1}$$

The next step is to calculate the expectation value. Since we consider the zero-temperature system in this section, the system that quasi-particles degenerate is treated. Hence, we must sum over momenta from zero to the value that single-quasi-particle energy is equal to the chemical potential. Sandwiching with a "bra" and "ket", we obtain

$$\begin{aligned} \Phi &= \frac{1}{V}\langle {\rm F.D.}| \left(H_{\rm MFA} - \mu\int d^3x\, \mathcal{N}\right) |{\rm F.D.}\rangle \\ &= \frac{1}{V}\sum_{\vec{p},\eta,\tau,\alpha}^{E_{\vec{p}}^{(\eta)}\le\mu,\vec{p}^2\le\Lambda^2} \left(E_{\vec{p}}^{(\eta)} - \mu\right) + \frac{M^2}{4G_{\rm S}} + \frac{F^2}{2G_{\rm T}}, \end{aligned}$$

where $|{\rm F.D.}\rangle$ means the state that quasi-particles degenerate and Λ is a three-momentum cutoff parameter for the integration over momenta. Here, τ and α are indeces for isospin and quark color, respectively. The upper limit of the integration is imposed by two conditions, which are $E_{\vec{p}}^{(\eta)} \le \mu$ and $\vec{p}^2 \le \Lambda^2$. We would like to discuss spin polarization and chiral condensate simultaneously. However, the above expression does not have the contribution from Dirac sea. Since chiral condensate occurs by the effect of Dirac sea, let us add the contribution from it. Thus, we get

$$\Phi(M,F,\mu) = \frac{1}{V}\sum_{\vec{p},\eta,\tau,\alpha}^{E_{\vec{p}}^{(\eta)}\le\mu,\vec{p}^2\le\Lambda^2} \left(E_{\vec{p}}^{(\eta)} - \mu\right) - \frac{1}{V}\sum_{\vec{p},\eta,\tau,\alpha}^{\vec{p}^2\le\Lambda^2} E_{\vec{p}}^{(\eta)} + \frac{M^2}{4G_{\rm S}} + \frac{F^2}{2G_{\rm T}}, \tag{3.2}$$

where, the second term represents the contribution from the Dirac sea (negative energy sea). We change the sum $\frac{1}{V}\sum_{\vec{p}}$ into the integration $\int \frac{d^3p}{(2\pi)^3}$. Then, the thermodynamic potential can be expressed as

$$\Phi(M,F,\mu) = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4, \tag{3.3}$$

where,

$$\Phi_1(F,M,\mu) = \sum_{\tau,\alpha}\int_{\Gamma_1}\frac{d^3p}{(2\pi)^3}\left\{\sqrt{p_3^2+(\sqrt{p_1^2+p_2^2+M_{\rm q}^2}+F)^2}-\mu\right\},$$
$$\Gamma_1 = \left\{E_{\vec{p}}^{(+1)} \le \mu,\ \vec{p}^2 \le \Lambda^2\right\} \tag{3.4}$$

$$\Phi_2(M,F,\mu) = \sum_{\tau,\alpha}\int_{\Gamma_2}\frac{d^3p}{(2\pi)^3}\left\{\sqrt{p_3^2+(\sqrt{p_1^2+p_2^2+M_{\rm q}^2}-F)^2}-\mu\right\},$$
$$\Gamma_2 = \left\{E_{\vec{p}}^{(-1)} \le \mu,\ \vec{p}^2 \le \Lambda^2\right\} \tag{3.5}$$

$$\Phi_3(M,F,\mu) = -\sum_{\eta,\tau,\alpha}\int_{\Gamma_3}\frac{d^3p}{(2\pi)^3}\sqrt{p_3^2+(\sqrt{p_1^2+p_2^2+M_{\rm q}^2}+\eta F)^2},$$
$$\Gamma_3 = \left\{\vec{p}^2 \le \Lambda^2\right\} \tag{3.6}$$

$$\Phi_4(M,F,\mu) = \frac{M^2}{4G_{\rm S}} + \frac{F^2}{2G_{\rm T}}. \tag{3.7}$$

Here, Φ_i $(i=1,2,3,4)$ means the contribution from positive energy for $\eta=+1$, positive energy for $\eta=-1$, vacuum and mean field, respectively. Γ_i $(i=1,2,3)$ is the interval of integration over momenta. Since these integrands do not depend on τ or α, the summations over τ and α give factors 2 and 3, respectively. We assume $M \ge 0$, $F \ge 0$ and $\Lambda > \mu$, and introduce polar coordinates (p_T, θ):

$$p_1 = p_T\cos\theta,\ p_2 = p_T\sin\theta.$$

Moreover, we define $q \equiv \sqrt{p_T^2+M_{\rm q}^2}$ in order to integrate over momenta. After integrating over θ, we obtain the thermodynamic potential $\Phi = \sum_{i=1}^4 \Phi_i$ as follows:

$$\Phi_1(M,F,\mu) = \frac{3}{2\pi^2}\int_{\Gamma_1} dqdp_3\, q\left(\sqrt{p_3^2+(q+F)^2}-\mu\right),$$
$$\Gamma_1 = \left\{p_3^2+(q+F)^2 \le \mu^2, p_3^2+q^2 \le \Lambda^2+M_{\rm q}^2, q \ge M_{\rm q}\right\}$$

$$\Phi_2(M,F,\mu) = \frac{3}{2\pi^2}\int_{\Gamma_2} dqdp_3\, q\left(\sqrt{p_3^2+(q-F)^2}-\mu\right),$$
$$\Gamma_2 = \left\{p_3^2+(q-F)^2 \le \mu^2, p_3^2+q^2 \le \Lambda^2+M_{\rm q}^2, q \ge M_{\rm q}\right\}$$

$$\Phi_3(M,F,\mu) = -\frac{3}{2\pi^2}\sum_{\eta}\int_{\Gamma_3} dqdp_3\, q\sqrt{p_3^2+(q+\eta F)^2},$$
$$\Gamma_3 = \left\{p_3^2+q^2 \le \Lambda^2+M_{\rm q}^2, q \ge M_{\rm q}\right\}$$
$$\Phi_4(M,F,\mu) = \frac{M^2}{4G_{\rm S}} + \frac{F^2}{2G_{\rm T}}.$$

§4. Calculation of the thermodynamic potential at zero temperature

In this section we will integrate the thermodynamic potential over p_3 analytically. To do this we must discuss the interval of the integration.

