Comparison of Qubit and Qutrit like Entangled Squeezed and Coherent States of Light

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Abstract

Squeezed state of light is one of the important subjects in quantum optics which is generated by optical nonlinear interactions. In this paper, we especially focus on qubit like entangled squeezed states (ESS's) generated by beam splitters, phase-shifter and cross Kerr nonlinearity. Moreover the Wigner function of two-mode qubit and qutrit like ESS are investigated. We will show that the distances of peaks of Wigner functions for two-mode ESS are entanglement sensitive and can be a witness for entanglement. Unlike the qubit cases, it is shown that there are qutrit like ESS violating monogamy inequality. These trends are compared with those obtained for qubit and qutrit like entangled coherent state (ECS).

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1 Introduction

Squeezed states are a general class of minimum-uncertainty states, for which the noise in one quadrature is reduced compared with a coherent state. The squeezed state of the electromagnetic field can be generated in many optical processes [1, 2, 3]. The first two experiments used third-order nonlinear optical media. Squeezed state finds a wide range of applications in quantum information processing [4]. A superposition of odd photon number states for quantum information networks has been generated by photon subtraction from a squeezed vacuum state produced by a continuous wave optical parametric amplifier [5]. In Ref. [6], orthogonal Bell states with entangled squeezed vacuum states were constructed and a scheme for teleportation a superposition of squeezed states based on the Bell state measurement were presented. An analysis of squeezed single photon states as a resource for teleportation of coherent state qubits was investigated in Ref. [7]. In [8] it was shown that non-Gaussian entangled states are good resources for quantum information processing protocols, such as, quantum teleportation. The problem of generating ESS's was discussed in [9, 10, 11, 12, 13, 14]. The new physical interpretation of the generalized two-mode squeezing operator has been studied in [15] which was useful to design of optical devices for generating various squeezed states of light.

Coherent states, originally introduced by Schrodinger in 1926 [16]. In recent years, there has been the considerable interest in studying multi-mode quantum states of radiation fields because they have widely role in quantum information theory [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Entanglement concentration for W-type entangled coherent states was investigated in Ref.[32]. Generation of multipartite ECS's and entanglement of multipartite states constructed by linearly independent coherent states are investigated in [33, 34]. In [35] it was considered the production and entanglement properties of the generalized balanced

N-mode Glauber coherent states of the form

$$|\Psi_N^{(d)}\rangle_c = \frac{1}{\sqrt{M_N^{(d)}}} \sum_{i=0}^{d-1} f_i \underbrace{|\alpha_i\rangle \cdots |\alpha_i\rangle}_{\text{N modes}}, \tag{1.1}$$

which is a general form of the balanced two-mode ECS $|\Psi^{(2)}\rangle_c = \frac{1}{\sqrt{M_c^{(2)}}}(|\alpha\rangle|\alpha\rangle + f|\beta\rangle|\beta\rangle)$ and $|\Psi^{(3)}\rangle_c = \frac{1}{\sqrt{M_c^{(3)}}}(|\alpha\rangle|\alpha\rangle + f_1|\beta\rangle|\beta\rangle + f_2|\gamma\rangle|\gamma\rangle)$. By assumption that the coherent states are linearly independent, these states recast in two qubit and qutrit form respectively. Then the entanglement of this states was evaluated by concurrence measure. In Ref. [36], the effect of noise on entanglement between modes 1 and 2 in qubit and qutrit like ECS's was investigated.

In 1932, Wigner introduced a distribution function in mechanics that permitted a description of mechanical phenomena in a phase space [37, 38]. Wigner functions have been especially used for describing the quadratures of the electrical field with coherent and squeezed states or single photon states [39, 40, 41]. The Bell inequality based on a generalized quasi probability Wigner function and its violation for single photon entangled states and two-mode squeezed vacuum states were investigated in Ref. [42]. In [43] the negativity of the Wigner function was discussed as a measure of the non-classicality which is a reason why the Wigner function can not be regarded as a real probability distribution but it is a quasi-probability distribution function. This character is a good indication of the possibility of the occurrence of nonclassical properties of quantum states [44]. The Wigner function of two-mode qubit and qutrit like ECS's was investigated in [45].

In this paper we consider two-mode qubit like ESS $|\Psi^{(2)}\rangle_s = \frac{1}{\sqrt{M_s^{(2)}}}(|\xi\xi\rangle + f|\eta\eta\rangle)$. As two squeezed states are in general nonorthogonal, they span a two dimensional qubit like Hilbert space $\{|0\rangle, |1\rangle\}$. Therefore, two-mode squeezed state $|\Psi^{(2)}\rangle_s$ can be recast in two qubit form. Moreover as an example we introduce a method for producing qubit like ESS using kerr medium and beam splitters [12]. The same argument can be formulated for other two-mode squeezed states such as $|\Psi^{(3)}\rangle_s = \frac{1}{\sqrt{M_s^{(3)}}}(|\xi\xi\rangle + f_1|\eta\eta\rangle + f_2|\tau\tau\rangle)$, in which the set $\{|\xi\rangle, |\eta\rangle, |\tau\rangle\}$ are linearly independent and span the three dimensional qutrit like Hilbert space $\{|0\rangle, |1\rangle, |2\rangle\}$, implying that $|\Psi^{(3)}\rangle_s$ can be recast in two qutrit form. The entanglement of the state $|\Psi^{(2)}\rangle_s$ can be calculated by concurrence measure introduced by Wootters [46, 47] and in the same manner, its generalized version [48] can be used to obtain the entanglement of $|\Psi^{(3)}\rangle_s$. Moreover we investigate the Wigner quasi-probability distribution function for qubit and qutrit like ESS. For both qubit like ESS and ECS the distance of peaks in Wigner function can be an evidence for amount of entanglement. While for qutrit like ESS this argument seems to be rather pale. On the other hand, the monogamy inequality problem for multi-qutrit like ESS and ECS are discussed and it is shown that there are qutrit like states violating monogamy inequality.

The outline of this paper is as follows: In section 2 we investigate the entanglement of two-mode qubit like ESS and compare it with ECS. Moreover the Wigner quasi-probability distribution function for qubit like ESS is studied in this section. The behavior of Wigner function and monogamy inequality for multi-qutrit like ESS are discussed in section 3. Our conclusions are summarized in section 4.

2 Two-mode Qubit like ESS's

A rather more exotic set of states of the electromagnetic field are the squeezed states. The squeezed state of light is two-photon coherent state that photons will be created or destroyed in pairs. They may be generated through the action of a squeeze operator defined as

$$\hat{S}(\xi) = \exp[\frac{1}{2}(\xi^* \hat{a}^2 - \xi \hat{a}^{\dagger 2})], \tag{2.2}$$

where $\xi = r_1 e^{i\theta_1}$, and r_1 is known as the squeeze parameter and $0 \leq r_1 < \infty$ and $0 \leq \theta_1 \leq 2\pi$. Note the squeeze operator is unitary and $\hat{S}^\dagger(\xi) = \hat{S}^{-1}(\xi) = \hat{S}(-\xi)$. The operator $\hat{S}(\xi)$ is a kind of two-photon generalization of the displacement operator used to define the usual coherent states of a single-mode field. Acting squeeze operator on vacuum states would create some sort of two photon coherent states:

$$|\xi\rangle = \hat{S}(\xi)|0\rangle, \tag{2.3}$$

namely squeezed states. One can write the squeezed states in terms of Fock states $|n\rangle$ as

$$|\xi\rangle = \frac{1}{\sqrt{\cosh r_1}} \sum_{n=0}^{\infty} (-e^{i\theta_1} \tanh r_1)^n \frac{\sqrt{(2n)!}}{2^n n!} |2n\rangle. \tag{2.4}$$

The overlap of two squeezed state reads

$$\langle\xi|\eta\rangle = \frac{1}{\sqrt{\cosh r_1 \cosh r_2 (1 - e^{-i\Delta\theta} \tanh r_1 \tanh r_2)}}, \tag{2.5}$$

in which $\xi = r_1 e^{i\theta_1}$, $\eta = r_2 e^{i\theta_2}$ and $\Delta\theta = \theta_2 - \theta_1$.

