

The Schwinger Mechanism in (Anti) de Sitter Spacetimes

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Abstract

We present a short and novel derivation of the Schwinger mechanism for particle pair production in $1 + 1$ dimensional de Sitter and Anti de Sitter spacetimes. We work in the flat embedding space and derive the pair production rates in these spacetimes via instanton methods by directly exploiting the Davies-Unruh effect. The derivation is manifestly coordinate independent, and lends support to the deep connection between two conceptually disparate quantum phenomena - Schwinger effect and the Davies-Unruh effect.

1 Introduction

It is well known that in presence of an electric field, the vacuum is unstable and particle-antiparticle pair production occurs spontaneously [1]. In flat Minkowski space, the probability of this event occurring is given by

$$\Gamma \sim e^{\frac{-\pi M^2}{eE}} \quad (1)$$

where e is the particle charge and E is the constant external electric field. This probability is non-perturbative in the coupling “ e ” as can be seen from (1). This effect, due to Schwinger, has been extensively explored in various contexts - especially in non-trivial backgrounds to understand the effects of temperature and/or spacetime curvature on the Schwinger mechanism [2, 3]. For instance, the Schwinger mechanism has been applied to particle production and false vacuum decay in de Sitter space [3, 4]. The de Sitter case is interesting from the cosmological perspective, and the computation essentially revolves around finding the one-loop effective action, formally via the heat kernel method. The Schwinger mechanism has also been investigated in Anti de Sitter backgrounds, in the context of charged Reissner-Nordström (RN) black holes. RN black holes emit particles via Hawking radiation and since the near horizon geometry is $AdS_2 \times S^2$, there is an inevitable interplay between the Hawking process and Schwinger effect. [5, 6].

In addition to the Schwinger effect, quantum field theory predicts another remarkable and far reaching result - the Davies-Unruh effect [7]. Stated simply, according to this effect a uniformly accelerating observer perceives a thermal bath with temperature proportional to its acceleration. Besides the fact that under the influence of a constant electric field, charged particles move with constant acceleration $a = \frac{eE}{M}$, there does not seem to be any relation between the Schwinger and Davies-Unruh effects. However, the first hint of a relation between the two effects surfaces while studying Euclidean instantons in the context of Schwinger pair production. The instanton is characterized as a solution to the classical equations of motion, albeit in Euclidean time. Such a solution describes a closed circular orbit in flat Minkowski space. The corresponding action for an instanton in the presence of a constant electric field in flat space is given by $S_{Euc} = \frac{\pi M^2}{eE}$, and the proper Euclidean time to complete this closed orbit is given by Hamilton-Jacobi relation $\tau_{Euc} = \partial_M S_{Euc} = \frac{2\pi}{a} = \frac{1}{T_{DU}}$, where T_{DU} is the Davies-Unruh temperature. This points to a possible connection between Schwinger and the Davies-Unruh effect. In this paper we advance this connection by deriving Schwinger pair production rate using the Davies-Unruh effect, in both de Sitter and Anti de Sitter spacetimes. By working in the embedding space, we shall present a coordinate independent and unified treatment of deriving Schwinger effect in these spacetimes.

We shall work in the semiclassical approximation using instanton methods. Additionally, we restrict ourselves to $1 + 1$ dimensions for simplicity. The qualitative picture for the Schwinger mechanism is as follows. Initially, there is just vacuum and the electric field is E_{out} everywhere. Suddenly a particle-antiparticle pair is spontaneously created and the electric field drops to E_{in} between the particles. Subsequently the pair accelerate apart, converting the electric field value to E_{in} as they move away. The closed Euclidean worldline divides the space into “inside” and “outside” regions. On the worldline, the electric field is defined as the average sum of E_{out} and E_{in} . In the instanton method, the charged particle couples to the electromagnetic field and by complexifying the time coordinate, the on-shell action (the coupling term and the surface term cancel each other on-shell) for the particle-field system is given by [8]

$$S_E = M \int_{\Sigma} ds + \frac{1}{4} \int_{Vol} F^{\mu\nu} F_{\mu\nu} \quad (2)$$