First we consider Φ_1 and Γ_1. If the first condition in Γ_1 is satisfied, the second condition in it will be satisfied automatically. So we could reduce Γ_1 to

$$\Gamma_1 = \left\{p_3^2+(q+F)^2 \le \mu^2, q \ge M_{\rm q}\right\}.$$

Furthermore, we could change the above condition to the following:

$$\Gamma_1 = \left\{-\sqrt{\mu^2-(q+F)^2} \le p_3 \le \sqrt{\mu^2-(q+F)^2},\ q \ge M_{\rm q}\right\}.$$

Since the contents in a square root must be positive, the final form is

$$\Gamma_1 = \left\{-\sqrt{\mu^2-(q+F)^2} \le p_3 \le \sqrt{\mu^2-(q+F)^2},\ M_{\rm q} \le q \le \mu - F\right\}.$$

However, we need the condition: $M_{\rm q} \le \mu - F$ in order to integrate over q. If $M_{\rm q} > \mu - F$, we can not perform integral. Using an integration formula:

$$\int dx\,\sqrt{x^2+a^2} = \frac{1}{2}\left\{x\sqrt{x^2+a^2} + a^2\log\left(x+\sqrt{x^2+a^2}\right)\right\},$$

we could integrate over p_3 easily. The final results are the following:
If $M_{\rm q} > \mu - F$,

$$\Phi_1(M,F,\mu) = 0. \tag{4.1}$$

If $M_{\rm q} \le \mu - F$,

$$\Phi_1(M,F,\mu) = \frac{3}{2\pi^2}\int_{M_{\rm q}}^{\mu-F} dq\, q\Big\{-\mu\sqrt{\mu^2-(q+F)^2} + (q+F)^2\log\left(\frac{\sqrt{\mu^2-(q+F)^2}+\mu}{q+F}\right)\Big\}. \tag{4.2}$$

Next we consider Φ_2 and Γ_2. This case is more complicated than the previous case. There are five cases for conditions to perform integral as follows:
If $F-\mu \le M_{\rm q} \le F+\mu \le \sqrt{\Lambda^2+M_{\rm q}^2}$,

$$\Phi_2(M,F,\mu) = \frac{3}{2\pi^2}\int_{M_{\rm q}}^{F+\mu} dq \int_{-\sqrt{\mu^2-(q-F)^2}}^{\sqrt{\mu^2-(q-F)^2}} dp_3\, q\left(\sqrt{p_3^2+(q-F)^2}-\mu\right).$$

If $M_{\rm q} \le F-\mu \le F+\mu \le \sqrt{\Lambda^2+M_{\rm q}^2}$,

$$\Phi_2(M,F,\mu) = \frac{3}{2\pi^2}\int_{F-\mu}^{F+\mu} dq \int_{-\sqrt{\mu^2-(q-F)^2}}^{\sqrt{\mu^2-(q-F)^2}} dp_3\, q\left(\sqrt{p_3^2+(q-F)^2}-\mu\right).$$

If $M_{\rm q} \le F-\mu \le b \le \sqrt{\Lambda^2+M_{\rm q}^2} \le F+\mu$,

$$\Phi_2(M,F,\mu) = \frac{3}{2\pi^2}\left(\int_{F-\mu}^{b}\int_{-\sqrt{\mu^2-(q-F)^2}}^{\sqrt{\mu^2-(q-F)^2}} + \int_{b}^{\sqrt{\Lambda^2+M_{\rm q}^2}}\int_{-\sqrt{\Lambda^2+M_{\rm q}^2-q^2}}^{\sqrt{\Lambda^2+M_{\rm q}^2-q^2}}\right) \times q\left(\sqrt{p_3^2+(q-F)^2}-\mu\right) dp_3\, dq.$$

If $F-\mu \le M_{\rm q} \le b \le \sqrt{\Lambda^2+M_{\rm q}^2} \le F+\mu$,

$$\Phi_2(M,F,\mu) = \frac{3}{2\pi^2}\left(\int_{M_{\rm q}}^{b}\int_{-\sqrt{\mu^2-(q-F)^2}}^{\sqrt{\mu^2-(q-F)^2}} + \int_{b}^{\sqrt{\Lambda^2+M_{\rm q}^2}}\int_{-\sqrt{\Lambda^2+M_{\rm q}^2-q^2}}^{\sqrt{\Lambda^2+M_{\rm q}^2-q^2}}\right) \times q\left(\sqrt{p_3^2+(q-F)^2}-\mu\right) dp_3\, dq.$$

If $F-\mu \le b \le M_{\rm q} \le \sqrt{\Lambda^2+M_{\rm q}^2} \le F+\mu$,

$$\Phi_2(M,F,\mu) = \frac{3}{2\pi^2}\int_{M_{\rm q}}^{\sqrt{\Lambda^2+M_{\rm q}^2}} dq \int_{-\sqrt{\Lambda^2+M_{\rm q}^2-q^2}}^{\sqrt{\Lambda^2+M_{\rm q}^2-q^2}} dp_3\, q\left(\sqrt{p_3^2+(q-F)^2}-\mu\right).$$

Here, b is the solution for q of the simultaneous equation:

$$\begin{cases} p_3^2+q^2 = \Lambda^2+M_{\rm q}^2 \\ p_3^2+(q-F)^2 = \mu^2 \end{cases}.$$

The solution is

$$q = \frac{\Lambda^2+M_{\rm q}^2+F^2-\mu^2}{2F} \equiv b.$$

We could perform integral over p_3, then define two functions for simplicity as follows:

$$\phi_1(q) \equiv \frac{3}{2\pi^2}q\left\{-\mu\sqrt{\mu^2-(F-q)^2}+(F-q)^2\log\left(\frac{\mu+\sqrt{\mu^2-(F-q)^2}}{|F-q|}\right)\right\},$$

$$\phi_2(q) \equiv \frac{3}{2\pi^2}q\Bigg\{\sqrt{\Lambda^2+M_{\rm q}^2-q^2}\left(-2\mu+\sqrt{F^2+\Lambda^2+M_{\rm q}^2-2Fq}\right) + (F-q)^2\log\left(\frac{\sqrt{F^2+\Lambda^2+M_{\rm q}^2-2Fq}+\sqrt{\Lambda^2+M_{\rm q}^2-q^2}}{|F-q|}\right)\Bigg\}.$$

Using the above expressions, the final results are summarized as follows:

