# **Launching jets from accretion belts**

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## **ABSTRACT**

We propose that sub-Keplerian accretion belts around stars might launch jets. The sub-Keplerian inflow does not form a rotationally supported accretion disk, but it rather reaches the accreting object from a wide solid angle. The basic ingredients of the flow are a turbulent region where the accretion belt interacts with the accreting object via a shear layer, and two avoidance regions on the poles where the accretion rate is very low. A dynamo that is developed in the shear layer amplifies magnetic fields to high values. It is likely that the amplified magnetic fields form polar outflows from the avoidance regions. Our speculative belt-launched jets model has implications to a rich variety of astrophysical objects, from the removal of common envelopes to the explosion of core collapse supernovae by jittering jets.

#### **1. INTRODUCTION**

<span id="page-0-0"></span>Jets are known to be launched by accretion disks around compact objects, such as supermassive black holes in active galactic nuclei, X-ray binaries, young stellar objects (YSOs), and in planetary nebulae (PNe; e.g., [Livio 2011](#page-7-0)). Many jet launching models are based on the operation of large scale magnetic fields, i.e., those with coherence scale larger than the radius of the disk at the considered location (e.g., [Zanni & Ferreira 2013](#page-8-0) and [Narayan et al. 2014](#page-7-1) and references therein). Some models do not rely on the magnetic fields of the accreting compact object, but rather assume an outflow from an extended disk region with large-scale magnetic fields (e.g., [Konigl & Pudritz](#page-7-2) [2000;](#page-7-2) [Shu et al. 2000;](#page-8-1) [Ferreira 2002](#page-7-3); [Krasnopolsky et al. 2003;](#page-7-4) [Ferreira & Casse 2004\)](#page-7-5).

However, there are several arguments hinting that jets can be launched without the presence of large scale disk magnetic fields. (1) There are well collimated jets in PNe, such as in Hen 2-90 [\(Sahai & Nyman 2000](#page-8-2)). No large scale magnetic fields are expected around the companion to the AGB star that is though to launch the jets in PNe. (2) DQ Her (intermediate polars) systems are cataclysmic variables where the accreting white dwarf (WD) has a sufficiently strong magnetic field to truncate the accretion disk close to the WD. Such a stellar-disk magnetic field geometry

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is assumed in some jet-launching models in YSOs (e.g., [Shu et](#page-8-3) al. [1991\)](#page-8-3). Despite that, no jets are observed in DQ Her systems. (3) The strong dynamo that operates in accretion disks is likely to modify the structure of the large scale magnetic fields, (e.g. [Hujeirat et al. 2003](#page-7-6)). (4) [Ferreira](#page-7-7) [\(2013\)](#page-7-7) concluded that steady state jets' launching models cannot spin down a protostar. The balance between the spinning-up accretion torque and the spinning-down magnetic torque in such models is expected to spin up the star on a time scale smaller than the disk lifetime. Observations, on the other hand, show that the rotation period of some YSOs is almost constant, at roughly 10% of their break-up speed. In contrast, unsteady models are better candidates for the removal of energy and angular momentum from the disk. The model studied in the present paper is a type of unsteady model. (5) In several YSOs the jets precess on a time scale of about 100 years (e.g. [Ybarra et al. 2006](#page-8-4)), much too rapid for a large scale magnetic field to change its geometry. Models that lunch jets from close to the accreting compact object might allow for such a rapid precession.

In the present study we examine cases where the accreted gas has a sub-Keplerian angular momentum, and hence cannot form a fully developed Keplerian accretion disk around the compact object. Instead, the gas that does posses a non-negligible specific angular momentum forms an accretion belt on the surface of the accreting body. We aim at two cases in particular: (1) The case where a main sequence (MS) star is spiraling-in inside the envelope of a giant star. The MS star can accrete mass from the envelope at a rate of the order of the Eddington limit. The specific angular momentum of the accreted gas,  $j_{\text{acc}}$ , might be in many cases below that required to form a Keplerian accretion disk around the MS star,  $j_{\text{Kep}}$ , but still be non-negligible  $j_{\text{acc}} \approx 0.1 - 1 j_{\text{Kep}}$ [\(Soker 2004](#page-8-5)). (2) An accretion by the newly born NS in core collapse supernovae (CCSNe). In some CCSNe the accreted gas might possess a rapidly varying non-negligible value of the specific angular momentum, but not sufficient to form a Keplerian accretion disk [\(Gilkis & Soker 2015](#page-7-8), [2016;](#page-7-9) [Papish et al. 2016](#page-7-10)). Both scenarios are most difficult for observational validation.