We assume a constant external electric field. Following the work of Brown and Teitelboim [9], we define the following quantities.

$$\begin{aligned} E_{out} - E_{in} &= +e \\ E_{out} + E_{in} &= 2E_{on} \end{aligned} \quad (3)$$

Using equations (2) and (3), the relevant instanton action is therefore given by

$$\begin{aligned} S_E[instanton] &= S_E^{in} - S_E^{out} \\ &= M \int_{\Sigma} ds + \frac{1}{2} (E_{in}^2 - E_{out}^2) \int_{Vol} \\ &= M \int_{\Sigma} ds - e E_{on} \int_{Vol} \end{aligned} \quad (4)$$

Now consider a constant external electric field in dS_2/AdS_2 space given by

$$F_{\mu\nu} = -E_{out} \sqrt{-g} \epsilon_{\mu\nu} \quad (5)$$

where $\epsilon_{01} = -\epsilon_{10} = 1$. Under the action of the above field, charged particles trace out worldlines according to the equations of motion

$$a_2^i = \frac{e F_{on}^{ij} u_j}{M} \quad (6)$$

where a_2^i is the 2-acceleration of the particle, F_{on}^{ij} the field strength defined on the worldline, and u^i the usual 2-velocity. Using (5) and (6), and considering the fact that $u^i u_i = -1$, the magnitude of 2-acceleration is given by

$$a_2^2 = g_{ij} u^i u^j = \frac{e^2 E_{on}^2}{M^2} \quad (7)$$

As defined before, E_{on} is the electric field on the worldline.

2 Schwinger Effect in dS_2

Consider de Sitter space with scale R defined by the hyperboloid

$$-X_0^2 + X_1^2 + X_2^2 = R^2 \quad (8)$$

From the perspective of embedding space, this hyperboloid exists in 2+1 dimensional flat Minkowski space. Consider a constant electric field along the direction X_1 . The charged particle then accelerates along this direction keeping the coordinate $X_2 = X_c = \text{constant}$. Therefore, the trajectory in embedding space is given by

$$-X_0^2 + X_1^2 = R^2 - X_c^2 \quad (9)$$

suggesting that in embedding space the particle has an accelerating trajectory with “3-acceleration” given by

$$a_3^2 = 1/(R^2 - X_c^2) \quad (10)$$

In an elegant paper by Deser and Levin [10], it was shown that acceleration in embedding space is what determines the Davies-Unruh temperature in the target space as well. This equivalence of temperature is the key step. Therefore, in our case of dS_2 , the Davies-Unruh temperature in terms of acceleration is given by

$$\begin{aligned} a_3 &= \sqrt{\frac{1}{R^2} + a_2^2} \\ &= 2\pi T_{Unruh} \end{aligned} \quad (11)$$

Therefore, from equations (7), (10) and (11), we have

$$X_c = \frac{e E_{on} R^2}{\sqrt{M^2 + e^2 E_{on}^2 R^2}} \quad (12)$$

We can now compute the instanton action (4) in the embedding space. We first complexify the time coordinate as $X_0 \rightarrow iX_{0E}$, and thus from Eq.(9) the worldline radius becomes $R_0 = \sqrt{R^2 - X_c^2} = \frac{MR}{\sqrt{M^2 + e^2 E_{on}^2 R^2}}$. The instanton action (4) for dS_2 can be calculated as

$$\begin{aligned} S_E[instanton] &= 2\pi MR_0 - eE_{on} \iiint_D \delta(\sqrt{X_{0E}^2 + X_1^2 + X_2^2} - R) dX_2 dX_1 dX_{0E} \\ &= 2\pi R[\sqrt{M^2 + e^2 E_{on}^2 R^2} - eE_{on}R] \end{aligned} \quad (13)$$

where the domain of integration $D : -\sqrt{R^2 - X_c^2} \leq X_{0E} \leq \sqrt{R^2 - X_c^2}$; $-\sqrt{R^2 - X_c^2 - X_{0E}^2} \leq X_1 \leq \sqrt{R^2 - X_c^2 - X_{0E}^2}$; $0 \leq X_2 \leq \sqrt{R^2 - X_1^2 - X_{0E}^2}$. Using (13), the Schwinger pair creation rate is given by

$$\Gamma_{dS_2} \sim e^{-2\pi R[\sqrt{M^2 + e^2 E_{on}^2 R^2} - eE_{on}R]} \quad (14)$$