2.1 Entanglement of Two-Mode Qubit like ESS

Let us consider two-mode qubit like ESS as

$$|\Psi^{(2)}\rangle_s = \frac{1}{\sqrt{M_s^{(2)}}} (|\xi\xi\rangle + f|\eta\eta\rangle), \tag{2.6}$$

where f, ξ and η are generally complex numbers and $M_s^{(2)}$ is a normalization factor, i.e.

$$M_s^{(2)} = 1 + |f|^2 + 2Re(fp^2), \tag{2.7}$$

in which $p = \langle\xi|\eta\rangle$ is equal to Eq.(2.5). Note that we used the superscript (2) for qubit-like states to distinguish it from that of qutrit like states in the next section and superscript s is referred to squeezed states to distinguish it from coherent states. Two nonorthogonal squeezed states $|\xi\rangle$ and $|\eta\rangle$ are assumed to be linearly independent and span a two-dimensional subspace of the Hilbert space $\{|0\rangle, |1\rangle\}$. The state $|\Psi^{(2)}\rangle_s$ is in general an entangled state. To show this avowal we use a measure of entanglement called concurrence which is introduced by Wooters [46, 47]. For any two qubit pure state in the form $|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$, the concurrence is defined as $C = 2|a_{00}a_{11} - a_{01}a_{10}|$. By defining the orthonormal basis as

$$|0\rangle = |\xi\rangle, \quad |1\rangle = \frac{1}{\sqrt{1 - |p|^2}} (|\eta\rangle - p|\xi\rangle), \tag{2.8}$$

the state $|\Psi^{(2)}\rangle_s$ is reduced to the state $|\psi\rangle$. Therefore the concurrence is obtained as

$$C^{(2)} = \frac{2|f|(1 - \frac{1}{\cosh r_1 \cosh r_2 \sqrt{1 + \tanh^2 r_1 \tanh^2 r_2 - 2\cos\Delta\theta \tanh r_1 \tanh r_2}})}{1 + f^2 + \frac{2f(1 - \cos\Delta\theta \tanh r_1 \tanh r_2)}{\cosh r_1 \cosh r_2 ((1 - \cos\Delta\theta \tanh r_1 \tanh r_2)^2 + \sin^2\Delta\theta \tanh^2 r_1 \tanh^2 r_2)}}, \tag{2.9}$$

where for simplicity we assume that f to be a real number. The behavior of concurrence as a function of f is shown in figure 1 in which for comparison we also plot the behavior of the concurrence of two-mode qubit like ECS, $|\Psi^{(2)}\rangle_c = \frac{1}{\sqrt{M_c^{(2)}}}(|\alpha\alpha\rangle + f|\beta\beta\rangle)$ (for further details see [35]). A comparison between full and dashed line in figure 1 shows that for $f > 0$ the

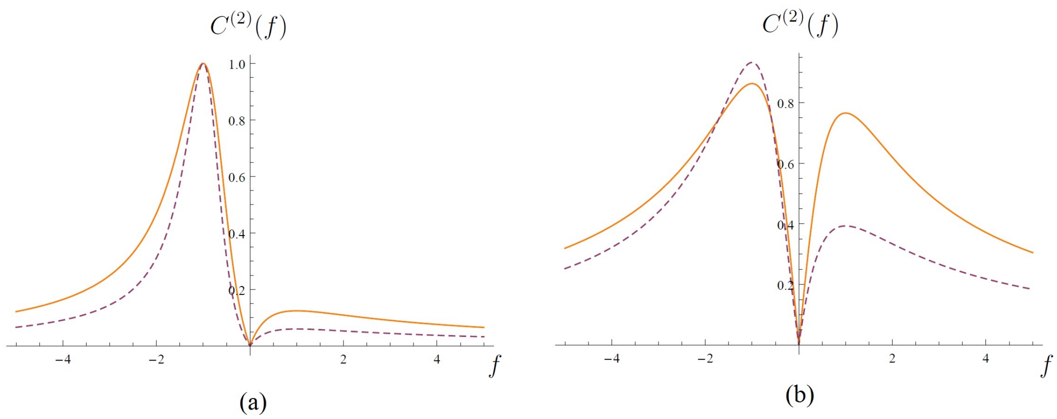


Figure 1: (Color online) Concurrence of $|\Psi^{(2)}\rangle_s$ (dashed line) and $|\Psi^{(2)}\rangle_c$ (full line) as a function of f for $r_1 = 1.5$: (a) $r_2 = 2$ and $\Delta\theta = 0$ and (b) $r_2 = 0.5$ and $\Delta\theta = 1.68\pi$.

concurrence of squeezed state is less than the concurrence of coherent state. On the other hand for $f < 0$, there are r_1, r_2 and $\Delta\theta$ for which the cross over occurs. Moreover figure shows that only for $f = 0$ the state $|\Psi^{(2)}\rangle_s$ is separable which is confirmed by Eq.(2.9). If we assume that all parameters are real and $f = 1$ the concurrence (2.9) is rewritten as

$$C^{(2)}(\Delta_s) = \frac{1 - \text{sech}[\Delta_s]}{1 + \text{sech}[\Delta_s]}, \tag{2.10}$$

where $\Delta_s = \xi - \eta$. This equation shows that the concurrence is a monotone function of Δ_s (see figure 2). If $\Delta_s \to \infty$, the concurrence tends to its maximum ($C^{(2)}_{max} = 1$), while for small separation (i.e. $\Delta_s \to 0$) the concurrence tends to zero and the state becomes separable. Such ESS can be generated by beam splitters (BS), phase-shifter and cross Kerr nonlinearity [12]. The interaction Hamiltonian for a cross Kerr nonlinearity is $\hat{H} = \hbar k \hat{n}_a \hat{n}_b$ where $\hat{n}_a$ and $\hat{n}_b$ are photon number operators of mode a and mode b, respectively. k is proportional to the