If $F-\mu \le M_{\rm q} \le F+\mu \le \sqrt{\Lambda^2+M_{\rm q}^2}$,

$$\Phi_2(M,F,\mu) = \int_{M_{\rm q}}^{F+\mu} dq\, \phi_1(q). \tag{4.3}$$

If $M_{\rm q} \le F-\mu \le F+\mu \le \sqrt{\Lambda^2+M_{\rm q}^2}$,

$$\Phi_2(M,F,\mu) = \int_{F-\mu}^{F+\mu} dq\, \phi_1(q). \tag{4.4}$$

If $M_{\rm q} \le F-\mu \le b \le \sqrt{\Lambda^2+M_{\rm q}^2} \le F+\mu$,

$$\Phi_2(M,F,\mu) = \int_{F-\mu}^{b} dq\, \phi_1(q) + \int_{b}^{\sqrt{\Lambda^2+M_{\rm q}^2}} dq\, \phi_2(q). \tag{4.5}$$

If $F-\mu \le M_{\rm q} \le b \le \sqrt{\Lambda^2+M_{\rm q}^2} \le F+\mu$,

$$\Phi_2(M,F,\mu) = \int_{M_{\rm q}}^{b} dq\, \phi_1(q) + \int_{b}^{\sqrt{\Lambda^2+M_{\rm q}^2}} dq\, \phi_2(q). \tag{4.6}$$

If $F-\mu \le b \le M_{\rm q} \le \sqrt{\Lambda^2+M_{\rm q}} \le F+\mu$,

$$\Phi_2(M,F,\mu) = \int_{M_{\rm q}}^{\sqrt{\Lambda^2+M_{\rm q}^2}} d\,q\phi_2(q). \tag{4.7}$$

Finally, we discuss Φ_3 and Γ_3. We could derive the interval of integration easily in this case. The interval is re-expressed as

$$\Gamma_3 = \left\{ -\sqrt{\Lambda^2+M_{\rm q}^2-q^2} \le p_3 \le \sqrt{\Lambda^2+M_{\rm q}^2-q^2},\ M_{\rm q} \le q \le \sqrt{\Lambda^2+M_{\rm q}^2} \right\}.$$

After integrating over p_3, we obtain the final result:

$$\Phi_3(M,F,\mu) = -\frac{3}{2\pi^2}\sum_{\eta}\int_{M_{\rm q}}^{\sqrt{\Lambda^2+M_{\rm q}^2}} dq\, q \Bigg\{ \sqrt{\Lambda^2+M_{\rm q}^2-q^2}\sqrt{\Lambda^2+M_{\rm q}^2+2\eta qF+F^2} + (q+\eta F)^2 \log\left(\frac{\sqrt{\Lambda^2+M_{\rm q}^2-q^2}+\sqrt{\Lambda^2+M_{\rm q}^2+2\eta qF+F^2}}{|q+\eta F|} \right) \Bigg\}. \tag{4.8}$$

§5. Thermodynamic potential at finite temperature and density

We have discussed the thermodynamic potential at zero temperature in the previous section. In this section let us consider the thermodynamic potential at finite temperature. We define a thermodynamic potential at finite temperature, $\Omega(M,F,\mu,T)$, as follows:

$$\Omega(M,F,\mu,T) \equiv \mathcal{H}'_{\rm MFA} - \mu(\mathcal{N}_{\rm P}-\mathcal{N}_{\rm AP}) + V_{\rm vacuum} - TS, \tag{5.1}$$

$$\mathcal{H}'_{\rm MFA} \equiv \sum_{\vec{p},\eta,\tau,\alpha} E^{(\eta)}_{\vec{p}} \left(n^{(\eta)}_{\vec{p}} + \bar{n}^{(\eta)}_{\vec{p}} \right) + \frac{M^2}{4G_{\rm S}} + \frac{F^2}{2G_{\rm T}},$$
$$\mathcal{N}_{\rm P} \equiv \sum_{\vec{p},\eta,\tau,\alpha} n^{(\eta)}_{\vec{p}}, \mathcal{N}_{\rm AP} \equiv \sum_{\vec{p},\eta,\tau,\alpha} \bar{n}^{(\eta)}_{\vec{p}},$$
$$n^{(\eta)}_{\vec{p}} = \frac{1}{1+\exp\left((E^{(\eta)}_{\vec{p}} - \mu)/T\right)}, \bar{n}^{(\eta)}_{\vec{p}} = \frac{1}{1+\exp\left((E^{(\eta)}_{\vec{p}} + \mu)/T\right)},$$
$$V_{\rm vacuum} \equiv - \sum_{\vec{p},\eta,\tau,\alpha} E^{(\eta)}_{\vec{p}},$$

where T means temperature of the system, $n^{(\eta)}_{\vec{p}}$ and $\bar{n}^{(\eta)}_{\vec{p}}$ are the distribution functions for particle and anti-particle, respectively. Entropy S is given as follows:

$$S = - \sum_{\vec{p},\eta,\tau,\alpha} \Big\{ n^{(\eta)}_{\vec{p}} \log n^{(\eta)}_{\vec{p}} + (1 - n^{(\eta)}_{\vec{p}}) \log(1 - n^{(\eta)}_{\vec{p}}) + \bar{n}^{(\eta)}_{\vec{p}} \log \bar{n}^{(\eta)}_{\vec{p}} + (1 - \bar{n}^{(\eta)}_{\vec{p}}) \log(1 - \bar{n}^{(\eta)}_{\vec{p}}) \Big\}.$$

Using the following identities

$$n^{(\eta)}_{\vec{p}} \log n^{(\eta)}_{\vec{p}} = -n^{(\eta)}_{\vec{p}} \times \frac{E^{(\eta)}_{\vec{p}} - \mu}{T} - n^{(\eta)}_{\vec{p}} \log(1 - n^{(\eta)}_{\vec{p}}),$$
$$\bar{n}^{(\eta)}_{\vec{p}} \log \bar{n}^{(\eta)}_{\vec{p}} = -\bar{n}^{(\eta)}_{\vec{p}} \times \frac{E^{(\eta)}_{\vec{p}} + \mu}{T} - \bar{n}^{(\eta)}_{\vec{p}} \log(1 - \bar{n}^{(\eta)}_{\vec{p}}),$$

the above Ω can be recast into

$$\Omega = - \sum_{\vec{p},\eta,\tau,\alpha} \left\{ E^{(\eta)}_{\vec{p}} + T \log \left(1 + \exp \left(- \frac{E^{(\eta)}_{\vec{p}} - \mu}{T} \right) \right) + T \log \left(1 + \exp \left(- \frac{E^{(\eta)}_{\vec{p}} + \mu}{T} \right) \right) \right\} + \frac{M^2}{4G_{\rm S}} + \frac{F^2}{2G_{\rm T}}.$$