We observe that even in the absence of any electric field, pair production occurs with rate $\Gamma_{dS_2} \sim e^{-2\pi MR}$. This feature is the well known cosmological particle production of heavy fields in de Sitter space [3, 4, 8].

3 Schwinger Effect in AdS₂

We now turn to Anti de Sitter space. Consider AdS₂ with scale R defined by

$$-X_0^2 + X_1^2 - X_2^2 = -R^2 \quad (15)$$

From the perspective of embedding space, this hyperboloid exists in 1+2 dimensional flat Minkowski space. This immediately raises a concern regarding closed time-like curves since both X_0 and X_2 behave like time coordinates. However, in our present work we sidestep this issue by taking X_0 as the “relevant” time coordinate. We now consider a constant electric field along the direction X_1 . The charged particle then accelerates along this direction keeping the coordinate $X_2 = X_c = \text{constant}$. Therefore, the trajectory is given by

$$-X_0^2 + X_1^2 = X_c^2 - R^2 \quad (16)$$

This implies that in embedding space the particle has an accelerating trajectory with “3-acceleration” given by

$$a_3^2 = 1/(X_c^2 - R^2) \quad (17)$$

Again following [10], the Davies-Unruh temperature in AdS₂ in terms of acceleration is given by

$$\begin{aligned} a_3 &= \sqrt{\frac{-1}{R^2} + a_2^2} \\ &= 2\pi T_{Unruh} \end{aligned} \quad (18)$$

Therefore, from equations (7), (17) and (18), we have

$$X_c = \frac{eE_{on}R^2}{\sqrt{e^2E_{on}^2R^2 - M^2}} \quad (19)$$

The instanton action can now be computed as follows. As in the case of dS₂, the time coordinate is complexified as $X_0 \rightarrow iX_{0E}$, and thus from Eq.(16) the worldline radius becomes $R_0 = \sqrt{X_c^2 - R^2} = \frac{MR}{\sqrt{e^2E_{on}^2R^2 - M^2}}$. Therefore the instanton action (4) for AdS₂ can be calculated in the embedding space as

$$S_E[instanton] = 2\pi MR_0 - eE_{on} \iiint_D \delta(\sqrt{X_2^2 - X_1^2 - X_{0E}^2} - R) dX_2 dX_1 dX_{0E} \quad (20)$$

where now the domain of integration is $D : -\sqrt{X_c^2 - R^2} \leq X_{0E} \leq \sqrt{X_c^2 - R^2}; -\sqrt{X_c^2 - X_{0E}^2 - R^2} \leq X_1 \leq \sqrt{X_c^2 - X_{0E}^2 - R^2}; 0 \leq X_2 \leq \sqrt{X_1^2 + X_{0E}^2 + R^2}$.

Evaluating the above integral near the neighborhood $\frac{eE_{on}R}{\sqrt{e^2E_{on}^2R^2 - M^2}} \approx 1$, we get

$$S_E[instanton] \approx 2\pi R[eE_{on}R - \sqrt{e^2E_{on}^2R^2 - M^2}] \quad (21)$$

Therefore, the Schwinger pair creation rate is given by

$$\Gamma_{AdS_2} \sim e^{-2\pi R[eE_{on}R - \sqrt{e^2E_{on}^2R^2 - M^2}]} \quad (22)$$

However, unlike in the case of dS₂, there exists a critical threshold electric field $E_{cr}^2 = \frac{M^2}{e^2R^2}$ in AdS₂, below which there is no pair creation [5, 6]. This is due to the confining effects of AdS.