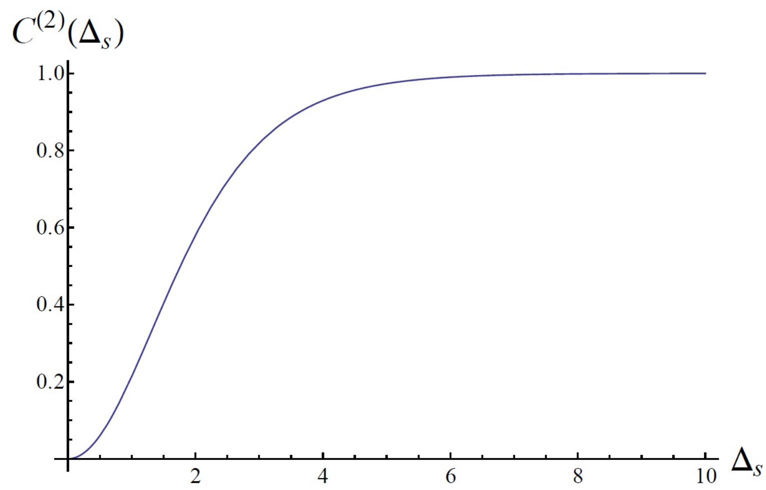


Figure 2: (Color online) Concurrence of $|\Psi^{(2)}\rangle_s$ as a function of $\Delta_s = \xi - \eta$.

third-order nonlinear susceptibility $\chi^{(3)}$. The time evolution operator is

$$\hat{U}(\tau) = e^{-i\tau\hat{n}_a\hat{n}_b}, \tag{2.11}$$

in which $\tau = kt = k(l/v)$, here l is the length of Kerr medium (KM) and v is the velocity of light in the Kerr medium. If we assume that mode a is initially in a squeezed vacuum state $|\xi\rangle_a$, and choose $\tau = \frac{\pi}{2}$, then we will obtain the superpositions of squeezed vacuum states $|\varphi\rangle \sim |\xi\rangle_a \pm |-\xi\rangle_a$. Assume that the input state to $BS1$ is $|1\rangle_b|0\rangle_c$, the output state becomes $\frac{1}{\sqrt{2}}(|10\rangle_{bc} + i|01\rangle_{bc})$ (see figure 3). On the other hand the state $|\xi\rangle_a$ after transmitting from Kerr medium and the phase shifter $\hat{\mathcal{P}} = e^{i\hat{N}\theta}$ is reduced to

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|\xi e^{-2i\tau}\rangle_a|10\rangle_{bc} + ie^{i\theta}|\xi\rangle_a|01\rangle_{bc}). \tag{2.12}$$

Now let mode b interacts with mode a' (in a squeezed vacuum state $|\eta\rangle_{a'}$), then the state $|\psi_1\rangle$ after the Kerr medium $KM2$ and $BS2$ is transformed to

$$\begin{aligned} |\psi_2\rangle = {} & \tfrac{1}{2}(|\xi e^{-2i\tau}\rangle_a|\eta e^{-2i\tau'}\rangle_{a'} - e^{i\theta}|\xi\rangle_a|\eta\rangle_{a'})|10\rangle_{bc} \\ & + \tfrac{i}{2}(|\xi e^{-2i\tau}\rangle_a|\eta e^{-2i\tau'}\rangle_{a'} + e^{i\theta}|\xi\rangle_a|\eta\rangle_{a'})|01\rangle_{bc}. \end{aligned} \tag{2.13}$$

Now if one of the detectors, D_c or D_b, fires then we have the entangled states

$$(|\xi e^{-2i\tau}\rangle_a|\eta e^{-2i\tau'}\rangle_{a'} - e^{i\theta}|\xi\rangle_a|\eta\rangle_{a'}), \tag{2.14}$$

or

$$i(|\xi e^{-2i\tau}\rangle_a|\eta e^{-2i\tau'}\rangle_{a'} + e^{i\theta}|\xi\rangle_a|\eta\rangle_{a'}), \tag{2.15}$$

in which by choosing different parameters τ, τ' and θ, different entangled states can be obtained [12]. By taking $\tau = \tau' = \frac{\pi}{2}$ and $\theta = 0$ we have $|\psi^{(2)}\rangle_s \sim |-\xi\rangle_a|-\eta\rangle_{a'} - |\xi\rangle_a|\eta\rangle_{a'}$ which is

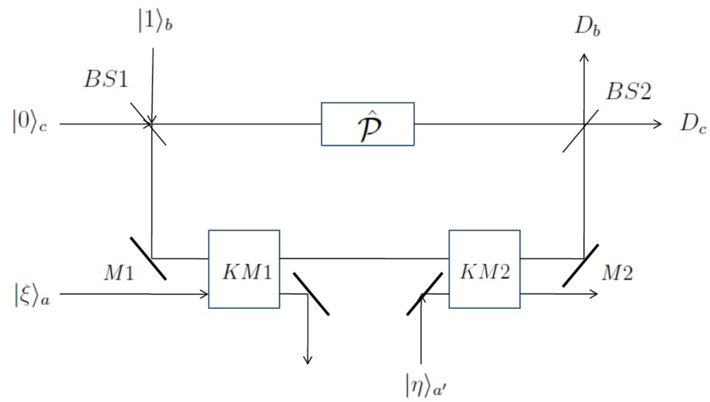


Figure 3: (Color online) Experimental set up for generation ESS.

in general an entangled state. Similarly two nonorthogonal squeezed states $\{|\xi\rangle, |-\xi\rangle\}$ and $\{|\eta\rangle, |-\eta\rangle\rangle$ are assumed to be linearly independent and span a two-dimensional subspace of the Hilbert space $\{|0\rangle, |1\rangle\}$ which are defined by

$$\begin{aligned} &|0\rangle = |\xi\rangle, \quad |1\rangle = \tfrac{|-\xi\rangle - p_1|\xi\rangle}{N_1} \quad \text{for system 1,} \\ &|0\rangle = |\eta\rangle, \quad |1\rangle = \tfrac{|-\eta\rangle - p_2|\eta\rangle}{N_2} \quad \text{for system 2,} \end{aligned} \tag{2.16}$$

where

$$\begin{aligned} p_1 = \langle\xi|-\xi\rangle, \quad N_1 = \sqrt{1-p_1^2}, \\ p_2 = \langle\eta|-\eta\rangle, \quad N_2 = \sqrt{1-p_2^2}, \end{aligned} \tag{2.17}$$

where again for simplicity we assumed ξ and η are real parameters. Then the concurrence of the state

$$|\psi^{(2)}\rangle_s = \frac{1}{\sqrt{2(1-p_1p_2)}}\{(p_1p_2-1)|00\rangle + N_2p_1|01\rangle + N_1p_2|10\rangle + N_1N_2|11\rangle\}, \tag{2.18}$$

becomes

$$C^{(2)} = \frac{\sqrt{(1-p_1^2)(1-p_2^2)}}{1-p_1p_2}. \tag{2.19}$$

The behavior of concurrence as a function of r_1 and r_2 is shown in figure 4. Figure 4 shows that

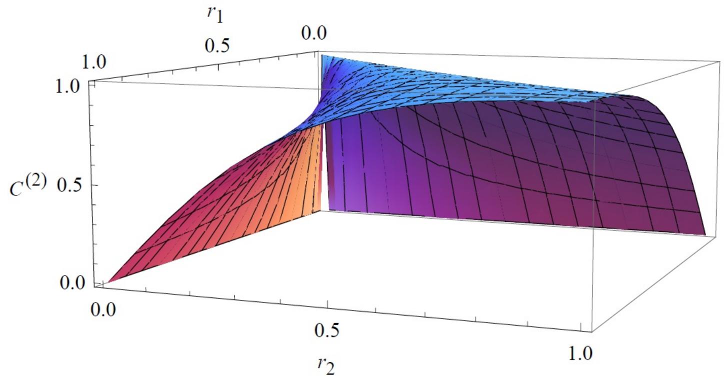


Figure 4: (Color online) Concurrence of $|\psi^{(2)}\rangle_s$ as a function of r_1 and r_2.

for $r_1 = r_2 \neq 0$ (i.e. $p_1 = p_2$) the state $|\psi^{(2)}\rangle_s$ is maximally entangled state (i.e. $C^{(2)}_{max} = 1$).