Let us change the summation over momenta into integration, and then let us introduce polar coordinates, $p_1 = p_T \cos\theta$, $p_2 = p_T \sin\theta$, so as to integrate. The interval of integration is obtained as follows:

$$-\sqrt{\Lambda^2 - p_T^2} \le p_3 \le \sqrt{\Lambda^2 - p_T^2}, \qquad 0 \le p_T \le \Lambda.$$

After integrating over θ, and summing over τ and α, we get the final form:

$$\Omega(M, F, \mu, T) = -\frac{3}{\pi^2} \sum_{\eta} \int_0^{\Lambda} dp_T \int_0^{\sqrt{\Lambda^2 - p_T^2}} dp_3 \, p_T \times \left\{ E^{(\eta)}_{\vec{p}} + T \log \left(1 + \exp \left(- \frac{E^{(\eta)}_{\vec{p}} - \mu}{T} \right) \right) + T \log \left(1 + \exp \left(- \frac{E^{(\eta)}_{\vec{p}} + \mu}{T} \right) \right) \right\}$$

$$+\frac{M^2}{4G_{\rm S}}+\frac{F^2}{2G_{\rm T}}. \tag{5.2}$$

§6. Numerical results and discussions

In this section we would like to discuss the thermodynamic potential at zero/finite temperature numerically. In order to evaluate it we use the three-momentum cutoff parameter and coupling constants in Table 1. Here, we adopt the strength of tensor interaction $G_{\rm T}$ as a rather small value compared with one used in our previous paper. The reason why we take $G_{\rm T}$ as a rather small value 11.0 GeV is that the vacuum polarization, namely the contribution of the negative energy sea, is taken into account. The detail discussion has already been given in appendix B in.[12)]

6.1. *Thermodynamic potential at zero temperature*

Let us discuss the thermodynamic potential at zero temperature. First we consider the chiral condensate M and the spin polarization F separately.

Figure 1 shows the thermodynamic potential in a special case where $F = 0$ and chiral limit, $m_0 = 0$. When the chemical potential has a value below 0.32 GeV and above 0.35 GeV, the thermodynamic potential has only one minimum. On the other hand, when $\mu = 0.33 \sim 0.34$ GeV, the thermodynamic potential has two local minima. This indicates that the order of phase transition for chiral condensate is the first.

Figure 2 shows the thermodynamic potential for $M = 0$ in chiral limit. When the chemical potential is small, the spin polarized phase does not exist. However, the chemical potential μ has a value above 0.42 GeV, the spin polarized phase appears. This figure shows that the order of phase transition for spin polarization is the

Table I. Parameter set

Λ/GeV	m_0/GeV	$G_{\rm S}$/GeV^{-2}	$G_{\rm T}$/GeV^{-2}
0.631	0.01	5.5	11.0

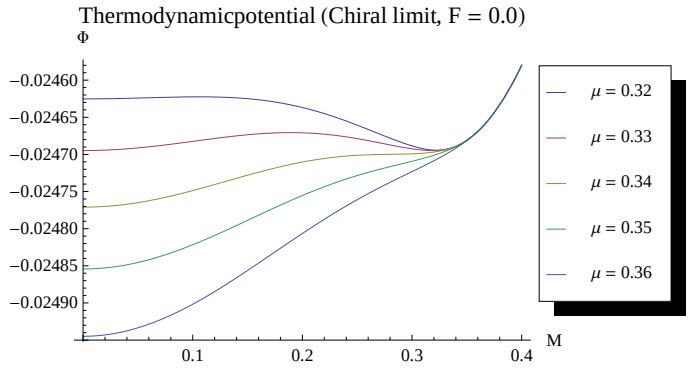

Fig. 1. The thermodynamic potential with $F = 0$ is depicted as a function of the constituent quark mass M in various quark chemical potential. .

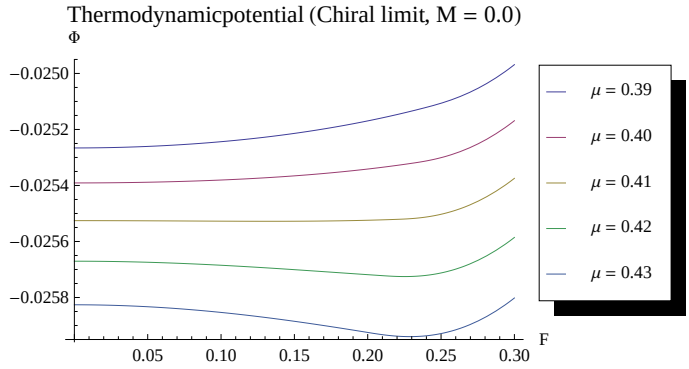

Fig. 2. The thermodynamic potential with $M = 0$ is depicted as a function of the spin polarized condensate F in various quark chemical potential. .

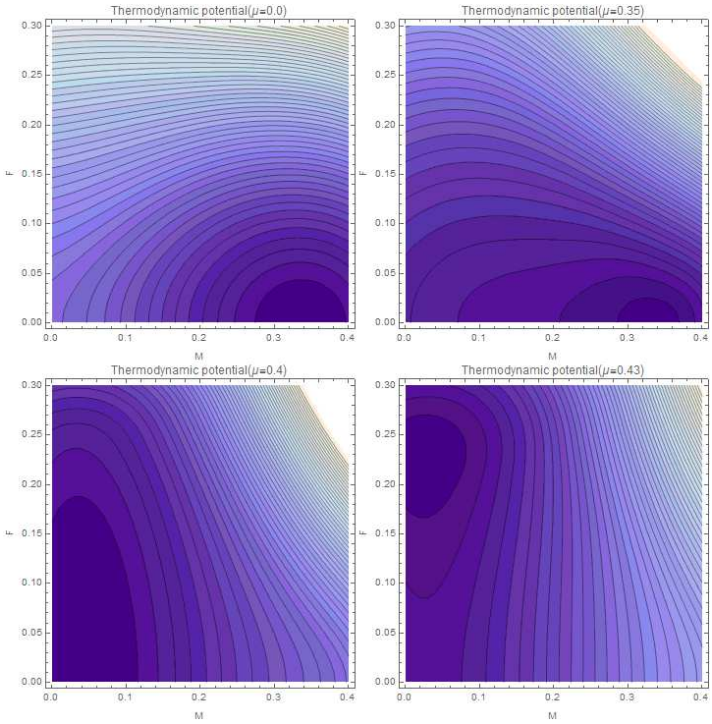

Fig. 3. The contour map of the thermodynamic potential is depicted as a function of the constituent quark mass M and the spin polarized condensate F in various quark chemical potential. Here, $m_0 = 0.01$ GeV is adopted. The horizontal and vertical axes represent M and F, respectively. As color becomes dark, the thermodynamic potential lowers.

second.