4 Conclusion

The derivation presented here is self-contained but a few concluding remarks are in order. In our calculation, we have ignored the gravitational backreaction, and have also implicitly

assumed that mass of the instanton is much greater than curvature scales, i.e. $M^2 \gg \frac{1}{R^2}$. If one relaxes this assumption, the instanton actions and pair production rates (13), (14), (21) and (22) will be modified by simply shifting the mass squared term to $M^2 \rightarrow M^2 \mp \frac{1}{4R^2}$ for de Sitter and Anti de Sitter spaces respectively. This shift can be understood by looking at the quadratic Casimirs for de Sitter and Anti de Sitter groups, $SO(1,2)$ and $Sl(2,\mathbb{R})$, respectively. Though operationally we have used the kinematic equalities in Eqs.(11) and (18) to derive our results, it is noteworthy to belabor on their interpretation. The first equality in both these equations is a classical relation, and in fact, the relation between accelerations in any embedding space and a submanifold of it (a_D and a_{D-1}) is a purely differential geometric result (see [11] for a pedagogical discussion). However, the appearance of temperature, which has its origins in quantum statistics, presents a conceptually different interpretation of Eqs.(11) and (18). There is no general theorem that establishes the equivalence of Unruh temperature for a general embedding space and its submanifold. Such an equivalence has only been established for (Anti) de Sitter spacetimes [10], where the detector response for target spaces as well as their corresponding embedding spaces was explicitly calculated using quantum field theory. We therefore posit that Eqs.(11) and (18) also hold true at the quantum level, thereby latently encapsulating the Davies-Unruh effect in our derivation.

Summarizing, our present derivation of the Schwinger pair creation rate rests on two ingredients - 1) Equivalence of Euclidean action at the level of both embedding as well as target space. 2) The Davies-Unruh effect, and the equivalence of temperature at the level of both embedding as well as target space. This strongly suggests that instead of working in target space, it should be possible to demonstrate Schwinger pair creation in AdS/dS by employing a quantum field theoretic approach in their flat embedding spaces. However, unlike in the case of de Sitter, one may run into technical difficulties while attempting a field theoretic derivation in the case Anti de Sitter spacetime. This is due to the presence of two time-like coordinates at the embedding level. Therefore, additional assumptions and boundary conditions must be specified for the derivation to go through. Additionally, while the Davies-Unruh effect is essentially thermodynamic in nature, the Schwinger effect is not. The fact that the Davies-Unruh effect was used to derive the pair production rate alludes to a deep connection between the two phenomena. Though this connection has been explored previously in the literature [3, 5, 12, 13], we believe our present derivation to be the most concrete realization of this connection by invoking the Davies-Unruh effect.

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References

- [1] J. Schwinger, Phys. Rev. D **82**(1951) 664
- [2] A. R. Brown, arXiv:1512.05716
- [3] R. G. Cai and S. P. Kim, JHEP **09** (2014) 072
- [4] M. B. Frob, J. Garriga, S. Kanno, M. Sasaki, J. Soda, T. Tanaka, and A. Vilenkin, JCAP **1404** (2014) 009
- [5] S. P. Kim, H. K. Lee and Y. Yoon, arXiv:1503.00218
- [6] B. Pioline and J. Troost, JHEP **03** (2005) 043
- [7] P. C. W. Davies, J.Phys. A **8** (1975) 609
- [8] J. Garriga, Phys. Rev. D **49** (1994) 6327
- [9] J. D. Brown and C. Teitelboim, Nucl. Phys. B **297** (1988) 787
- [10] S. Deser and O. Levin, Class. Quant. Grav. **14** (1997) L163
- [11] D. Kothawala, Am. J. Phys. **79** (2011) 6
- [12] N. P. Myhrvold, Ann. of Phys. **160** (1985) 102
- [13] R. Parentani and S. Massar, Phys. Rev. D **55**(1997) 3603