Incorporating ingredients from accretion disks formed at high accretion rates we build the belt-launched jets model. The basic ingredients of the model are described in section [2,](#page-1-0) and the dynamo is discussed in section [3.](#page-3-0) The implications for some specific astrophysical objects are discussed in section [4](#page-5-0) that also contains our summery.

#### **2. BASIC INGREDIENTS**

<span id="page-1-0"></span>A geometrically-thin Keplerian accretion disk is connected to the accreting body, if it is not a black hole and if the accreting body magnetic field is not too strong, via a thin high-shear layer termed the boundary layer (e.g., [Regev 1983;](#page-7-11) [Regev & Bertout](#page-8-6) [1995](#page-8-6)). The matter in the boundary layer expands to higher latitudes along the surface of the accreting body. [Inogamov & Sunyaev](#page-7-12) [\(1999\)](#page-7-12) built a model for cases of high accretion rates onto neutron stars (NS), and proposed that the meridional spreading brings the accreted mass close to the poles. In that model the luminosity from the spreading-layer, or belt, reaches a maximum away from the equatorial plane. The accreted gas spins-down to the rotation velocity of the accreting body through a turbulent friction that exists in the spreading-layer [\(Inogamov & Sunyaev 1999](#page-7-12), [2010\)](#page-7-13). This type of flow is depicted in the right side of Fig. [1.](#page-9-0) The left half-side of Fig. [1](#page-9-0) schematically presents the flow structure proposed in the present study.

It is not possible to directly apply the results of [Inogamov &](#page-7-12) Sunyaev [\(1999\)](#page-7-12) to the cases we study here (listed in section [1\)](#page-0-0). We study MS stars accreting at very high rates, a process that is different than accretion onto a NS. For example, radiation pressure might not be as important as in accretion onto a NS. In the flow structure onto a newly born NS in CCSNe, cooling is mainly via neutrinos, and not by photon emission. The flow structures we discuss here are more complicated, and 3D numerical studies will be required to simulate such flows. The study of [Inogamov & Sunyaev](#page-7-12) [\(1999\)](#page-7-12) non the less suggests that an energetic wide accretion belt can be formed on the surface of the accreting body, and that turbulence is expected in such a belt.

Neglecting magnetic fields, [Inogamov & Sunyaev \(1999](#page-7-12)) find that the meridional extent of the hot part of the spreading layer (the belt)  $\theta_{\star}$ , is approximately given by

<span id="page-2-0"></span>
$$
\sin \theta_{\star} \approx \frac{\dot{M}_{\text{acc}} v_K^2}{2L_{\text{edd}}},\tag{1}
$$

where  $L_{\text{edd}}$  is the Eddington luminosity limit,  $\dot{M}_{\text{acc}}$  is the accretion rate, and  $v_K$  is the Keplerian velocity on the surface of the accreting body. One can find that for accreting rates larger than about  $10^{18}$  g s<sup>-1</sup>  $\simeq 10^{-8} M_{\odot}$  yr<sup>-1</sup> onto a NS with a mass of  $M_{\rm NS} \approx 1.4 M_{\odot}$  and a radius of  $R = 12$  km, the spreading layer extends to the polar regions. We here also apply equation [\(1\)](#page-2-0) to accretion onto a MS star. We find that for the spreading layer to reach the polar region of an accreting solar-like star, the accretion rate should be larger than approximately  $10^{-3} M_{\odot}$  yr<sup>-1</sup>. Such accretion rates can be achieved during the common envelope evolution.

We note that equation [\(1\)](#page-2-0) was developed for a belt formed by an accretion disk. We here investigate cases where no geometrically-thin accretion disk is formed, but rather the accreted gas streams from a wide angle, with an avoidance region near the poles.