Next, let us consider M and F simultaneously. In Fig.3, the contour map for the thermodynamic potential with the current quark mass $m_0 = 0.01$ GeV, is depicted with various quark chemical potential. The horizontal and the vertical axes represent the constituent quark mass M and the spin polarized condensate F, respectively.

When μ is from 0 GeV to 0.33 GeV, the chiral condensed phase is realized. However, when μ reaches 0.40 GeV, it vanishes. If $\mu = 0.43$ GeV, spin polarized phase become to be realized. From these contour maps, it is indicated that two phases, the chiral condensed and the spin polarized phases, do not coexist.

6.2. *thermodynamic potential at finite temperature*

Let us consider the thermodynamic potential at finite temperature. First, let us treat two cases without M or F separately. Figure 4 shows the thermodynamic

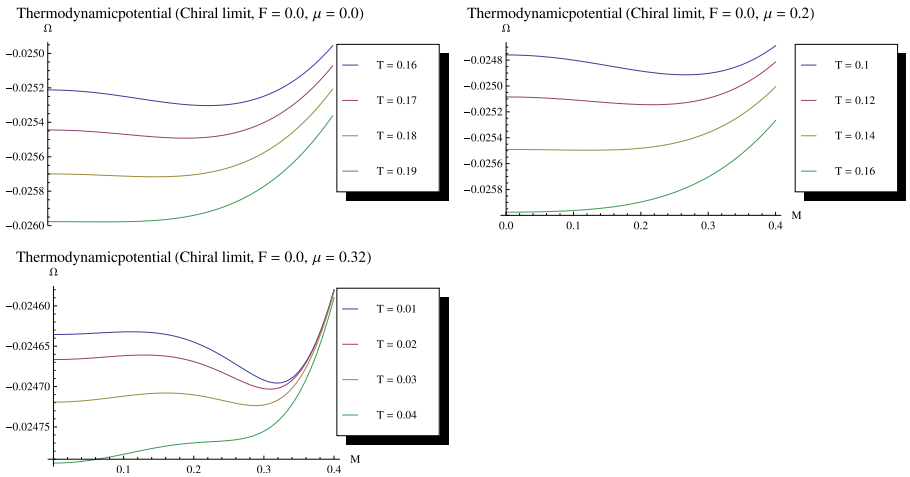

Fig. 4. The thermodynamic potential with $F = 0$ is depicted in the various temperature T in the case with the chemical potential $\mu = 0$, 0.2 GeV and 0.32 GeV. The horizontal and vertical axes represent the constituent quark mass M and the thermodynamic potential $\Omega(M, \mu, T)$, respectively.

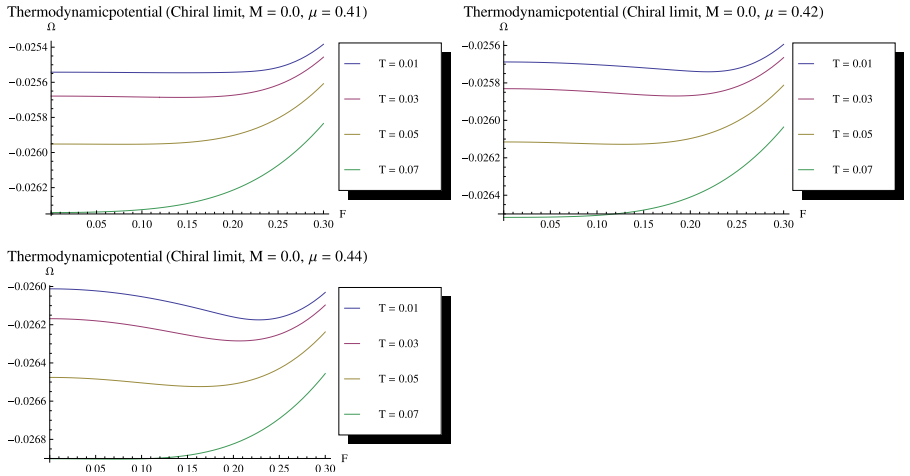

Fig. 5. The thermodynamic potential with $M = 0$ is depicted in the various temperature T in the case with the chemical potential $\mu = 0.41$ GeV, 0.42 GeV and 0.44 GeV. The horizontal and vertical axes represent the spin polarized condensate F and the thermodynamic potential $\Omega(F, \mu, T)$, respectively. .