2.2 Wigner Function for Qubit like ESS

One of the important quasi-probability distribution over phase space is the Wigner function defined as [49]

$$W(\gamma) = \frac{1}{\pi^2} \int d^2\lambda C_W(\lambda) e^{\lambda^*\gamma - \lambda\gamma^*}, \tag{2.20}$$

in which $C_W(\lambda) = Tr(\rho D(\lambda))$. $D(\lambda) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ is displacement operator and ρ is reduced density matrix which is obtained by partially tracing out second mode. Here we investigate the behavior of Wigner function for qubit like ESS, Eq.(2.6). Wigner characteristic function reads

$$C^{(2)}_W(\lambda) = \frac{1}{M^{(2)}_s} \{\langle\xi|D(\lambda)|\xi\rangle + f^2\langle\eta|D(\lambda)|\eta\rangle + fp(\langle\xi|D(\lambda)|\eta\rangle + \langle\eta|D(\lambda)|\xi\rangle)\}. \tag{2.21}$$

For calculating the Wigner characteristic function, we use the fact $S(\xi)D(\lambda)|0\rangle = |\lambda, \xi\rangle$ is a squeezed-coherent state. Moreover $S(\xi)$ and $D(\lambda)$ may commutate each other for appropriate parameters i.e.,

$$S(\xi)D(\lambda) = D(\lambda')S(\xi), \tag{2.22}$$

where $\lambda' = \mu\lambda + \nu\lambda^*$ and

$$\mu = \cosh r_1, \nu = e^{i\theta_1}\sinh r_1. \tag{2.23}$$

By substituting $C_W^{(2)}(\lambda)$ in Eq.(2.20) the Wigner function can be obtained as

$$\begin{aligned} W^{(2)}(\gamma) = \tfrac{1}{M^{(2)}}\{\tfrac{2}{\pi}(e^{(\frac{-\gamma^2(\mu-\nu)^2}{2})} + f^2 e^{(\frac{-\gamma^2(\mu'-\nu')^2}{2})}) \\ + \tfrac{fp}{\pi\sqrt{2R(\mu\mu'-\nu\nu')(A-B-D)}}\exp[\tfrac{\gamma^2}{A-B-D}(-1+\tfrac{(B-D)^2}{2R(A-B-D)})] \\ + \tfrac{fp}{\pi\sqrt{2R'(\mu\mu'-\nu\nu')(A'-B'-D')}}\exp[\tfrac{\gamma^2}{A'-B'-D'}(-1+\tfrac{(B'-D')^2}{2R'(A'-B'-D')})]\}, \end{aligned} \tag{2.24}$$

in which

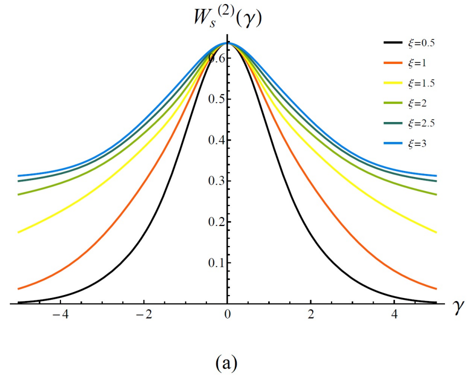
$$\begin{aligned} A = \tfrac{1}{2}(\mu'^2 + \nu'^2 + \tfrac{2\mu'\nu'(\nu'\mu-\mu'\nu)}{\mu\mu'-\nu\nu'}), \quad A' = \tfrac{1}{2}(\mu^2 + \nu^2 + \tfrac{2\mu\nu(\nu\mu'-\mu\nu')}{\mu\mu'-\nu\nu'}), \\ B = \tfrac{1}{2}(-\mu'\nu' - \tfrac{\mu'^2(\nu'\mu-\mu'\nu)}{\mu\mu'-\nu\nu'}), \quad B' = \tfrac{1}{2}(-\mu\nu - \tfrac{\mu^2(\nu\mu'-\mu\nu')}{\mu\mu'-\nu\nu'}), \\ D = \tfrac{1}{2}(-\mu'\nu' - \tfrac{\nu'^2(\nu'\mu-\mu'\nu)}{\mu\mu'-\nu\nu'}), \quad D' = \tfrac{1}{2}(-\mu\nu - \tfrac{\nu^2(\nu\mu'-\mu\nu')}{\mu\mu'-\nu\nu'}), \\ R = \tfrac{1}{2}(A + B + D + \tfrac{(B-D)^2}{A-B-D}), \quad R' = \tfrac{1}{2}(A' + B' + D' + \tfrac{(B'-D')^2}{A'-B'-D'}), \end{aligned} \tag{2.25}$$

and $\mu' = \cosh r_2, \nu' = e^{i\theta_2}\sinh r_2$. Note that for simplicity we assume that all parameters except λ are real i.e.

$$\begin{aligned} \mu = \cosh\xi, \quad \nu = \sinh\xi, \\ \mu' = \cosh\eta, \quad \nu' = \sinh\eta. \end{aligned} \tag{2.26}$$

Now we consider two special cases $f = \pm 1$:

For the case $f = 1$ we plot the diagram of Wigner distribution as a function of γ for given ξ and η (see figure 5). Figure 5 shows the behavior of Wigner function for squeezed and coherent states. By increasing the difference of ξ and η the width of squeezed Wigner function $W_s^{(2)}(\gamma)$ and its concurrence increase. On the other hand for ECS the distance of peaks of $W_c^{(2)}(\gamma)$ increases. For $f = -1$ the behavior of $W_s^{(2)}(\gamma)$ and $W_c^{(2)}(\gamma)$ are rather dissimilar. Unlike the $W_c^{(2)}(\gamma)$, the peaks of $W_s^{(2)}(\gamma)$ approach to each other by increasing the difference of $\Delta_s = \xi - \eta$. This affectation is different for $W_c^{(2)}(\gamma)$, that is the peaks of $W_c^{(2)}(\gamma)$ recede from each other by increasing the $\Delta_c = \alpha - \beta$ (see figure 6). As an another example we study the Wigner function for the qubit like ESS which is generated by beam splitters, phase-shifter and cross



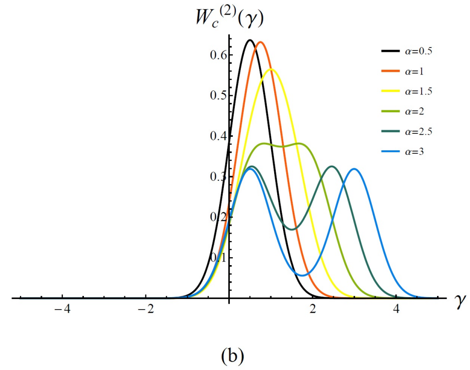


Figure 5: (Color online) Wigner distribution as a function of γ with $f=1$ (a) $W_s^{(2)}(\gamma)$ for given ξ, $\eta=0.5$, (b) $W_c^{(2)}(\gamma)$ for given α, $\beta=0.5$.