In the sub-Keplerian inflow scenario the accreted gas has an average specific angular momentum of  $j_{\text{acc}} < j_{\text{Kep}}$ , where  $j_{\text{Kep}} = (GMR)^{1/2}$  is the Keplerian specific angular momentum on the surface of the accreting object of mass  $M$  and radius  $R$ . A parcel of gas with such an angular momentum cannot be accreted to the surface of the star within an angle of  $\theta_a$  from the poles, given by

<span id="page-2-1"></span>
$$
\theta_a = \sin^{-1} \sqrt{\frac{j_{\text{acc}}}{j_{\text{Kep}}}}.
$$
\n(2)

This is termed the avoidance angle.

In section [3](#page-3-0) we speculate that the extended belt formed in the sub-Keplerian accretion flow can launch jets. We suggest there that the strong radial shear due to the powerful differential rotation together with the turbulence, amplify magnetic fields through the dynamo action. MHD effects then launch a polar-collimated outflow. The point to make here is that in the case of a belt formed by an accretion disk it is expected that more mass will be launched by the accretion disk than by the belt. In the sub-Keplerian wide inflow case, most, or even all, of the outflowing gas is launched by the belt.

The basic ingredients of the belt-launched jets scenario are presented in Table 1 alongside with those of jets launched by accretion disks. Our proposed launching mechanism does not require large scale magnetic fields and does not require a thin Keplerian disk. The most important ingredient is the operation of a dynamo, the belt dynamo.



Table 1: Jet launching cases

The different symbols have the following meaning: R is the radius of the accreting body;  $\dot{M}_{\rm Edd}$  is the Eddington accretion limit;  $j_{\text{Kep}}$  and  $\Omega_{\text{Kep}}$  are the Keplerian specific angular momentum and angular velocity very close to the accreting body surface, respectively.

## **3. THE BELT DYNAMO**

<span id="page-3-0"></span>There are several approaches to estimate the amplification of magnetic fields in sheared layers in stellar interiors, both in convective and non-convective regions. In the present preliminary study we take one calculation referring a non-convective region. We then argue that in a convective regions of the accretion belt the amplification will be more efficient even.

[Spruit](#page-8-7) [\(2002\)](#page-8-7) gave the analytic ground for quantifying the magnetic fields created in nonconvective layers in a differentially rotating star. The differential rotation stretches poloidal field lines into toroidal fields. Magnetic instabilities in the amplified toroidal magnetic fields replace the role of convection in creating more poloidal magnetic field from the toroidal field. This process, that is relatively slow in stellar interior, is expected to be more efficient in the more rapidly spinning belt. The turbulent regions of the belt are even more efficient in amplifying the magnetic field. We turn to show that even in the non-turbulent regions of the belt the dynamo can be effective.

Due to the strong differential rotation (shear) occurring in the belt the radial component of the field,  $B_r$ , is twisted into the azimuthal direction,  $B_{\phi}$ , so that after few rotations it becomes the dominant component. Its strength increases linearly with time, until it becomes unstable. The amplitude of the dynamo-generated field in the non-turbulent regions is given by [\(Spruit 2002](#page-8-7))

<span id="page-4-0"></span>
$$
B_{\phi} = r (4\pi \rho)^{1/2} \Omega_{\text{belt}} q^{1/2} \left(\frac{\Omega_{\text{belt}}}{N}\right)^{1/8} \left(\frac{\kappa}{r^2 N}\right)^{1/8} \tag{3}
$$

<span id="page-4-1"></span>
$$
B_r = B_\phi \left(\frac{\Omega_{\text{belt}}}{N}\right)^{1/4} \left(\frac{\kappa}{r^2 N}\right)^{1/4},\tag{4}
$$

where the density in the belt is estimated from mass conservation of the inflowing gas