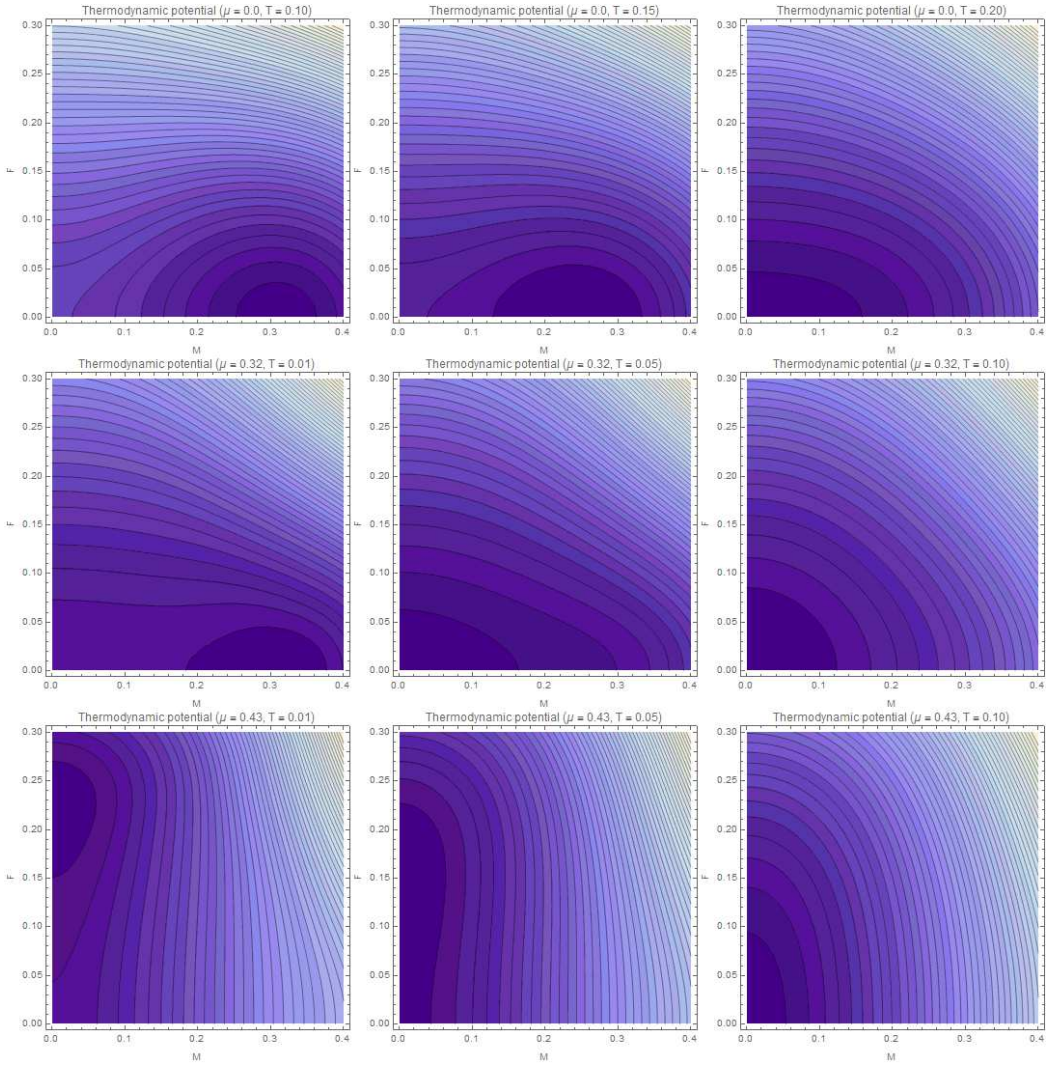

Fig. 6. The contour maps of the thermodynamic potential are depicted as a function of the constituent quark mass M and the spin polarized condensate F with various quark chemical potential and temperature. The horizontal and vertical axes represent M and F, respectively. Here, $m_0 = 0.01$ GeV is adopted.

potential at finite temperature for $F = 0$ in the case of chiral limit. If temperature T is not so high, the chiral condensed phase is realized. However, in high temperature region the chiral condensed phase disappears. It should be noted that, in the cases with $\mu = 0$ GeV and $\mu = 0.2$ GeV, the order of phase transition is the second, while the order of phase transition is the first in the case $\mu = 0.32$ GeV.

Secondly, we discuss the case for $M = 0$ at chiral limit. In Fig. 5, it is shown that the spin polarized phase is realized in only low temperature region. If temperature rises, the spin polarization disappears soon. In this case, the order of phase transition from the spin polarized phase to normal phase is the second order one.

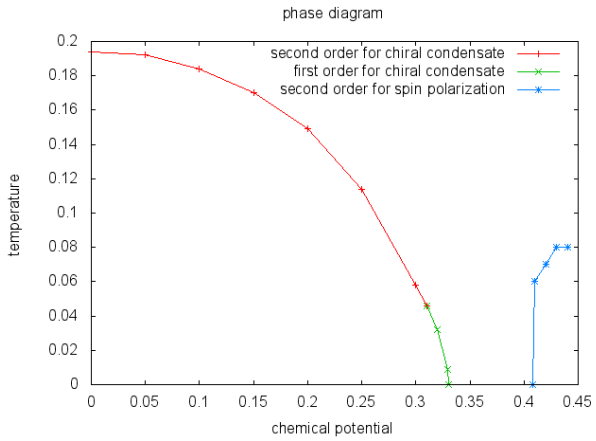

Fig. 7. The phase diagram in this model is depicted. The horizontal and vertical axes represent the quark chemical potential and the temperature, respectively.

Finally, let us consider M and F simultaneously. It is shown in Fig. 6 that there is only chiral condensate in small chemical potential and low temperature region. However, if the chemical potential or temperature becomes high, the chiral condensate disappears. In the large chemical potential and low temperature region, the spin polarized condensate appears. However, when we make temperature higher, it disappears soon. According to these contour maps, it may be concluded that two phases, namely the chiral condensed phase and the spin polarized phase, do not coexist at finite temperature.

6.3. *Phase diagram on T-μ plane*

As a summary, it is possible to show the regions of the chiral condensed phase and the spin polarized phase on the plane with the temperature T and the quark chemical potential μ and also to draw the phase boundary indicating the order of phase transition under the chiral limit, $m_0 = 0$. In Fig. 7, the phase diagram in this model is presented . As is shown in this phase diagram, the chiral condensed phase exists in the left side on the T-μ plane and the spin polarized phase exists in the right side. It is indicated that, for the boundary of the chiral condensed and the normal phases, there is the critical endpoint for the phase transition near $\mu = 0.31$ GeV and $T = 0.046$ GeV.

§7. Summary and concluding remarks

In this paper, it has been shown that the spin polarized phase appears in the region with the large quark chemical potential and low temperature by using the NJL model with tensor-type four-point interaction between quarks. We have consider the chiral condensate and spin polarized condensate simultaneously. In the rather low density, the chiral condensate exists and spin polarized condensate does not exist. As increasing the quark chemical potential, the chiral condensate disappears and further,

the spin polarized condensate becomes to appear. Thus, the spin polarized phase may exist in the high density and low temperature region in QCD phase diagram. However, it may be concluded that two phases do not coexist in this model under the parameter set adopted here.

It should be also indicated that the color superconducting phase may be realized in the region with high density and low temperature. However, at zero temperature, the spin polarized phase may be realized against the two-flavor color superconducting phase in the case with only two flavors.[9] It is interesting that the spin polarized phase survives or not at finite temperature against the color superconducting phase. It is one of future important problems to investigate. Furthermore, in this paper we do not consider the electromagnetic field at all. It is also important to study the electromagnetic properties in the spin polarized phase.