Kerr nonlinearity (i.e. $|\psi^{(2)}\rangle_s \sim |-\xi\rangle_a|-\eta\rangle_{a'} - |\xi\rangle_a|\eta\rangle_{a'}$). The Wigner function is given by

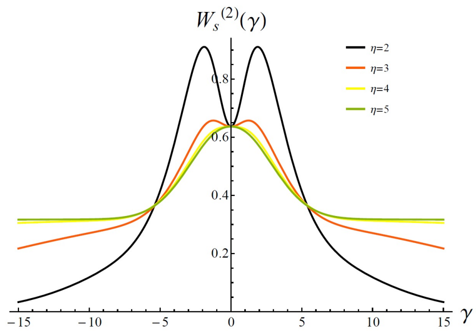
$$\begin{aligned} W_s^{(2)}(\xi,\eta,\gamma) = \frac{e^{-\frac{\gamma^2}{2}(\cosh 2\xi+\sinh 2\eta)}\sqrt{\cosh\xi}}{\pi\sqrt{\cosh 2\xi}(-1+\sqrt{\cosh\xi\cosh\eta})}\{-e^{-\frac{\gamma^2}{4\cosh 2\xi}(-1+7\cosh 4\xi-9\sinh 4\xi)} \\ - e^{-\frac{\gamma^2}{2}\cosh 2\xi(1+\tanh 2\xi)(3+4\tanh 2\xi)} + (1+e^{\gamma^2\sinh 2\xi})\sqrt{\cosh\eta\cosh 2\xi}\}. \end{aligned} \tag{2.27}$$

The behavior of Wigner function is illustrated in figure 7. Figure 7 confirmed the above results i.e. by increasing Δ_s the two peaks in Wigner function approach each other. Furthermore, figure 7 shows that the Wigner function begin to be negative for large values of γ. Negativity of the Wigner function is the reason why the Wigner function can not be regarded as a real probability distribution but a quasi-probability distribution function and it is a good indication of the possibility of the occurrence of nonclassical properties [43].

3 Qutrit like ESS's

In this section we consider two and three-mode qutrit like ESS's. First consider two-mode case:

$$|\Psi^{(3)}\rangle_s = \frac{1}{\sqrt{M_s^{(3)}}}(|\xi\xi\rangle + f_1|\eta\eta\rangle + f_2|\tau\tau\rangle), \tag{3.28}$$



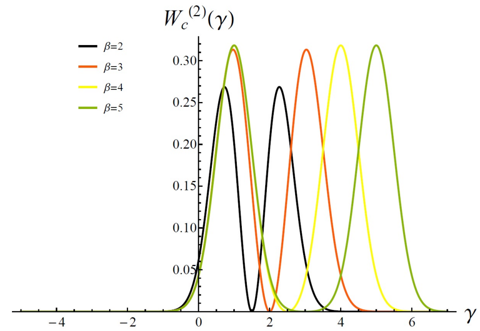


Figure 6: (Color online) Wigner distribution as a function of γ with $f=-1$ (a) $W_s^{(2)}(\gamma)$ for given η, $\xi=1$, (b) $W_c^{(2)}(\gamma)$ for given β, $\alpha=1$.

where $\xi=r_1e^{i\theta_1}$, $\eta=r_2e^{i\theta_2}$ and $\tau=r_3e^{i\theta 3}$ are generally complex numbers and the normalization factor $M_s^{(3)}=1+f_1^2+f_2^2+2f_1Re(p_1^2)+2f_2Re(p_3^2)+2f_1f_2Re(p_2^2)$ with $p_1=\langle\xi|\eta\rangle$, $p_2=\langle\tau|\eta\rangle$ and $p_3=\langle\tau|\xi\rangle$. For simplicity we assume that all parameters are real. From linearly independent of three squeezed states $|\xi\rangle$, $|\eta\rangle$ and $|\tau\rangle$, one can define three-dimensional space spanned by the set $\{|0\rangle,|1\rangle,|2\rangle\}$ as

$$\begin{aligned}
&|0\rangle=|\xi\rangle,\\
&|1\rangle=\tfrac{1}{\sqrt{1-p_1^2}}(|\eta\rangle-p_1|\xi\rangle),\\
&|2\rangle=\sqrt{\tfrac{1-p_1^2}{1-p_1^2-p_2^2-p_3^2+2p_1p_2p_3}}\left(|\tau\rangle+(\tfrac{p_1p_3-p_2}{1-p_1^2})|\eta\rangle+(\tfrac{p_1p_2-p_3}{1-p_1^2})|\xi\rangle\right).
\end{aligned}\tag{3.29}$$

By substitution these basis into Eq.(3.28) the state $|\Psi^{(3)}\rangle_s$ is reduced to a qutrit like ESS. We use the general concurrence measure for bipartite state $|\psi\rangle=\sum_{i=1}^{d_1}\sum_{j=1}^{d_2}a_{ij}|e_i\otimes e_j\rangle$ [48]. The norm of concurrence vector is obtained as $C=2(\sum\limits_{i<j}^{d_1}\sum\limits_{k<l}^{d_2}|a_{ik}a_{jl}-a_{il}a_{jk}|^2)^{1/2}$, where d_1 and d_2 are dimensions of first and second part respectively. The behavior of concurrence $|\Psi^{(3)}\rangle_s$ is represented in figure 8. In order to compare this result we also plot the concurrence of $|\Psi^{(3)}\rangle_c$ in the same figure 8. By $|\Psi^{(3)}\rangle_c$ we mean that in Eq.(3.28) the squeezed states $|\xi\rangle,|\eta\rangle$ and $|\tau\rangle$ must be replaced by coherent states $|\alpha\rangle,|\beta\rangle$ and $|\gamma\rangle$. Figure 9(a) displays the Wigner function of $|\Psi^{(3)}\rangle_s$ as a function of γ with $f_1=f_2=1$ for a given ξ,η and τ and figure 9(b) shows the

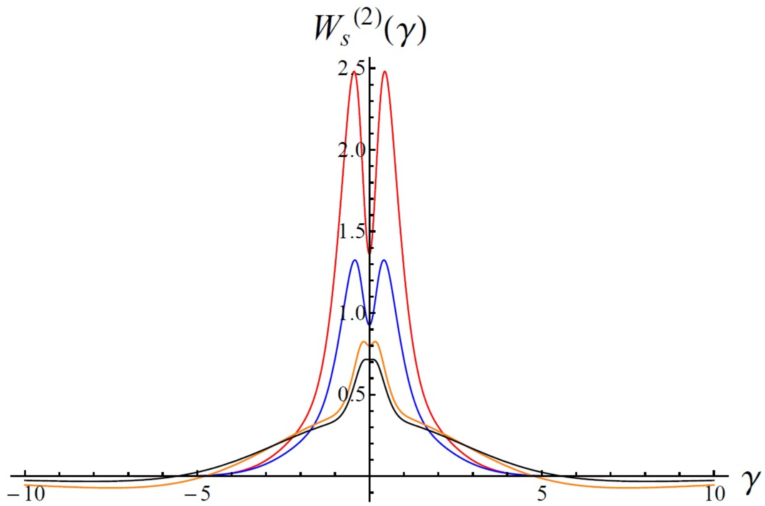


Figure 7: (Color online) $W_s^{(2)}(\gamma)$ as a function of γ for given: $\xi = \eta = 0.5$ (red), $\xi = 0.5$, $\eta = 1$ (blue), $\xi = 1$, $\eta = 2$ (orange), $\xi = 1$, $\eta = 3$ (black).

behavior of concurrence $C^{(3)}(\xi)$ as a function of ξ for a given $\eta = \tau = 0.5$. By comparison we deduce that by increasing the differences of ξ, η and τ both the width of Wigner function and the concurrence are raising. Once again we assume $f_1 = f_2 = -1$ and investigate the behavior of Wigner distribution as a function of γ and ξ for given η and τ (see figure 10). On the other hand figure 11 shows that for $\xi = 1.5$, the Wigner function takes negative values which is a good indication of the possibility of the occurrence of nonclassical properties.