$$
\rho = \dot{M} \left( 4\pi r^2 v_{\rm in} \right)^{-1},\tag{5}
$$

and  $v_{\text{in}}$  is the radial inflow speed into the belt, which we take as the free-fall velocity. The temperature of the accreted plasma when it is stopped on the accreting object is calculated from  $3/2kT = 1/2 m_p v_{\text{in}}^2$ . Here  $\Omega_{\text{belt}}$  is the belt angular velocity, and N is the buoyancy (Brunt-Vaisala) frequency that is slightly less than the Keplerian frequency. We can crudely take  $N \approx \Omega_{\text{belt}}$ . The thermal diffusivity is  $\kappa = 16\sigma T^3/(3\kappa_R \rho^2 c_p)$ , where  $c_p$  is the heat capacity per unit mass,  $\kappa_R$  is the opacity, and  $q = (r \partial_r \Omega_{\text{belt}})/\Omega_{\text{belt}} \simeq 1-3$  is the dimensionless differential rotation rate.

As the cooling in accreting NS in CCSNe is due to neutrinos, we apply [\(3\)](#page-4-0) and [\(4\)](#page-4-1) to accreting MS stars. We take an accretion rate of  $\dot{M}_{\rm acc} = 10^{-3} M_{\odot} \text{ yr}^{-1}$  onto a solar like star, from which we find the density of the accretion inflow near the surface, and the temperature of the gas in the belt to be  $T = 10^6$  K. We find that  $\kappa \approx 10^{22}$  cm<sup>2</sup> s<sup>-1</sup>, and so  $\kappa/(r^2N)$ <sup>1/8</sup>  $\approx 1$ . We can derive from [\(3\)](#page-4-0) the ratio of the magnetic field energy density to that of the kinetic and thermal gas density, for an accreting MS star at a rate of  $\dot{M}_{\rm acc} \approx 10^{-2} - 10^{-4} M_\odot \text{ yr}^{-1}$ . For the gas energy density we take  $\rho v_{\rm esc}^2/2$ , where  $v_{\rm esc}$  is the escape velocity from the star

<span id="page-4-2"></span>
$$
\frac{B^2/4\pi}{e_{\text{gas}}} \approx 0.2q \left(\frac{\Omega_{\text{belt}}/\Omega_{\text{Kep}}}{0.3}\right)^2 \left(\frac{\Omega_{\text{belt}}}{N}\right)^{1/4} \left(\frac{\kappa}{r^2 N}\right)^{1/4}.
$$
 (6)

As stated, this is for the non-turbulence part of the accretion belt. As turbulence is expected in the strongly sheared belt, the amplification can be more efficient event. We conclude that a fraction of  $\approx$  few  $\times$  0.1 of the accreted energy is channelled to magnetic energy. In our proposed scenario the magnetic field lines are further winded and stretched by the rotating belt as they are dragged by the outflowing gas. The magnetic field reconnect and launch jets.

In claiming that an efficient dynamo can be operate in the accretion belt we are encouraged by the recent results of Mösta et al. [\(2015\)](#page-7-14) who conducted very high resolution simulations of CCSNe with pre-collpase rapidly rotating cores. They show that rapidly rotating material around the newly born NS can amplify tremendously an initial magnetic field, and leads to jets lunching. In their simulations the turbulent kinetic energy in the accreted gas is converted into electromagnetic energy. The timescale for an e-fold increase in the magnetic field in the accretion disk,  $\tau_e \approx$ 0.5 ms, is about half an orbital period in the relevant part of the disk. Interestingly, only veryhigh resolution simulations were able to demonstrate the tremendous magnetic field amplification. These results are very supportive for the jittering-jets model for the explosion of all CCSNe with explosion energies of  $\geq 2 \times 10^{50}$  erg.

Intermittent accretion belts in CCSNe are expected to last for a time of  $\tau_b \approx 0.03 - 0.1$  s, equals to tens of Keplerian orbits at  $\sim 25$  km from the NS. A belt with a specific angular momentum of  $\eta \equiv j_{\text{acc}}/j_{\text{Kep}}$  has an orbital period on the equator of the newly born NS of  $\approx$  $30(\eta/0.05)^{-1}$  ms. Closer to the poles the period will be shorter. This shows that the sub-Keplerian disk has sufficient time to substantially amplify the magnetic fields. We conjecture that the strong magnetic fields with the preferred axis of rotation and the avoidance angle (eq. [2\)](#page-2-1), lead to the launching of two opposite jets in the polar directions, where the ram pressure of the accreted gas is very low.