Acknowledgment

One of the authors (Y.T.) is partially supported by the Grants-in-Aid of the Scientific Research (No.26400277) from the Ministry of Education, Culture, Sports, Science and Technology in Japan.

Appendix A

—— Derivation of the thermodynamic potential at zero temperature from that at finite temperature ——

In this appendix we derive the thermodynamic potential at zero temperature from that at finite temperature. The thermodynamic potential at finite temperature is as follows:

$$\Omega(M,F,\mu,T) = -\sum_{\eta,\tau,\alpha}\int_{\vec{p}^2\le\Lambda^2}\frac{d^3p}{(2\pi)^3}\left\{E_{\vec{p}}^{(\eta)} + T\log\left(1+\exp\left(-\frac{E_{\vec{p}}^{(\eta)}-\mu}{T}\right)\right) + T\log\left(1+\exp\left(-\frac{E_{\vec{p}}^{(\eta)}+\mu}{T}\right)\right)\right\} + \frac{F^2}{2G_{\rm T}} + \frac{M^2}{4G_{\rm S}}. \tag{A·1}$$

If we assume $T \ll 1$, we can carry out the Taylor expansion for the logarithmic function in the following way:

$$T\log\left(1+\exp\left(-\frac{E_{\vec{p}}^{(\eta)}-\mu}{T}\right)\right) \to \begin{cases} -(E_{\vec{p}}^{(\eta)}-\mu) + T\exp\left(\frac{E_{\vec{p}}^{(\eta)}-\mu}{T}\right) & \text{for } E_{\vec{p}}^{(\eta)} \le \mu \\ T\exp\left(-\frac{E_{\vec{p}}^{(\eta)}-\mu}{T}\right) & \text{for } E_{\vec{p}}^{(\eta)} > \mu, \end{cases}$$

$$T\log\left(1+\exp\left(-\frac{E_{\vec{p}}^{(\eta)}+\mu}{T}\right)\right)\to T\exp\left(-\frac{E_{\vec{p}}^{(\eta)}+\mu}{T}\right). \tag{A·2}$$

Furthermore, in the region where $T\to 0$ we can reduce the above expressions into

$$T\log\left(1+\exp\left(-\frac{E_{\vec{p}}^{(\eta)}-\mu}{T}\right)\right)\to -(E_{\vec{p}}^{(\eta)}-\mu)\,\theta(\mu-E_{\vec{p}}^{(\eta)}),$$

$$T\log\left(1+\exp\left(-\frac{E_{\vec{p}}^{(\eta)}+\mu}{T}\right)\right)\to 0, \tag{A·3}$$

where $\theta(x)$ is the step function. Using these results, we could rewrite $\Omega(M,F,\mu,T)$ into

$$\Omega(M,F,\mu,T)\to -\sum_{\eta,\tau,\alpha}\int_{\vec{p}^2\le\Lambda^2}\frac{d^3p}{(2\pi)^3}\left\{E_{\vec{p}}^{(\eta)}-(E_{\vec{p}}^{(\eta)}-\mu)\,\theta(\mu-E_{\vec{p}}^{(\eta)})\right\} + \frac{F^2}{2G_{\rm T}}+\frac{M^2}{4G_{\rm S}}. \tag{A·4}$$

This expression is just one for the thermodynamic potential at zero temperature.

Appendix B

—— Derivation for the effective potential with functional method ——

Let us start with the following Lagrangian density in order to derive the effective potential by using the functional method:

$$\begin{aligned}\mathcal{L}&=\bar\psi(i\gamma^\mu\partial_\mu-m_0)\psi+G_S(\bar\psi\psi)^2-\frac{G_T}{2}(\bar\psi\gamma^1\gamma^2\tau_3\psi)(\bar\psi\gamma_1\gamma_2\tau_3\psi)\\&=\bar\psi(i\gamma^\mu\partial_\mu-m_0)\psi+G_S(\bar\psi\psi)^2+\frac{G_T}{2}(\bar\psi\Sigma_3\psi)^2,\end{aligned} \tag{B·1}$$

where we define $\Sigma_3\equiv -i\gamma^1\gamma^2$. In order to perform functional integral, let us introduce two auxiliary fields, M' and F, and use a relation of unit:

$$\begin{aligned}1=\int\mathcal{D}M'\mathcal{D}F\exp\left[-i\int d^4x\left\{M'+G_S(\bar\psi\psi)\right\}G_S^{-1}\left\{M'+G_S(\bar\psi\psi)\right\}\right]\\ \times\exp\left[-\frac{i}{2}\int d^4x\left\{F+G_T(\bar\psi\Sigma_3\psi)\right\}G_T^{-1}\left\{F+G_T(\bar\psi\Sigma_3\psi)\right\}\right].\end{aligned} \tag{B·2}$$

The generating functional Z for the Lagrangian density (B1) is given as follows:

$$Z\propto\int\mathcal{D}\bar\psi\mathcal{D}\psi\exp\left[i\int d^4x\left\{\bar\psi(i\gamma^\mu\partial_\mu-m_0)\psi+G_S(\bar\psi\psi)^2+\frac{G_T}{2}(\bar\psi\Sigma_3\psi)^2\right\}\right]. \tag{B·3}$$

Inserting the relation of unit, (B2), into Z and setting $M' = M/2$, we obtain

$$Z \propto \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}M\mathcal{D}F \exp\left[i \int d^4x \left\{ \bar{\psi}(i\gamma^\mu\partial_\mu - M_q - F\Sigma_3)\psi - \frac{M^2}{4G_S} - \frac{F^2}{2G_T} \right\} \right], \tag{B·4}$$

where we define $M_q \equiv m_0 + M$. Thus, we can integrate out with respect to ψ and $\bar{\psi}$ easily. After some calculations, we get

$$\begin{aligned} Z &\propto \int \mathcal{D}M\mathcal{D}F \ \mathrm{Det}(i\gamma^\mu\partial_\mu - M_q - F\Sigma_3) \exp\left[-i \int d^4x \left(\frac{M^2}{4G_S} + \frac{F^2}{2G_T} \right) \right] \\ &= \int \mathcal{D}M\mathcal{D}F \exp\left[\mathrm{Tr}\log\det(i\gamma^\mu\partial_\mu - M_q - F\Sigma_3) - i \int d^4x \left(\frac{M^2}{4G_S} + \frac{F^2}{2G_T} \right) \right], \end{aligned} \tag{B·5}$$

where in the second line the determinant, det, operates on gamma matrices. In order to compute the trace, Tr, we change to momentum space.