3.1 Monogamy Inequality for Multi-Qutrit like ESS

Another problem that arises in multipartite states is monogamy inequality. Here in this section we examine the concurrence based monogamy inequality for qutrit like ESS. To this end consider multi qutrit like ESS:

$$|\Phi^{(3)}\rangle_s = \frac{1}{\sqrt{M_N'^{(3)}}}(|\xi\rangle ... |\xi\rangle + f_1|\eta\rangle ... |\eta\rangle + f_2|\tau\rangle ... |\tau\rangle), \tag{3.30}$$

in which

$$M_N^{(3)} = 1 + f_1^2 + f_2^2 + 2f_1 p_1^N + 2f_2 p_3^N + 2f_1 f_2 p_2^N, \tag{3.31}$$

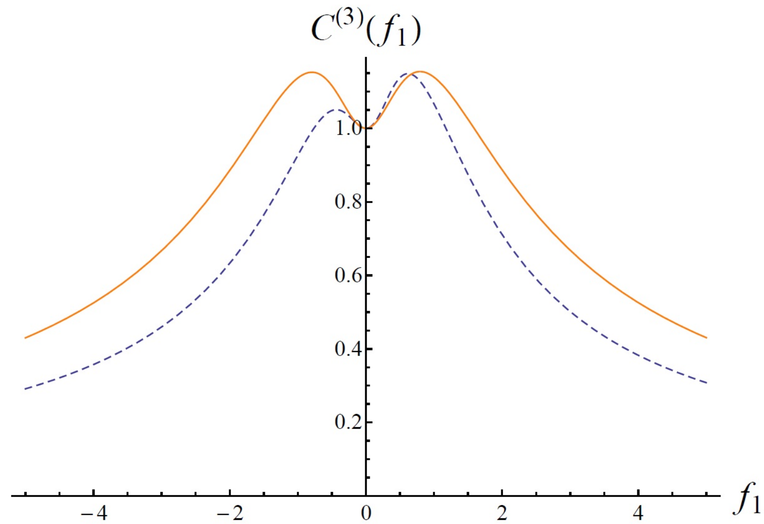


Figure 8: (Color online) Concurrence of $|\Psi^{(3)}\rangle_s$ (dashed line) and $|\Psi^{(3)}\rangle_c$ (full line) as a function of f_1 for $\xi = 6$, $\eta = 2.5$, $\tau = 5$ and $f_2 = -1$.

is normalization factor and for simplicity we assume that the all parameters are real. Next imagine the state $|\Phi^{(3)}\rangle_s$ as tripartite A, B and D including m_1, m_2 and $m_3 = N - m_1 - m_2$ modes respectively. For the moment, suppose that $\tau \to \infty$, i.e. $p_2, p_3 = 0$ and $p_1 \neq 0$. One can easily obtain the reduced density matrices $\rho_{AB} = Tr_{_D}(|\Phi_N^{(3)}\rangle_{ABD}\langle\Phi_N^{(3)}|)$ and $\rho_{AD} = Tr_{_B}(|\Phi_N^{(3)}\rangle_{ABD}\langle\Phi_N^{(3)}|)$ by partially tracing out subsystems D and B respectively. For general mixed states the concurrence is defined as $|C|^2 = \sum_{\alpha\beta}|C_{\alpha\beta}|^2$ where $C^{\alpha\beta} = \lambda_1^{\alpha\beta} - \sum_{i=2}^{n}\lambda_i^{\alpha\beta}$ with $\lambda_1 = \max\{\lambda_i, i = 1, ..., d^2\}$ and $\lambda_i^{\alpha\beta}$ are the nonnegative eigenvalues of $\tau\tau^*$ defined as [50]

$$\tau^{\alpha\beta}\tau^{\alpha\beta *} = \sqrt{\rho}(E_\alpha - E_{-\alpha}) \otimes (E_\beta - E_{-\beta})\rho^*(E_\alpha - E_{-\alpha}) \otimes (E_\beta - E_{-\beta})\sqrt{\rho}. \tag{3.32}$$

where α's are positive roots of the lie group (here $SU(3)$) and $E_{\pm\alpha}$'s are corresponding raising/lowering operators (like $J_\pm$ of the angular momentum operator). Let us consider $N = 20, m_1 = 1, m_2 = 2$ and $f_2 = 0.4$ and explore monogamy inequality [51, 52, 53]

$$C^2_{A(BD)} \geq C^2_{AB} + C^2_{AD}, \tag{3.33}$$

where C_{AB} and C_{AD} are the concurrences of the reduced density matrices of ρ_{AB} and ρ_{AD} respectively and $C_{A(BD)}$ is the concurrence of pure state $|\Phi^{(3)}\rangle_s$ with respect to the partitions

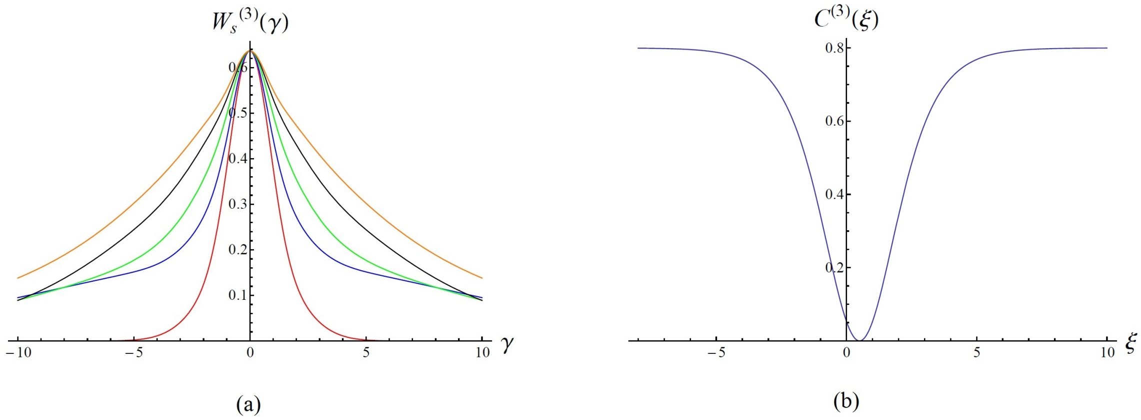


Figure 9: (Color online) (a) $W_s^{(3)}(\gamma)$ as a function of γ with $f_1 = f_2 = 1$ and $\xi = 0.5$ for given $\eta = \tau = 0.5$ (red), $\eta = 0.5, \tau = 2$ (blue), $\eta = 1, \tau = 2$ (green), $\eta = 1.5, \tau = 2$ (black), $\eta = \tau = 2$ (orange), (b) $C^{(3)}(\xi)$ as a function of ξ for given $\eta = \tau = 0.5$.