#### **4. DISCUSSION AND SUMMARY**

<span id="page-5-0"></span>We conducted a preliminary study that led us to argue that sub-Keplerian accretion flows onto compact objects can launch jets. The sub-Keplerian accreted gas forms an accretion belt rather than an accretion disk (left half side of Fig. [1\)](#page-9-0). Within the avoidance angle  $\theta_a$  from the polar directions (eq. [2\)](#page-2-1) the accretion rate is very low. The dynamo amplification of the magnetic field within the belt (section [3\)](#page-3-0) can lead to very strong magnetic fields, as can be seen from equation [\(6\)](#page-4-2) that gives the ratio of the amplified magnetic field energy density to that in the accreted gas.

We speculate that reconnection of the magnetic field lines can lead to an outflow through the two opposite polar avoidance regions. Winding of the magnetic field lines frozen to the polar outflow can further channel rotation energy to outflow kinetic energy. The main differences and common properties of the proposed belt-launched jets scenario from those of the common disklaunched jets are presented in Table 1.

Our conclusion is that accretion belts that are formed by sub-Keplerian accretion flows might lunch jets. If true, these results might have far reaching implications for the removal of common envelopes by jets. Numerical studies point at a limited efficiency of the common envelope removal by the gravitational energy released during the spiraling-in process (e.g., [De Marco et al.](#page-7-15) [2011;](#page-7-15) [Passy et al. 2012;](#page-7-16) [Ricker & Taam 2012](#page-8-8); [Ohlmann et al. 2015](#page-7-17)). It has been suggested that jets can assist it removing the common envelope (e.g. [Soker 2014](#page-8-9)). However, the specific angular momentum of the gas accreted by a MS star spiraling-in inside a giant envelope is sub-Keplerian  $j_{\text{acc}} \approx 0.1 - 1 j_{\text{Kep}}$  [\(Soker 2004\)](#page-8-5). A belt is expected to be formed around the MS star during the common envelope phase. Jets that might be launched by the belt, as argued in the present study, can assist in removing the common envelope.

As well, our results might have implication to CCSNe explosion scenarios. The neutrinodelayed mechanism has severe problems in accounting for explosions with kinetic energy of more than about  $2 - 5 \times 10^{50}$  erg [\(Papish et al. 2015\)](#page-7-18). An intermittent accretion belt is expected to be formed around the newly born NS during the first several seconds of the explosion [\(Gilkis & Soker](#page-7-8) [2015,](#page-7-8) [2016](#page-7-9); [Papish et al. 2015](#page-7-18)). If the present results hold to that flow structure, then the jets can explode the star, up to explosion energies of above  $10^{52}$  erg [\(Gilkis et al. 2016\)](#page-7-19). We note that because of the stochastic nature of the angular momentum of the accreted mass in many CCSNe, in the jittering jets model the spin of the NS might be inclined to the momentarily angular momentum of the belt. The effect of this will have to be studied in future numerical simulations.

In both cases, of common envelope removal and CCSNe, the origin region of the jets is completely obscured. In cases where the origin of jets is seen, such as in young stellar objects, active galactic nuclei, and some binary systems, the accretion rate is low and an accretion disk is required to be formed. We here argued that in cases where accretion rate is very high, hence the entire region is heavily obscured, even accretion belt formed by sub-Keplerian inflow can launch jets (see also [Shiber et al. 2015\)](#page-8-10).

We can summarize our study by stating that the possibility for accretion belts to launch collimated outflows, or jets, opens a rich variety of processes that can account for some puzzles in astrophysics, such as the explosion of massive stars and in some cases in the removal of the common envelope.

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<span id="page-9-0"></span>Fig. 1.— The flow structure studied here is presented schematically in the left half of the figure. It has some similarities with the spreading-layer that was proposed by [\(Inogamov & Sunyaev 1999](#page-7-12)) that is presented on the right half. The meridional extent of the hot part of the spreading layer  $\theta_{\star}$  is given by equation [\(1\)](#page-2-0). The angle of avoidance region  $\theta_a$  is given by equation [\(2\)](#page-2-1).