$$\begin{aligned} Z \propto \int \mathcal{D}M\mathcal{D}F \exp\left[iN_C N_F \int d^4x \frac{d^4p}{i(2\pi)^4} \log\det(\not{p} - M_q - F\Sigma_3) \right] \\ \times \exp\left[-i \int d^4x \left(\frac{M^2}{4G_S} + \frac{F^2}{2G_T} \right) \right], \end{aligned} \tag{B·6}$$

where N_C and N_F mean the number of color and flavor, respectively. Our next step is to calculate the determinant. We can do it as follows:

$$\begin{aligned} &\det(\not{p} - M_q - F\Sigma_3) \\ &= \det\gamma^0(\not{p} - M_q - F\Sigma_3) \\ &= \det\left[p^0 - \gamma^0(\vec{\gamma}\cdot\vec{p} + M_q + F\Sigma_3) \right] \\ &= \det\left[p^0 - \begin{pmatrix} E_{\vec{p}}^{(+)} & & & \\ & E_{\vec{p}}^{(-)} & & \\ & & -E_{\vec{p}}^{(+)} & \\ & & & -E_{\vec{p}}^{(-)} \end{pmatrix} \right] \\ &= (p^0 - E_{\vec{p}}^{(+)})(p^0 - E_{\vec{p}}^{(-)})(p^0 + E_{\vec{p}}^{(+)})(p^0 + E_{\vec{p}}^{(-)}) \ . \end{aligned} \tag{B·7}$$

Substituting the above result into Z in (B6), we obtain

$$\begin{aligned} Z \propto \int &\mathcal{D}M\mathcal{D}F \\ &\times \exp\left[iN_C N_F \int d^4x \frac{d^4p}{i(2\pi)^4} \log(p^0 - E_{\vec{p}}^{(+)})(p^0 - E_{\vec{p}}^{(-)})(p^0 + E_{\vec{p}}^{(+)})(p^0 + E_{\vec{p}}^{(-)}) \right] \\ &\times \exp\left[-i \int d^4x \left(\frac{M^2}{4G_S} + \frac{F^2}{2G_T} \right) \right]. \end{aligned} \tag{B·8}$$

For a little while, we consider the only contents of exponential in the second line in (B8):

$$\int \frac{d^4p}{i(2\pi)^4} \log(p^0 - E_{\vec{p}}^{(+)})(p^0 - E_{\vec{p}}^{(-)})(p^0 + E_{\vec{p}}^{(+)})(p^0 + E_{\vec{p}}^{(-)}). \tag{B·9}$$

Let us differentiate and integrate the above expression with respect to $E_{\vec{p}}^{(+)}$ and $E_{\vec{p}}^{(-)}$. As a result, (B9) can be recast into

$$\int \frac{d^4p}{i(2\pi)^4} \int dE_{\vec{p}}^{(+)} \left(\frac{1}{p^0 + E_{\vec{p}}^{(+)}} - \frac{1}{p^0 - E_{\vec{p}}^{(+)}} \right) + \int \frac{d^4p}{i(2\pi)^4} \int dE_{\vec{p}}^{(-)} \left(\frac{1}{p^0 + E_{\vec{p}}^{(-)}} - \frac{1}{p^0 - E_{\vec{p}}^{(-)}} \right). \tag{B·10}$$

We would like to discuss a system at finite temperature and density. So let us change the integration to the summation by using the Matsubara method as follows:

$$\int \frac{d^4p}{i(2\pi)^4} f(p^0, \vec{p}) \to T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} f(i\omega_n + \mu, \vec{p}), \tag{B·11}$$

where ω_n is the Matsubara frequency and μ is chemical potential. Using a formula

$$\lim_{\epsilon \to +0} \sum_n \frac{e^{i\omega_n \epsilon}}{i\omega_n - x} = \lim_{\epsilon \to +0} \frac{e^{i\omega_n \epsilon}}{e^{x/T} + 1} = \frac{1}{e^{x/T} + 1}, \tag{B·12}$$

we can calculate the summation following the standard technique. As a result, we obtain

$$\sum_{\eta=\pm} \int \frac{d^3p}{(2\pi)^3} \left[E_{\vec{p}}^{(\eta)} + \mu + T \log \left\{ 1 + \exp\left(-\frac{E_{\vec{p}}^{(\eta)} + \mu}{T} \right) \right\} + T \log \left\{ 1 + \exp\left(-\frac{E_{\vec{p}}^{(\eta)} - \mu}{T} \right) \right\} \right]. \tag{B·13}$$

Substituting the above result into Z, Z can be expressed as

$$Z \propto \int \mathcal{D}M\mathcal{D}F \exp i \left[N_C N_F \int d^4x \frac{d^3p}{(2\pi)^3} \sum_{\eta} \left(E_{\vec{p}}^{(\eta)} + T \log \left\{ 1 + \exp\left(-\frac{E_{\vec{p}}^{(\eta)} + \mu}{T} \right) \right\} + T \log \left\{ 1 + \exp\left(-\frac{E_{\vec{p}}^{(\eta)} - \mu}{T} \right) \right\} \right) - \int d^4x \left(\frac{M^2}{4G_S} + \frac{F^2}{2G_T} \right) \right], \tag{B·14}$$

where we neglect a constant term. In general, the effective action Γ and effective potential V are defined as follows:

$$Z = \exp\left(i\Gamma[M, F, T, \mu] \right), \qquad V[M, F, T, \mu] = -\frac{\Gamma[M, F, T, \mu]}{\int d^4x}. \tag{B·15}$$

Thus, finally, we obtain the effective potential V as

$$V[M,F,T,\mu] = -N_C N_F \int \frac{d^3p}{(2\pi)^3} \sum_{\eta} \left[E_{\vec{p}}^{(\eta)} + T \log \left\{ 1 + \exp\left(-\frac{E_{\vec{p}}^{(\eta)} + \mu}{T} \right) \right\} \right.$$
$$\left. + T \log \left\{ 1 + \exp\left(-\frac{E_{\vec{p}}^{(\eta)} - \mu}{T} \right) \right\} \right] + \frac{M^2}{4G_S} + \frac{F^2}{2G_T}. \qquad \text{(B·16)}$$

This is identical with the thermodynamic potential (5.2).

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