A and BD. The concurrence vectors C_{AB} and C_{AD} read

$$
\begin{aligned}
C_{AB} &= \frac{\sqrt{(1-p^2)^2(1+p^2)(4f_1^2p^{34}+f_1^4(p^2-p^6))}}{1.16+f_1^2+2f_1p^{20}}, \\
C_{AD} &= \frac{\sqrt{5f_1^2(p^4-p^6-p^{38}+p^{40})}}{1.16+f_1^2+2f_1p^{20}},
\end{aligned}
\tag{3.34}
$$

where $p = \frac{1}{\sqrt{\cosh r_1 \cosh r_2(1-\tanh r_1 \tanh r_2)}}$. On the other hand, one finds that

$$
C_{A(BD)} = \frac{2\sqrt{0.16f_1^2(1-p)p^{38} + 1.16f_1^2(1-p)(1-p^{19}) + 0.16f_1^2p^2(1-p^{19}) + 0.16(1+f_1p^{20})^2)}}{1.16 + 2f_1p^{20} + f_1^2}.
\tag{3.35}
$$

The behavior of $\tau_{ABD} = C^2_{A(BD)} - C^2_{AB} - C^2_{AD}$ as a function of f_1 for given ξ and η is shown in figure 12. One can immediately deduce that τ_{ABD} becomes negative for some values of f_1 which is a violation of the monogamy inequality. In order to compare this results with ECS we also display the results of $\tau^{(c)}_{ABD}$ calculated in Ref. [35].

4 Conclusion

In summary, we introduced two-mode qubit and qutrit like ESS's. For qubit like state $|\Psi^{(2)}\rangle_s$, a comparison between ESS and ECS showed that for $f > 0$ the concurrence of squeezed state

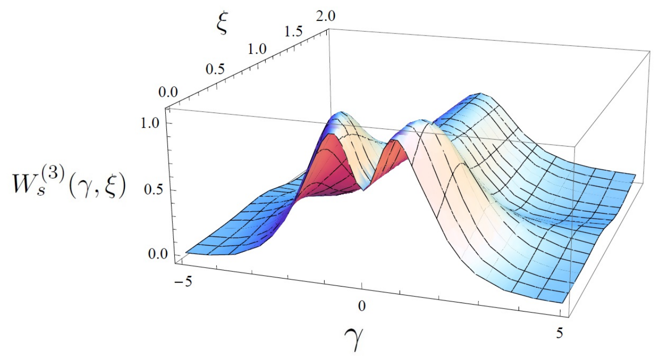


Figure 10: (Color online) $W_s^{(3)}(\gamma,\xi)$ as a function of γ and ξ with $f_1 = f_2 = -1$ for given $\eta = 0.2$ and $\tau = 0.4$.

is less than the concurrence of coherent state. On the other hand for $f < 0$, there were r_1, r_2 and $\Delta\theta$ for which the cross over occurred. It was shown that considering all parameters to be real and $f = 1$ the concurrence is a monotone function of Δ_s with the properties that for $\Delta_s \to \infty$, the concurrence tends to its maximum value ($C^{(2)}_{max} = 1$), while for small separation (i.e. $\Delta_s \to 0$) the concurrence tends to zero and the state becomes separable. Moreover we investigated the ESS generated by beam splitters, phase-shifter and cross Kerr nonlinearity and showed that for $r_1 = r_2 \neq 0$ (i.e. $p_1 = p_2$) the state $|\psi^{(2)}\rangle_s$ is maximally entangled state (i.e. $C^{(2)}_{max} = 1$). We also compared the Wigner functions $W_s^{(2)}(\gamma)$ $W_c^{(2)}(\gamma)$ of ESS and ECS respectively. For $f = -1$, unlike the $W_c^{(2)}(\gamma)$, the peaks of $W_s^{(2)}(\gamma)$ approached to each other by increasing the difference of $\Delta_s = \xi - \eta$. This affectation was different for $W_c^{(2)}(\gamma)$, i.e. the peaks of $W_c^{(2)}(\gamma)$ receded from each other by increasing the $\Delta_c = \alpha - \beta$. A similar result was discussed for qutrit like ESS, $|\Psi^{(3)}\rangle_s$. Comparison the concurrences of ESS and ECS illustrated that for $f_2 = -1$ there are ξ, η and τ in which cross over occurs. Furthermore for $f_1 = f_2 = 1$ it was shown that that by increasing the differences of ξ, η and τ, the width of $W^{(3)}(\gamma)_s$ and the concurrence $C^{(3)}(\xi)$ are raising. On the other hand for $f_1 = f_2 = -1$ the Wigner function $W_s^{(3)}(\gamma)$ took negative values which was a good indication of the possibility of the occurrence of nonclassical properties. Finally we showed that it is possible to find some f_1, for which

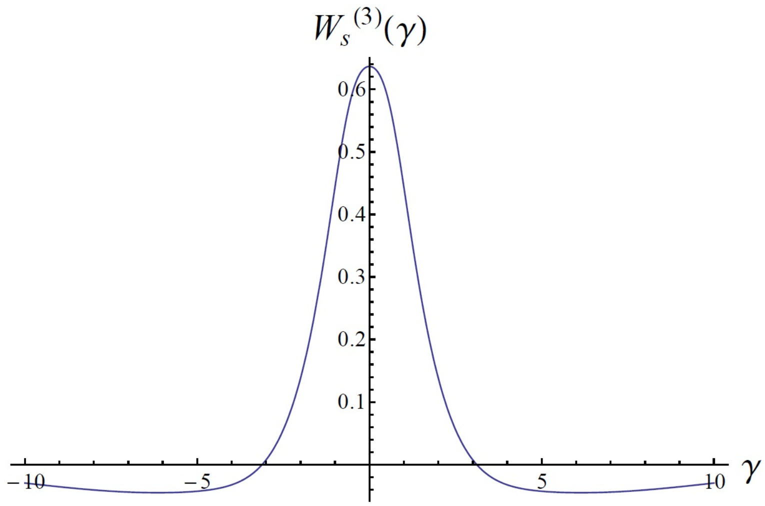


Figure 11: (Color online) $W_s^{(3)}(\gamma)$ as a function of γ with $f_1 = f_2 = -1$ for given $\xi = 1.5, \eta = 0.2$ and $\tau = 0.4$.

$\tau_{ABD} < 0$ implying that the monogamy inequality may be violated for qutrit like ESS.

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References

[1] R.E. Slusher, L.W. Hollberg, B. Yurke, J.C. Mertz, J.F. Valleys, Phys. Rev. Lett. **55**, 2409 (1985).

[2] R.M. Shelby, M.D. Levenson, S.H. Perlmutter, R.G. DeVoe, D.F. Walls, Phys. Rev. Lett. **57**, 691 (1986).

[3] L.A. Wu, H.J. Kimble, J.L. Hall, H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).

[4] S.J. van Enk, Phys. Rev. A **60**, 5095 (1999).

[5] J.S. Neergaard-Nielsen, B.M. Nielsen, C. Hettich, K. Moelmer, E.S. Polzik, Phys. Rev. Lett. **97**, 083604 (2006).

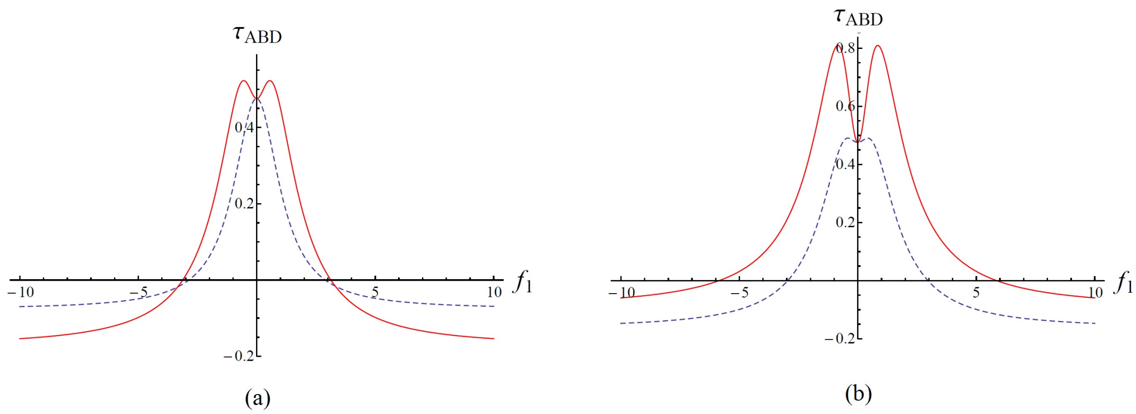

Figure 12: (Color online) $\tau^{(s)}_{ABD}$ for qutrit like ESS (dashed line) and $\tau^{(c)}_{ABD}$ for qutrit like ECS (full line) as a function of f_1 for given $N = 20, m_1 = 1, m_2 = 2$ and $f_2 = 0.4$ for (a) $\xi = 3$, $\eta = 2$ and (b) $\xi = 1$, $\eta = 2.5$.

[6] H. Lu, L. Chen, J. Lin, Chinese Optics Letters, Vol. **2**, No. 10 (2004).

[7] A.M. Branczyk, T.C. Ralph, Phys. Rev. A **78**, 052304 (2008).

[8] K.P. Seshadreesan, J.P. Dowling, G.S. Agarwal, Physica Scripta, Vol. **90**, No. 7, 074029 (2015).

[9] E.A. Sete, Optics Communications **281**, 6124 (2008).

[10] M.S. Kim, W. Son, V. Buzek, P.L. Knight , Phys. Rev. A **65**, 032323 (2002).

[11] L.M. Kuang, A.H. Zeng, Z.H. Kuang, Phys. Lett. A, Vol. **319** 24-31 (2003).

[12] Z.M. Zhang, arxiv:quant-ph/0604128v1 (2006).

[13] A. Tipsmark, J.S. Neergaard-Nielsen, U.L. Andersen, Optics Express, Vol. **21** 6670-6680 (2013).

[14] X. Wang, Phys. Rev. A, **66**, 064304 (2002).

[15] F. Gao, Y.X. Wang, H.Y. Fan, X.B. Tang, arxiv: quant-ph/1511.04833v1 (2015).

[16] E. Schrödinger Naturwissenschaften **14**, 664 (1926).

[17] P.T. Cochrane, G.J. Milburn, and W.J. Munro, Phys. Rev. A **59**, 2631 (1999).

[18] M.C. de Oliveira and W. J. Munro, Phys. Rev. A **61**, 042309 (2000).

[19] H. Jeong and M.S. Kim, Phys. Rev. A **65**, 042305 (2002).

[20] T.C. Ralph, A. Gilchrist and G.J. Milburn, Phys. Rev. A **68**, 042319 (2003).

[21] W.J. Munro, G.J. Milburn, and B.C. Sanders, Phys. Rev. A **62**, 052108 (2001).

[22] X. Wang, B.C. Sanders , S.H. Pan, J. Phys. A **33**, 7451 (2000).

[23] X. Wang, Phys. Rev. A **64**, 022302 (2001).

[24] X. Wang, B.C. Sanders, Phys. Rev. A **65**, 012303 (2002).

[25] W. Vogel, J. Sperling, Phys. Rev. A **89**, 052302 (2014).

[26] S. Salimi, A. Mohammadzade, Int J. Theor. Phys **50**: 1444-1450 (2011).

[27] S.J. van Enk, O. Hirota, Phys. Rev. A **67**, 022313 (2001).

[28] X. Wang, J. Phys. A, Math. Gen. **35**(1), 165173 (2002).

[29] H. Fu, X. Wang, A.I. Solomon, Phys. Lett. A **291**, 7376 (2001).

[30] G. Najarbashi, Y. Maleki, Int J. Theor. Phys. **50**, 2601-2608 (2011).

[31] Y.B. Sheng, C.C. Qu, O.Y. Yang, Z.F. Feng, L. Zhou, Int J. Theor. Phys. **53**, 20332040 (2014).

[32] Y.B. Sheng, J. Liu, S.Y. Zhao , L. Wang, L. Zhou, Chin. Phys. B, Vol. **23**, No. 8, 080305 (2014).

[33] B.C. Sanders, Phys. Rev. A **45**, 9 (1992).

[34] S. J. van Enk, Phys. Rev. Lett. , Vol. **91**, No. 1 (2003).

[35] G. Najarbashi, S. Mirzaei, Int J. Theor. Phys, Vol. **54**, No. 10 (2015).

[36] G. Najarbashi, S. Mirzaei, Int J. Theor. Phys, DOI: 10.1007/s10773-015-2869-7 (2015).

[37] E. Wigner, Phys. Rev, Vol. **40**, 749 (1932).

[38] M. Hillery, R.F. O'Connell, M.O. Scully, E.P. Wigner, Physics Reports, **106**, 121-167 (1984).

[39] G. Breitenbach, S. Schiller, J. Mlynek, Nature London, **387**, 471 (1997).

[40] A.I. Lvovsky, H. Hansen, T. Aichele, O. Benson, J. Mlynek, S. Schiller, Phys. Rev. Lett. **87**, 050402 (2001).

[41] G.S. Agarwal, New J. Phys. **13**, 073008 (2011).

[42] S.W. Lee, H. Jeong, D. Jaksch, Phys. Rev. A **80**, 022104 (2009).

[43] A. Banerji, R.P. Singh, A. Bandyopadhyay, Optics Communications **330**, 8590 (2014).

[44] Faisal A. A. El-Orany, M. H. Mahran, and M. R. B. Wahiddin and A. M. Hashim, Optics Communications **6**, 169-184 (2004).

[45] G. Najarbashi, S. Mirzaei, arxiv: quant-ph/1601.00143v1 (2016).

[46] W.K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).

[47] W.K. Wootters, Quantum Information and Computation, Vol. **1**, No. 1 (2001).

[48] S.J. Akhtarshenas, J. Phys. A: Math. Gen. **38**, 6777-6784 (2005).

[49] C. Gerry, P. Knight, Introductory Quantum Optics, Cambridge University Press (2005).

[50] Y. Q. Li, G. Q. Zhu, Front. Phys. China, **3(3)**: 250257 (2008).

[51] V. Coffman, J. Kundu and W. K. Wootters, Phys. Rev. A **61**, 052306 (2000).

[52] J. S. Kim, A. Das and B. C. Sanders, Phys. Rev. A **79**, 012329 (2009).

[53] B. C. Sanders, J. S. Kim, Applied Mathematics and Information Sciences **4(3)** (2010).